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Black-Scholes Model under G-Lévy Process

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Abstract

In this paper, we study the option price theory of stochastic differential equations under G-Lévy process. By using G-Itô formula and G-expectation property, we give the proof of Black-Scholes equations (Integro-PDE) under G-Lévy process. Finally, we give the simulation of G-Lévy process and the explicit solution of Black-Scholes under G-Lévy process.

Keywords

G-Lévy Process, G-Itô Formula, Integro-PDE

1. Introduction

Nowadays, many studies are interested in stochastic differential equations (SDEs). And SDEs have been widely applied to economics and finance fields, such as option pricing in stock market see [1]. In the 1970s, Black and Scholes propose the famous option pricing model and promote the development of stocks, bonds, currencies, products. Subsequently, the famous Black-Scholes formula was paid more attention by many scholars. In 1976, Merton [2] proposed the logarithmic jump-diffusion models in stock price, which was described as a combination of Brownian motion and compound Poisson process.

Although option pricing formula has developed for a long time, there are many uncertainty problems in stock market. Many scholars have been studied the uncertainty problem. For example, Peng [3] [4] proposed the sublinear expectation space to solve the uncertainty problem. In particular, the G-expectation space plays an important role in solving them. Then the G-Brownian motion, G-Itô formula and G-center limit theorem are proposed for us in G-expectation framework. In this paper, we consider the following stock price S_t such that:

$$\mathrm{d}S_t = aS_t\mathrm{d}t + bS_t\mathrm{d}W_t + cS_t\mathrm{d}L_t, \quad t \in [0,T], \tag{1}$$

where a is the interest rate, b is the volatility and c is the jump range of asset price, W_t is a G-Brownian motion and L_t is a G-Lévy process under the

G-framework.

Yang and Zhao [5] introduce the simulation of G-Brownian and G-normal distribution under G-expectation and Chai studies the option pricing for stochastic differential equation under G-framework. Although G-Brownian motion solved many financial issues, some financial models that depend on the Lévy processes remain unresolved. Therefore, Peng and Hu [6] studied the G-Lévy process, which is the generalization of G-Brownian motion. And Krzysztof [7] introduced G-Itô formula and G-martingale representation for G-Lévy process.

In this paper, we study Black-Scholes model under G-Lévy process and prove the Integro-PDE by using G-Itô formula, option pricing formula and G-expectation property. Then we simulate the G-Lévy process and the stock price S_t by using the new algorithms. Meanwhile, we give a numerical example to verify the result of simulation.

We introduce some notation as follows:

- $C_b^k\left(\mathbb{R}^q\right)$: the space of functions $\varphi : x \in \mathbb{R}^q \to \mathbb{R}$ with uniformly bounded partial derivatives $\partial_x^{k_1}\varphi$ for $1 \le k_1 \le k$.
- C: a generic constant depending only on the upper bounds of derivatives of a, b, c and h, and C can be different from line to line.

The outline of the paper is as follows. In Section 2, we introduce some necessary notations and theorems, such as the G-Lévy process and G-Itô formula. In Section 3, we propose a new theorem that gives the proof of Black-Scholes equations (Integro-PDE) under G-Lévy process. Finally, some numerical simulations for G-Lévy process and stock price are given in Section 4.

2. Preliminaries

In this section, we will introduce some basic knowledge and notation that is the focus of this paper. Throughout this paper, we will give the definition of G-Lévy process. Unless otherwise specified, we use the following notations. Let $|x| = \langle x, x\rangle^{\frac{1}{2}}$ be the Euclidean norm in $\mathbb{R}^q$ and $\langle x, y\rangle$ is the scalar product of x, y. If A is a vector or matrix, its transpose is denoted by A^{T}. Next, we will give the definition of Sublinear expectation and G-Lévy process.

Definition 1. [6] (**Sublinear expectation**) Let $\mathbb{H}$ is a linear space and $X_1, X_2 \in \mathbb{H}$, we give the definition of sublinear expectation $\mathbb{E} : \mathbb{H} \to \mathbb{R}$

- monotonicity: $\mathbb{E}\left[X_1\right] \ge \mathbb{E}\left[X_2\right]$ for $X_1 \ge X_2$.
- constant preserving: $\mathbb{E}[c] = c$ with $c \in \mathbb{R}$.
- sub-additivity: $\mathbb{E}\left[X_1 + X_2\right] \le \mathbb{E}\left[X_1\right] + \mathbb{E}\left[X_2\right]$.
- positive homogeneity: $\mathbb{E}\left[\lambda X_1\right] = \lambda \mathbb{E}\left[X_1\right]$ for $\lambda \ge 0$.

Therefore, we call the triple $(\Omega, \mathbb{H}, \mathbb{E})$ a sublinear expectation space.

Definition 2. [6] (**G-Lévy process**) Assume $X = \left(X_s\right)_{s\ge 0}$ is a Lévy process, X_s^f is a generalized G-Brownian motion and X_s^g is of finite variation. We say the X is a G-Lévy process if satisfy the following conditions:

- for $s \ge 0$, there exists a Lévy process $\left(X_s^f, X_s^g\right)$ satisfies $X_s = X_s^f + X_s^g$.
- process X_s^f and X_s^g satisfy the following growth conditions:

$$\lim_{s\downarrow 0}\mathbb{E}\left[\left|X_s^f\right|^3\right]s^{-1}=0;\quad \mathbb{E}\left[\left|X_s^g\right|\right]<Cs\quad \text{for all } s\geq 0,$$

where C is a positive constant.

Lemma 1. [7] (**G-Itô formula**) For $1\leq i\leq q$, X_t^i is the *k*-th component of X_t and it satisfies the following form:

$$X_t^i=X_0^i+\int_0^t a_s^i\mathrm{d}s+\sum_{j=1}^q\int_0^t b_s^{i,j}\mathrm{d}W_s^j+\int_0^t\int_{\mathcal{E}}c(e,s)L(\mathrm{d}e,\mathrm{d}s),$$

where $\mathcal{E}\in\mathbb{R}^q\setminus\{0\}$, W_s is a G-Brownian motion and $L(\mathrm{d}e,\mathrm{d}s)$ is a G-Lévy process. For $h\in C_b^2(\mathbb{R}^q)$, we deduce

$$\begin{aligned}h(X_t)=h(X_0)&+\sum_{i=1}^q\int_0^t a_s^i\frac{\partial h(X_s)}{\partial x_i}\mathrm{d}s+\frac{1}{2}\sum_{i,k=1}^q\sum_{j=1}^q\int_0^t b_s^{i,j}b_s^{k,j}\frac{\partial^2 h(X_s)}{\partial x_i\partial x_k}\mathrm{d}\langle W\rangle_s\\&+\sum_{i=1}^q\sum_{j=1}^q\int_0^t b_s^{i,j}\frac{\partial h(X_s)}{\partial x_i}\mathrm{d}W_s^j+\int_0^t\int_{\mathcal{E}}\left[h(X_{s-}+c(e,s))-h(X_{s-})\right]L(\mathrm{d}e,\mathrm{d}s).\end{aligned}$$

Lemma 2. [6] (**Lévy-Khintchine representation**) Assume X is a G-Lévy process in $\mathbb{R}^q$, we have the following form

$$G_X\left[h(\cdot)\right]:=\lim_{t\downarrow 0}\mathbb{E}\left[h(X_t)\right]t^{-1},\tag{2}$$

where $h\in C_b^3(\mathbb{R}^q)$. If Equation (2) is true, we have the following Lévy-Khintchine representation

$$G_X\left[h(\cdot)\right]=\sup_{(\lambda,a,b)\in\mathbb{U}}\left\{\langle Dh(0),a\rangle+\frac{1}{2}tr\left[D^2h(0)bb^{\mathrm{T}}\right]+\int_{\mathcal{E}}h(e)\lambda(\mathrm{d}e)\right\},$$

where $h(0)=0$, $\mathcal{E}=\mathbb{R}^q\setminus\{0\}$, $\mathbb{U}\subset\mathcal{V}\times\mathbb{R}^q\times\mathbb{Q}$, $\mathcal{V}$ is a set of all Borel measures of $\mathcal{E}$ and $\mathbb{Q}$ is a set of all positive definite symmetric matrix.

Lemma 3. [6] (**Integro-PDE**) Assume X is a G-Lévy process and the functions $u=u(x,t)$, and by using the Lemma 2 (Lévy-Khintchine representation), we have the following Integro-PDE:

$$\frac{\partial u}{\partial t}-\sup_{(\lambda,a,b)\in\mathbb{U}}\left\{\langle Du,a\rangle+\frac{1}{2}tr\left[D^2ubb^{\mathrm{T}}\right]+\int_{\mathcal{E}}\left(u(x+c(e),t)-u(x,t)\right)\lambda(\mathrm{d}e)\right\}=0$$

where D^2u is the Hessian matrix of u and $a\in\mathbb{R}^q$, $b\in\mathbb{R}^{q\times q}$.

3. Black-Scholes Equations under G-Lévy Process

In this section, we will give the Black-Scholes equations under G-Lévy process, and prove the Integro-PDE by combining the G-Itô formula and the option pricing formula.

Theorem 1. (**Black-Scholes equations**) Assume $u=u(S_t,t)$ is the option price and S_t is the stock price. For Equation (1), we can obtain the following integral partial differential Equation (Integro-PDE) under G-Lévy process

$$\begin{aligned}\frac{\partial u}{\partial t}+\sup_{(\lambda,a,b,c)\in\mathbb{U}}&\left\{aS\frac{\partial u}{\partial S}+\frac{b^2S^2}{2}\frac{\partial^2 u}{\partial S^2}+\ln(1+c)\lambda(\mathcal{E})S\frac{\partial u}{\partial S}\right.\\&\left.+\ln^2(1+c)\lambda(\mathcal{E})\left(\frac{b^2S^2}{2}\frac{\partial^2 u}{\partial S^2}+\frac{b^2S}{2}\frac{\partial u}{\partial S}\right)\right\}-au=0,\end{aligned}$$

where $a,b,c \in \mathbb{R}$, $\mathbb{U} \subset \mathcal{V} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, $\mathcal{V}$ is a set of all Borel measures of $\mathcal{E}$ and $\lambda(\mathcal{E}) = \int_{\mathcal{E}} \lambda(\mathrm{d}e)$.

Proof. We define a uniform time partition on time interval $[0,T]$ and $0 = t_0 < t_1 < \cdots < t_n < \cdots < t_N = T$, $\Delta t = t_{n+1} - t_n$ for $0 \le n \le N$. Let the function $u(S,t)$ be sufficiently smooth, $\Delta \langle W \rangle_n = \langle W \rangle_{t_{n+1}} - \langle W \rangle_{t_n}$ and $\Delta W_n = W_{t_{n+1}} - W_{t_n}$. Using the G-Itô formula, we can obtain the explicit solution of Equation (1):

$$S_{t_{n+1}} = S_a \exp\left\{ a\Delta t - \frac{1}{2} b^2 \Delta \langle W \rangle_n + b\Delta W_n + \int_{t_n}^{t_{n+1}} \int_{\mathcal{E}} \left[\ln(1+c) \right] L(\mathrm{d}e, \mathrm{d}s) \right\}. \tag{3}$$

In the G-expectation space, we have the following product rule:

$$\mathrm{d}W_t \cdot \mathrm{d}W_t = \mathrm{d}\langle W \rangle_t, \mathrm{d}L_t \cdot \mathrm{d}L_t = \lambda(\mathcal{E})\mathrm{d}t + (\lambda(\mathcal{E})\mathrm{d}t)^2, \mathrm{d}L_t \cdot \mathrm{d}t = 0, \mathrm{d}W_t \cdot \mathrm{d}L_t = 0.$$

Then, it is well known that the option pricing formula following form

$$u(S_a, t_n) = \frac{1}{r} \mathbb{E}\left(\left[u(S_{t_{n+1}}, t_{n+1}) - u(S_n, t_n) \right] | S_{t_n} = S_a \right) + \frac{1}{r} u(S_a, t_n). \tag{4}$$

Next, we introduce the Black-Scholes model under G-Lévy process. Using Taylor formula for $u(S_{t_{n+1}}, t_{n+1})$, we have

$$\begin{aligned} & u(S_{t_{n+1}}, t_{n+1}) - u(S_a, t_n) \\ & = \frac{\partial u(S_a, t_n)}{\partial t} \Delta t + \frac{\partial u(S_a, t_n)}{\partial S} (S_{t_{n+1}} - S_a) \\ & + \frac{1}{2} \frac{\partial^2 u(S_a, t_n)}{\partial S^2} (S_{t_{n+1}} - S_a)^2 + O(\Delta t)^{\frac{3}{2}}. \end{aligned} \tag{5}$$

Substituting Equation (3) into (5), we obtain

$$\begin{aligned} & u(S_{t_{n+1}}, t_{n+1}) - u(S_a, t_n) \\ & = \frac{\partial u(S_a, t_n)}{\partial t} \Delta t + \frac{\partial u(S_a, t_n)}{\partial S} (S_a \exp\{X^n\} - S_a) \\ & + \frac{1}{2} \frac{\partial^2 u(S_a, t_n)}{\partial S^2} (S_a \exp\{X^n\} - S_a)^2 + O(\Delta t)^{\frac{3}{2}}, \end{aligned}$$

where $X^n = a\Delta t - \frac{1}{2} b^2 \Delta \langle W \rangle_n + b\Delta W_n + \int_{t_n}^{t_{n+1}} \int_{\mathcal{E}} \ln(1+c) L(\mathrm{d}e, \mathrm{d}s)$. Let $\lambda(\mathcal{E}) = \int_{\mathcal{E}} \lambda(\mathrm{d}e)$, it induces from Taylor expansion for $\exp\{X^n\}$ that

$$\begin{aligned} & u(S_{t_{n+1}}, t_{n+1}) - u(S_a, t_n) \\ & = \left[\frac{\partial u}{\partial t} + aS_a \frac{\partial u}{\partial S} \right] \Delta t - S_a \frac{b^2}{2} \frac{\partial u}{\partial S} \Delta \langle W \rangle_n + S_a \ln(1+c) \frac{\partial u}{\partial S} L_{\mathcal{E}} \\ & + S_a b \frac{\partial u}{\partial S} \Delta W_n + \left[S_a \frac{\partial u}{\partial S} + S_a^2 \frac{\partial^2 u}{\partial S^2} \right] \frac{1}{2} (X^n)^2 + O(\Delta t)^{\frac{3}{2}} \\ & = \left[\frac{\partial u}{\partial t} + aS_a \frac{\partial u}{\partial S} \right] \Delta t - S_a \frac{b^2}{2} \frac{\partial u}{\partial S} \Delta \langle W \rangle_n + S_a \ln(1+c) \frac{\partial u}{\partial S} L_{\mathcal{E}} \\ & + S_a b \frac{\partial u}{\partial S} \Delta W_n + \left[S_a \frac{\partial u}{\partial S} + S_a^2 \frac{\partial^2 u}{\partial S^2} \right] \frac{1}{2} \left(\frac{b^4}{4} (\Delta \langle W \rangle_n)^2 + b^2 (\Delta W_n)^2 \right. \\ & \left. - b^3 \Delta W_n \Delta \langle W \rangle_n + \ln^2(1+c) \lambda(\mathcal{E}) \Delta t + \ln(1+c) \lambda(\mathcal{E}) (\Delta t)^2 \right) + O(\Delta t)^{\frac{3}{2}}, \end{aligned}$$

where $L_{\mathcal{E}} = \int_{t_n}^{t_{n+1}} \int_{\mathcal{E}} L(de, ds)$. Inserting the above result into Equation (4), we can deduce

$$u = \frac{1}{r}\mathbb{E}\Bigg[\left(\frac{\partial u}{\partial t} + aS_a\frac{\partial u}{\partial S}\right)\Delta t - S_a\frac{b^2}{2}\frac{\partial u}{\partial S}\left(\Delta\langle W\rangle_n\right) + S_a b\frac{\partial u}{\partial S}(\Delta W_n)$$
$$+ S_a\ln(1+c)\frac{\partial u}{\partial S}L_{\mathcal{E}} + \left[S_a\frac{\partial u}{\partial S} + S_a^2\frac{\partial^2 u}{\partial S^2}\right]\frac{1}{2}\Bigg(\frac{b^4}{4}\left(\Delta\langle W\rangle_n\right)^2 + b^2(\Delta W_n)^2$$
$$-b^3\Delta W_n\Delta\langle W\rangle_n + \ln^2(1+c)\lambda(\mathcal{E})\Delta t + O(\Delta t)^{\frac{3}{2}}\Bigg) \mid S_{t_n} = S_a\Bigg] + \frac{1}{r}u.$$

It induces from the G-expectation property and the fact $u = u\left(S_{t_n}, t_n\right)$ and $\Delta W_n \sim N\left(0;\left[\underline{\sigma}^2\Delta t, \overline{\sigma}^2\Delta t\right]\right)$ that we can deduce

$$\left(1-\frac{1}{r}\right)u = \frac{\Delta t}{r}\Bigg(\frac{\partial u}{\partial t} + \sup_{(\lambda,a,b,c)\in\mathbb{U}}\Bigg\{aS_a\frac{\partial u}{\partial S} + \left(\frac{b^2S_a^2}{2}\frac{\partial^2 u}{\partial S^2}\right)^+\overline{\sigma}^2$$
$$-\left(\frac{b^2S_a^2}{2}\frac{\partial^2 u}{\partial S^2}\right)^-\underline{\sigma}^2 + \ln(1+c)S_a\frac{\partial u}{\partial S}\lambda(\mathcal{E})$$
$$+\ln^2(1+c)\left(\frac{b^2S^2}{2}\frac{\partial^2 u}{\partial S^2} + \frac{b^2S}{2}\frac{\partial u}{\partial S}\right)\lambda(\mathcal{E})\Bigg\} + O(\Delta t)^{\frac{1}{2}}\Bigg),$$

where $r = 1 + a\Delta t$ and a is risk-free rate. Consequently, we obtain the following integro-partial differential equation:

$$\frac{\partial u}{\partial t} + \sup_{(\lambda,a,b,c)\in\mathbb{U}}\Bigg\{aS\frac{\partial u}{\partial S} + \frac{b^2S^2}{2}\frac{\partial^2 u}{\partial S^2} + \ln(1+c)\lambda(\mathcal{E})S\frac{\partial u}{\partial S}$$
$$+\ln^2(1+c)\lambda(\mathcal{E})\left(\frac{b^2S^2}{2}\frac{\partial^2 u}{\partial S^2} + \frac{b^2S}{2}\frac{\partial u}{\partial S}\right)\Bigg\} - au = 0.$$

The proof is completed. □

4. Numerical Experiment

In this section, we will give a numerical example for option pricing in stock market. And we study the stock price S_t under G-Lévy process. Platen [1] introduces the application jump process in stock market of financial field. The simulation of G-Brown under G-framework see [5] [8] [9]. Next, we firstly study the simulation of Poisson jump process and G-Lévy process.

Algorithm 1. (**The simulation of Poisson jump**)

- Setting up the values of intensity λ and the terminal time *T*.
- Generating random number z_i obeying exponential distribution with parameter *lambda*.
- Then by the formula $t_k = \sum_{i=1}^{k} z_i$, we get the occurrence time t_k of *n* events.
- Plotting a ladder figure for the Poisson jump process.

Assume the intensity $\lambda = 1.5$ and the number of jumps are equal to 10. And Figure 1 shows that the simulation of Poisson jump process.

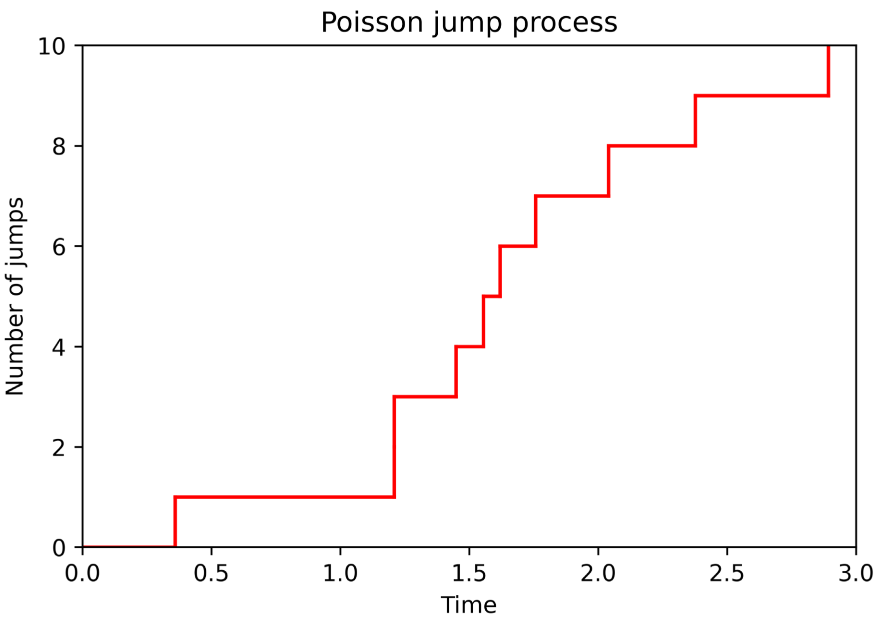

Figure 1. The plots of time and number of jumps for Poisson jump process.

Algorithm 2. (**The simulation of G-Lévy process**)

- Setting up the terminal time T and the intensity functions $\lambda(t)$, where $\lambda(t) \le \lambda$ with λ is a constant.
- Generating the Poisson jump process random number with intensity λ and obtaining the time of occurrence $s_1, s_2, \cdots, s_n$.
- Generating the uniformly distributed random number x_i on $(0,1)$. If $x_1 \le \lambda(s_i)/\lambda$, we retain the s_i, else we give up the time s_i.
- Plotting the time s_i which are obtained in the above step and the number of jumps.

Suppose the intensity function $\lambda(t) = \frac{t}{5}$ and the number of jumps are equal to 25. For $\lambda = 10$ and $\lambda = 15$, we simulate the G-Lévy process in Figure 2.

Next, we will introduce the Black-Scholes formula with jump under the G-Lévy process, and it is the generation of classical Black-Scholes formula. In [6], Peng and Hu use the option pricing formula under the G-Lévy process. There we will give the following examples.

Example 1. Consider the stock price S_t has the following form:

$$\frac{\mathrm{d}S_t}{S_t} = a\mathrm{d}t + b\mathrm{d}W_t + c\mathrm{d}L_t, \quad t \in [0,T], \tag{6}$$

where the initial value $S_0 = 0$, the interest rate a and volatility b are positive, W_t is a G-brownian motion and L_t is a G-Lévy process. Next, we give the explicit solution of Equation (6) on $t \in [0,1]$

$$S_t = S_0 \exp\left\{at - \frac{1}{2}b^2 \langle W \rangle_t + bW_t + \int_0^t \int_{\mathcal{E}} \left[\ln(1+c)\right] L(\mathrm{d}e, \mathrm{d}s)\right\}.$$

In this example, we firstly use three different coefficients $a_1 = 0.1$, $b_1 = 0.3$,

$c_1 = 0.1$, $a_2 = 0.2$, $b_2 = 0.2$, $c_2 = 0.2$ and $a_3 = 0.3$, $b_3 = 0.1$, $c_3 = 0.3$ to simulate the stock price S_t. And the simulation of S_t are given in **Figure 3** with three different coefficients.

Because the interest rate *a*, volatility *b* and jump intensity *c* are variable, we study the influence of volatility *b* and jump intensity *c* on stock price S_t. Let coefficients $a = 0.1$, $c = 0.1$, we plot the stock price S_t with the time *t* under the different coefficients $b = 0.1$, $b = 0.2$, $b = 0.3$ in **Figure 4**. And we obtain the stock price S_t will decrease with the increase of the volatility *b*.

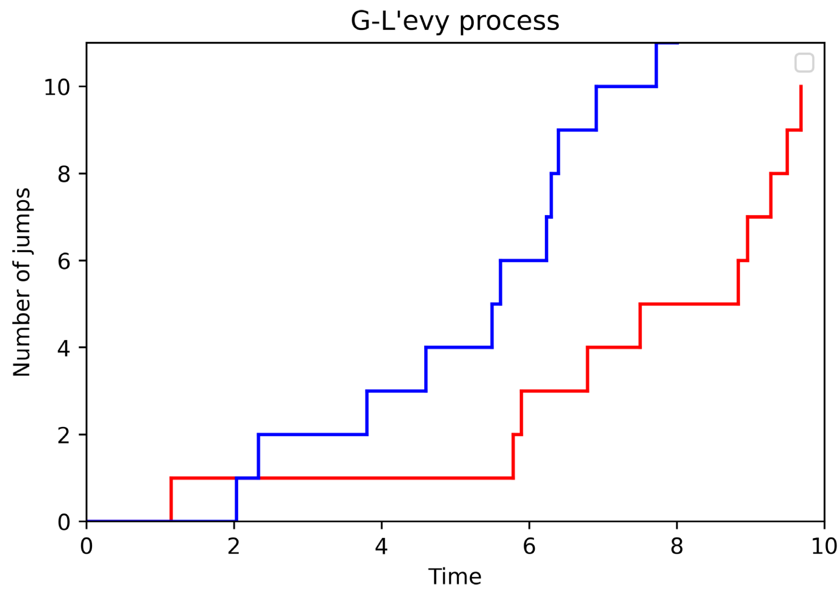

Figure 2. The simulation of G-Lévy process.

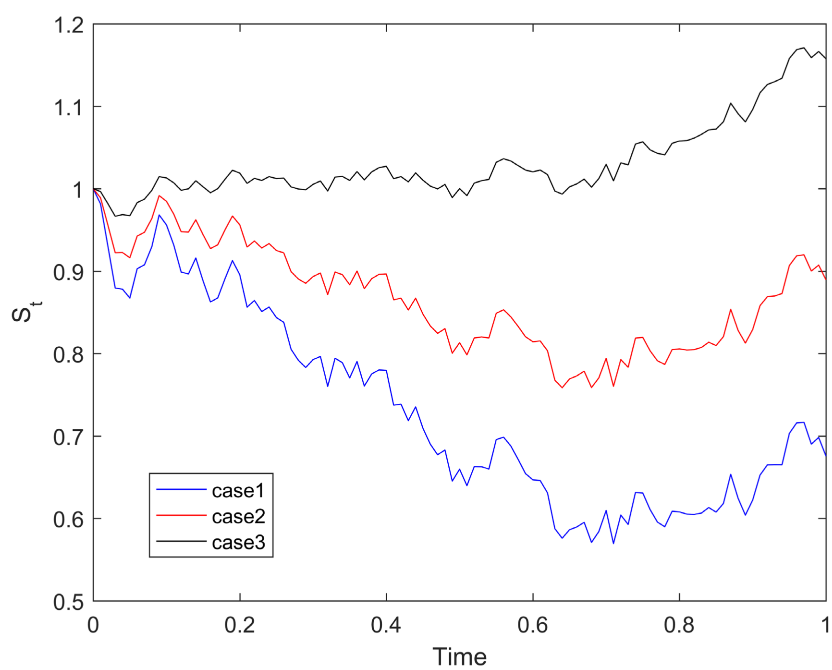

Figure 3. The simulation of stock price S_t with three different coefficients.

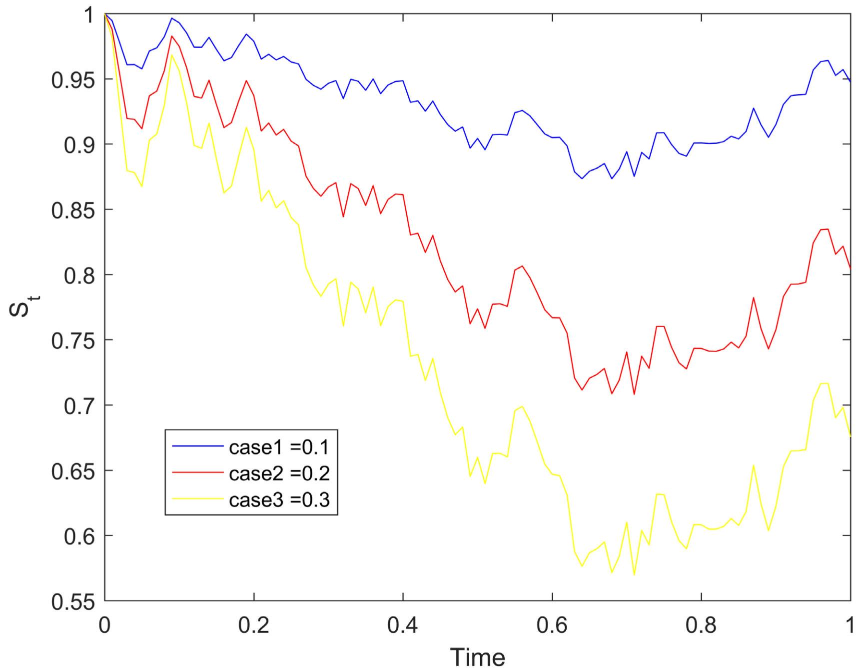

Figure 4. Stock price S_t with three different coefficients $b = 0.1$, $b = 0.2$, $b = 0.3$.

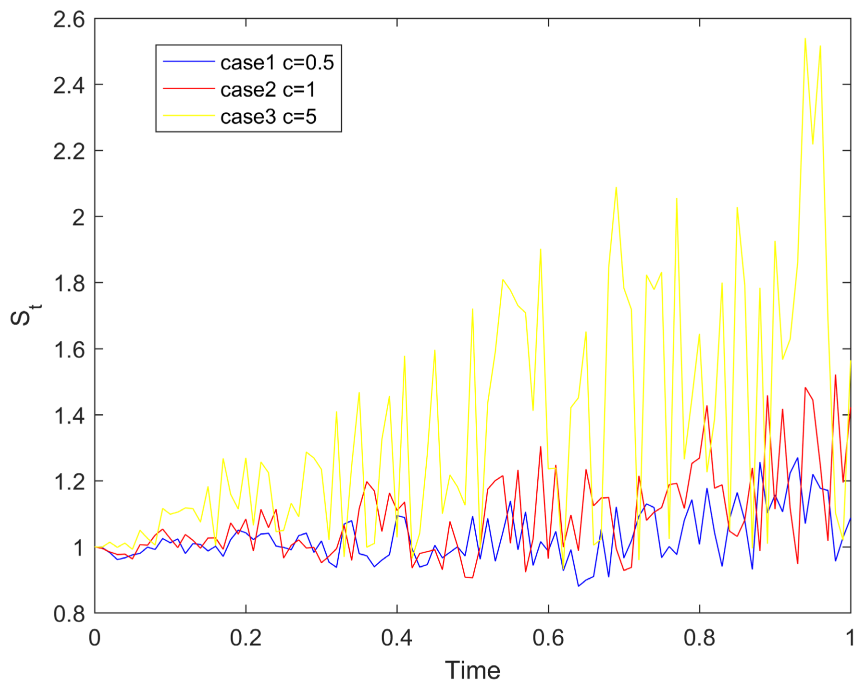

Figure 5. Stock price S_t with different coefficients $c = 0.5$, $c = 1$, $c = 5$.

Let coefficients $a = 0.1$, $b = 0.1$, we plot the stock price S_t with the time t under the jump intensity coefficients $c = 0.5$, $c = 1$, $c = 5$ in **Figure 5**.

By comparing **Figures 3-5**, we obtain that coefficients a and b have a great influence on stock price S_t than coefficient c. And the stock price S_t has a small variety when coefficient c changes.

5. Conclusion

In this paper, by using G-Itô formula and G-expectation property, we prove the Integro-PDE under G-Lévy process. Then we study the influence of coefficients on stock price S_t, and obtain the coefficients a, b that have a great influence on stock price S_t. In the future, we will study the numerical scheme for solving the Integro-PDE. And the numerical scheme is important in financial field.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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