Open Access Library Journal
2020, Volume 7, e6335
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Remarks on the Order of Quantum Equations of Motion

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How to cite this paper: Comay, E. (2020) Remarks on the Order of Quantum Equations of Motion. *Open Access Library Journal*, **7**: e6335.
https://doi.org/10.4236/oalib.1106335

Received: April 18, 2020
Accepted: May 9, 2020
Published: May 12, 2020

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Abstract

The paper examines the Dirac-Pauli differences concerning the order of the primary differential equation of an elementary massive quantum particle. The analysis relies on several self-evident constraints that an acceptable quantum theory must satisfy, like conservation laws, compatibility with Maxwellian electrodynamics and the correspondence principle. The dimension of the quantum function that is used in the Lagrangian density of a given quantum theory together with the corresponding differential equations play an important role in the reasoning procedure. The paper proves that Dirac was right and that second-order quantum theories like the Klein-Gordon equation and the electroweak theory of the $W^{\pm}$ bosons do not satisfy fundamental constraints. This outcome is inconsistent with the Standard Model of Particle Physics.

Subject Areas

Modern Physics

Keywords

Lagrangian Density, Noether Theorem, 4-Current, Dimension of Quantum Functions

1. Introduction

The history of science points out disagreements between leading figures of a given period. The present work is dedicated to a fundamental disagreement between Dirac and Pauli about the coherence of the *order* of the fundamental differential equations of a massive quantum particle. It is shown below that Dirac has supported the idea that the fundamental differential equations of an elementary massive quantum particle should be of the first order. By contrast, Pauli has argued that also second-order differential equations are acceptable.

The few quotations that are shown herein illustrate the problem. Very late in his life, Dirac has repeated his lifelong opinion and stated that "only equations linear in $\partial/\partial t$ give probabilities that are positive definite" (see [1], p. 3).

Pauli disagreed to Dirac's opinion and sent a letter to Heisenberg in 1934 where he states that "Dirac's assertion that a second-order time derivative in a wave equation contradicted the quantum mechanical transformation theory 'is pure rubbish and is refuted by application of our field quantization'" (see [2], p. 70).

These differences were not free of personal emotions. For example, Pauli stated that his interpretation of the second-order Klein-Gordon (KG) equation "has made me happy that I can again cast aspersion on my old enemy—the Dirac theory of the spinning electron" (see [2], p. 70). Below, these contradictory opinions are called *the Dirac-Pauli differences.* (It is explained later how a term of a Lagrangian density that contains a product of derivatives of the quantum function yields a second-order differential equation.)

An observation of the present literature indicates that Pauli has apparently won these differences. For example, contemporary textbooks on quantum field theory (QFT) devote specific sections to the second-order KG equation (see e.g. [3], chapter 2; [4], chapters 1-4). Furthermore, the Standard Model (SM) is the presently accepted theory of elementary particles and their interactions. The SM comprises several sectors, and its electroweak sector regards the three $W^{\pm}, Z$ massive bosons as elementary particles that satisfy a second order differential equation (see e.g. [4], p. 518; [5], pp. 305-318).

However, this state of affairs should not be regarded as the final word about the Dirac-Pauli differences. Indeed, physics is a mature science that takes a mathematical structure. Hence, an examination of mathematical aspects of the Dirac-Pauli differences may yield decisive arguments about this issue. This work relies on well established mathematical principles of theoretical physics and constraints that are derived from them. Later, these constraints are applied to the Dirac-Pauli differences and the outcome proves that it is Dirac who was right.

Units where $\hbar = c = 1$ are used. Greek indices run from 0 to 3. Most formulas take the standard form of relativistic covariant expressions. The metric is diagonal and its entries are $(1, -1, -1, -1)$. An upper dot denotes the time-derivative. Section 2 presents constraints that an acceptable quantum theory must abide with. The third section examines the coherence of quantum theories whose primary differential equations are of the first order or the second order, respectively. Further aspects of this topic are discussed in the fourth section. The last section summarized this work.

2. Constraints on Quantum Theories

Constraints play an important role in the construction of a new physical theory. They are used for a rejection of physical ideas that cannot be correct. In so doing they save a lot of time and effort that are doomed to be useless. This section presents well-known constraints that are organized in different groups. These constraints are found relevant to the Dirac-Pauli differences.

2.1. Constraints Related to Space-Time

Conservation of energy, momentum and angular momentum are well-known properties of a closed physical system. These issues are related to space-time homogeneity and spatial isotropy. Relativistic covariance is another requirement.

It is explained here how these requirements are satisfied by a QFT. Thus, the presently accepted approach to the construction of a QFT applies the variational principle to an action that takes the form

$$I(\psi) = \int \mathrm{d}^4 x \mathcal{L}(\psi, \psi_{,\mu}), \tag{1}$$

where $\mathcal{L}(\psi, \psi_{,\mu})$ is a Lagrangian density and ψ is the quantum function. The action $I(\psi)$ is a mathematically real Lorentz scalar. A well-known textbook supports this approach and states: "all field theories used in current theories of elementary particles have Lagrangians of this form" (see [6], p. 300). This approach is adopted in the present work.

The Noether theorem provides a good reason for using this form of the variational principle as a basis for a QFT. This theorem connects the invariance of a given Lagrangian density with respect to a specific transformation to a conservation law that the associated theory satisfies. In other words, the quantum equations of motion are the Euler-Lagrange equations of the Lagrangian density of (1) (see [6], p. 300)

$$\frac{\partial \mathcal{L}}{\partial \psi} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial (\partial \psi / \partial x^\mu)} = 0, \tag{2}$$

and these equations satisfy the conservation laws. In particular, the Noether theorem says that if the Lagrangian density of a mathematically coherent physical theory does not depend explicitly on the space-time coordinates then this theory satisfies the conservation of energy, momentum, and angular momentum (see [7], pp. 17-19). Evidently, the Lagrangian density of a quantum theory that takes the form of (1) does not depend *explicitly* on the space-time coordinates. Hence, the associated theory satisfies these important conservation laws. It is further proved that the action (1) yields a relativistic covariant theory (see [7], pp. 17-19). This outcome justifies the usage of (1) as the basis for any given quantum theory.

The action principle (1) yields three important properties that are used below.

- The Euler-Lagrange Equations (2) that are derived from the Lagrangian density are the differential equation of the quantum particle. The compatibility of these equations and of their solutions is analyzed.
- Using fundamental laws of differential calculus, one finds that the Lagrangian density $\mathcal{L}$ and the second term of the Euler-Lagrange Equations (2)

determine the order of the differential equation of a given theory. If a term of the Lagrangian density $\mathcal{L}$ contains a product of derivatives of the quantum function $\psi_{,\mu}\psi_{,\nu}$ then the differential equation is of the second order. On the other hand, if the Lagrangian density $\mathcal{L}$ is a linear function of the derivatives $\psi_{,\mu}$ then a first order differential equation is obtained.

- In the units used herein, the action (1) is a dimensionless Lorentz scalar. Therefore, the Lagrangian density $\mathcal{L}(\psi,\psi_{,\mu})$ is a Lorentz scalar whose dimension is [L^{-4}]. It follows that the quantum function ψ of every kind of a Lagrangian density has a well-defined dimension.

2.2. Constraints Related to Electrodynamics

Some elementary massive quantum particles carry an electric charge. As such, these particles interact with electromagnetic fields. For this reason, these particles should abide by fundamental laws of Maxwellian electrodynamics. The following laws are used later in this work.

EM.1 Charge conservation is a crucial element of Maxwellian electrodynamics. The continuity equation of the 4-current

$$j^{\mu}_{,\mu} = 0 \tag{3}$$

(see [8], pp. 73-78; [9], p. 169) is the theoretical manifestation of this law. Furthermore, this conservation law has a very solid experimental basis [10].

It turns out that the Noether theorem provides a prescription for constructing a conserved 4-current for a quantum theory of the form (1). The invariance of a quantum field Lagrangian density under a global phase transformation

$$\psi \to \exp(i\alpha)\psi \tag{4}$$

yields

$$0 = i\alpha\left[\frac{\partial\mathcal{L}}{\partial\psi} - \partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\psi)}\right)\right]\psi + i\alpha\partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\psi)}\psi\right) \tag{5}$$

(see [11], p. 314). The Euler-Lagrange Equation (2) proves that the quantity enclosed inside the square brackets vanishes. Hence, the expression that is written inside the last bracket of (5) is a conserved 4-current (3), where

$$j^{\mu} = \frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\psi)}\psi. \tag{6}$$

Here j^0 is the density of the quantum particle. The action (1) is a mathematically real quantity. Therefore, the invariance of the Lagrangian density $\mathcal{L}(\psi,\psi_{,\mu})$ of (1) under a global phase transformation is obtained if its terms take the form $\psi^{\dagger}\hat{O}\psi$, where $\hat{O}$ is a Hermitian operator.

The mathematical structure of the Noether theorem for the 4-current (6) enables the division of terms of a given Lagrangian density into three categories:

C.1 A term that depends on derivative-free quantum functions makes no contribution to the Noether expression for the conserved 4-current.

C.2 A term that contains just one factor $\partial_\mu \psi$ makes a contribution to the Noether expression for the conserved 4-current *and this 4-current is derivative-free.*

C.3. A term that contains more than one factor $\partial_\mu \psi$ makes a contribution to the Noether expression for the conserved 4-current *and this 4-current is not derivative-free.*

This classification is very useful for the analysis that is carried out below.

EM.2 Another property of Maxwellian electrodynamics is that the interaction strength of the electromagnetic fields with a charged particle is proportional to its electric charge *e*.

EM.3 A third property of Maxwell equations is that they are independent of the 4-potentials. This property is called gauge invariance. The Maxwell equations are the Euler-Lagrange equations that are derived from a Lagrangian density (see [8], pp. 78-80). Hence, the electromagnetic Lagrangian density should not contain terms where the power of the 4-potential is greater than unity.

2.3. Other Constraints

1) Physics is a science that aims to provide a good description of the state and the time-evolution of appropriate systems. The success of this objective is demonstrated by a comparison of solutions of differential equations with experimental data. The primary role of the variational principle (1) means that the required differential equations are the Euler-Lagrange equations of the theory's Lagrangian density. It means that a given QFT must satisfy these requirements:

A. The differential equations of the theory must take an explicit form.

B. Solutions of these equations are obtainable.

C. These solutions adequately describe the experimental properties of an actual physical system that belongs to the theory's domain of validity.

2) The Bohr correspondence principle relates the classical limit of quantum mechanics to classical physics. Here are few references that justify this important principle. The success of classical mechanics means that "classical mechanics must therefore be a limiting case of quantum mechanics." ([12], p. 84; see also [13], pp. 25-27, 137, 138). Furthermore, QFT corresponds to quantum mechanics. For example, a well-known textbook states clearly: "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics" (see [6], p. 49). This statement means that there are certain relationships between QFT and quantum mechanics. Below, these relationships are called the *Weinberg correspondence principle.* The combined meaning of these quotations is that QFT corresponds to classical physics. A general discussion of the correspondence between physical theories is presented on pp. 3-6 of [14].

These constraints are used below as criteria for the acceptability of quantum theories.

3. First-Order and Second-Order Quantum Theories

Mathematical textbooks discuss many aspects that pertain to the *order* of differential equations. It turns out that the order of the primary differential equation is an important physical issue because it determines the dimension of the quantum function ψ. Here a derivative ∂_μ is taken with respect to the space-time coordinates. Hence, the dimension of a first order derivative is [L^{-1}], whereas the dimension of a second order derivative is [L^{-2}]. This issue plays a key role in the analysis that is presented below.

Let us turn to the Dirac-Pauli differences and analyze the compatibility between the general physical laws that yield the above-mentioned constraints and the order of the differential equations of a quantum theory. The Dirac first-order theory of a massive spin-1/2 particle and second-order theories are examined separately. The first item of section 2 explains why the Lagrangian density of these theories is the cornerstone of the analysis. This analysis begins with the simplest case of a free particle.

The Lagrangian density of a free Dirac particle is

$$\mathcal{L}_D = \bar{\psi}\left(\gamma^\mu i\partial_\mu - m\right)\psi. \tag{7}$$

(see [3], p. 78; [7], p. 54). As shown in section 2, the Lagrangian density $\mathcal{L}$ must be a Lorentz scalar whose dimension is [L^{-4}]. In the case of the Dirac theory (7), the dimension of the derivative ∂_μ and the mass m is [L^{-1}]. Therefore, the dimension of the Dirac function ψ is [$L^{-3/2}$], and that of the product $\bar{\psi}\psi$ is [L^{-3}]. It follows that this product takes the dimension of density. In particular, the Noether theorem (6) yields the Dirac 4-current

$$j^\mu = \bar{\psi}\gamma^\mu\psi, \tag{8}$$

and this 4-current satisfies the continuity Equation (3) (see [15], pp. 9, 23, 24).

It is interesting to note that the entries of the Dirac γ^μ matrices are pure numbers and that these four matrices transform like a 4-vector. It is shown below that this is an important attribute of the Dirac theory of a spin-1/2 particle.

Let us turn to the simplest case of a second-order quantum equation, namely, the Pauli-Weisskopf theory of the KG equation [2]. This equation aims to describe the state and the time-evolution of a spin-0 quantum particle. The Lagrangian density of a free KG charged particle is

$$\mathcal{L}_{KG} = \phi^\dagger_{,\mu}\phi_{,\nu}g^{\mu\nu} - m^2\phi^\dagger\phi \tag{9}$$

(see [2], p. 191; [6], p. 21). As shown above, the dimension of a Lagrangian density is [L^{-4}]. Hence, the product of the derivatives as well as the term containing m^2 proves that the dimension of the KG quantum function ϕ is [L^{-1}]. Therefore, the dimension of the product $\phi^\dagger\phi$ is [L^{-2}]. Furthermore, the dimension of the 4-current is [L^{-3}]. Therefore, the 4-current of the KG quantum function must contain a derivative of the quantum function ϕ. And indeed, the Noether theorem yields the required 4-current of the KG function (9) (see [2], p. 193; [7], p. 40)

$$j_\mu = i\left(\phi^\dagger \phi_{,\mu} - \phi^\dagger_{,\mu}\phi\right). \tag{10}$$

The different dimension of Dirac quantum function and that of any second-order quantum theory proves that these theories are inherently different. In particular, the 4-current is an important element of Maxwellian electrodynamics. Here, the classification of Section 2 shows that the first-order Dirac equation belongs to the C.2 category, whereas second-order theories belong to the C.3 category of Section 2. This issue affects the coherence of the electromagnetic interactions of a charged particle.

At this point one already identifies a problem with a second-order quantum theory. The normalization of the Schroedinger function $\int \psi^* \psi \mathrm{d}^3 x = 1$ (see e.g. [16], pp. 53-57; [17], p. 117) means that the dimension of the Schroedinger function ψ is [$\mathrm{L}^{-3/2}$]. On the other hand, the dimension of a quantum function that satisfies a second-order differential equation is [L^{-1}]. Hence, the dimension discrepancy of these wave functions means that a second-order quantum theory violates the Weinberg correspondence principle of Section 2.

It is shown below that the dependence of the 4-current of a second-order quantum theory on a derivative is the root of inherent contradictions of its electromagnetic interactions. Here is a quotation from the literature that mentions associated troubles, and thereby supports this claim: "Indeed, they appear with a vengeance, since the coupling prescription (15.1) introduces interaction terms containing derivatives" (see [7], p. 87).

Let us examine the electromagnetic interactions of a quantum particle. In the case of the first-order Dirac equation of a charged spin-1/2 particle, one has the Lagrangian density of Quantum Electrodynamics (QED)

$$\mathcal{L}_{QED} = \bar{\psi}\left(\gamma^\mu i\partial_\mu - m\right)\psi - e\bar{\psi}\gamma^\mu A_\mu \psi - \frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} \tag{11}$$

(see [3], p. 78; [7], p. 84). Here the second term, which represents the interaction of a charge with electromagnetic fields, is free of derivatives. Hence, the Noether expression for the 4-current (6) proves that *the interaction term of the first order quantum theory does not alter the particle's* 4*-current* (8).

The dependence of the 4-current of a second-order equation on derivatives is an insurmountable obstacle for finding a coherent expression for electromagnetic interaction of the corresponding charged particle. The reason for this statement relies on the form $j^\mu A_\mu$ of the electromagnetic interaction term. Thus, as stated above, the dimension of a quantum function that satisfies a primary second order quantum equation is [L^{-1}]. The [L^{-3}] dimension of a 4-current means that the 4-current of such a quantum particle must contain a derivative. Therefore, in the case of a second-order equation, an application of the Noether theorem to the 4-current factor of the interaction term *alters the* 4*-current that is used in the interaction term.* It means that the 4-current of these equations takes an incoherent form. Here are two examples that illustrate this inherent contradiction.

1) The electromagnetic interaction terms of the Pauli-Weisskopf KG Lagrangian density is (see Equation (37) in [2], p. 198)

$$\mathcal{L} = \left(\phi_{,0}^{*} - ieV\phi^{*}\right)\left(\phi_{,0} + ieV\phi\right) - \sum_{k=1}^{3}\left(\phi_{,k}^{*} + ieA_k\phi^{*}\right)\left(\phi_{,k} - ieA_k\phi\right) - m^2\phi^{*}\phi, \quad (12)$$

where the 4-potential is $A^{\mu} = (V, \boldsymbol{A})$. Here one finds terms that contain a product of entries of the electromagnetic 4-potential and also a second power of the electric charge *e*. On the other hand, one should note that a coherent Lagrangian density of an electric charge and electromagnetic fields should also yield the Maxwell equations of the electromagnetic fields. The first and the second terms on the right hand side of (11) compose the interaction terms of the Lagrangian density of Maxwellian fields (see also the classical form in [8], p. 75). The constraints of Section 2 show that the second power of the 4-potential as well as that of the electric charge of the KG Lagrangian density (12) is inconsistent with Maxwellian electrodynamics.

Conclusion: The KG Lagrangian density (12) has been published more than 80 years ago, and the above-mentioned contradictions have not been settled yet. Therefore, one concludes that it is extremely unlikely that a consistent term that represents the interaction of a charged KG particle with electromagnetic fields can be constructed.

2) The SM electroweak theory treats the two $W^{\pm}$ charged bosons as elementary particles that mediate weak interactions. Its Lagrangian density takes a form that is analogous to the KG equation

$$\mathcal{L} = \phi_{,\mu}^{\dagger}\phi_{,\nu}g^{\mu\nu} - m^2\phi^{\dagger}\phi + OT, \quad (13)$$

where OT denotes other terms. This expression comprises a term which contains a factor that is a product of two derivatives of the quantum function ϕ, and a term that contains the factor m^2 (see e.g. [4], p. 518).

This general structure shows that the electroweak theory of the $W^{\pm}$ bosons suffers from the same problems as those of the KG theory. In particular, this theory cannot provide an electromagnetic interaction term where the 4-current of the $W^{\pm}$ satisfies charge conservation. And indeed, in spite of the fact that the electroweak theory is about fifty years old, major research centers, like Fermilab and CERN, use an effective expression for the $W^{\pm}$ electromagnetic interactions [18] [19]. This effective expression violates the Maxwellian charge conservation requirement because it belongs to the C.3 category of Section 2.

Conclusion: These arguments indicate that the above mentioned KG negative conclusion also applies to the $W^{\pm}$ particles: It is extremely unlikely that a consistent term for the interaction of the charged $W^{\pm}$ particles with electromagnetic fields can be constructed.

4. Discussion

It is shown in this section that the first-order Dirac theory has other favorable properties that are not shared by second-order quantum theories.

- The Dirac Lagrangian density (7) proves that the dimension of the product $\bar{\psi}\psi$ of the Dirac functions is [L^{-3}]. This is the dimension of density. Hence, the operators that are enclosed within $\bar{\psi}\psi$ take their original dimension. This is consistent with the notion of operators and their eigenvalues that are used by the Schroedinger theory (see e.g. [13], p. 47)

$$\langle O \rangle = \int \psi^* \hat{O} \psi \, \mathrm{d}^3 x, \tag{14}$$

where $\langle O \rangle$ denotes the expectation value of the variable that is represented by the operator $\hat{O}$. Hence, considering this aspect, one finds that the Dirac theory abides by the Weinberg correspondence principle. Second-order quantum equations do not share this property because the product of their functions $\phi^\dagger \phi$ has the dimension [L^{-2}].

- The Dirac analysis proves that a first-order theory of a massive quantum particle should use a 4-component spinor ψ (see e.g. [15], Chapters 1, 2). The required Lagrangian density takes the form of (11). This expression uses the Dirac γ^μ matrices. Entries of these matrices are pure numbers, and the γ^μ transform like a 4-vector. It means that the γ^μ of the first-order Dirac theory can be regarded as a mathematical tool: a dimensionless derivative-free 4-vector that can be used for a construction of the required Lorentz scalar terms of the Lagrangian density. The derivative-free Dirac 4-current (8) is an important result of this mathematical tool. In the units used herein, also the classical 4-velocity is a dimensionless 4-vector (see [8], p. 23). Hence, the Dirac γ^μ corresponds to the classical 4-velocity. However, unlike the classical velocity, the γ^μ are independent of derivatives.
- In 1941 Pauli examined a tentative interaction term of the electron (see [6], p. 14; [20], p. 223)

$$\mathcal{L}' = d\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi, \tag{15}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor, and the coefficient d has the dimension of length. This term is called tensor interaction, due to its dependence on

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right] \tag{16}$$

(see [15], p. 21). The interaction term (15) alters the electron's magnetic moment. However, the ordinary Dirac Lagrangian density (11) provides a very good prediction for the electron's magnetic moment. Therefore, it is concluded that "the additional term is unnecessary" (see [20], p. 223).

A further examination of the interaction term (15) proves that it "is consistent with all accepted invariance principles, including Lorentz invariance and gauge invariance, and so there is no reason why such a term should not be included in the field equations" (see [6], p. 14). And indeed, the usefulness of a term of the form (15) has recently been proved to be suitable for a description of weak interactions, where the associated Hamiltonian is compatible with their V-A attribute [21] [22] [23].

One conclusion that stems from this outcome is that the first-order Dirac equation is flexible enough for the construction of a consistent interaction term of the electron with an external electromagnetic-like second-rank antisymmetric field tensor, that takes the form of the fields tensor $F^{\mu\nu}$.

The case of the second-order quantum equation is different. For example, people working on the electroweak theory have introduced an analogous term for the W$^{\pm}$ bosons (see Equation (3) of [19])

$$\mathcal{L}_T = iW_\mu^\dagger W_\nu F^{\mu\nu}. \tag{17}$$

(The quantity $V^{\mu\nu}$ of [19] denotes the ordinary electromagnetic field tensor $F^{\mu\nu}$.) A self-evident argument explains why the electroweak expression (17) is inconsistent with the fundamental concept of dimension that is used in Maxwellian electrodynamics. Indeed, the electromagnetic field tensor $F^{\mu\nu}$ does not carry an electric charge. Furthermore, the dimension of each component W_ν is [L^{-1}]. Hence, the dimension of the product $W_\mu^\dagger W_\nu$ of (17) is [L^{-2}]. This is inconsistent with the notion of charge density whose dimension is [L^{-3}].

Dimension coherence is an extremely crucial requirement that every physical expression should satisfy. It turns out that the adherence to the unjustifiable second-order quantum equations of the electroweak theory is the reason that explains why major research centers, like Fermilab and CERN, use an expression like (17) that violates the fundamental concept of dimensional coherence.

- The QFT differential equations are the last but not the least subject that is considered here. The differential equation of the Dirac theory is a well-known expression that is explicitly presented in textbooks (see e.g. [15], chapters 1-4). Solutions of this equation appropriately describe experimental data. Hence, the first-order Dirac equation satisfies the requirements A-C of subsection 2.3.

By contrast, no textbook that discusses the electroweak theory shows the *explicit* form of the differential equation of the W$^{\pm}$ elementary particles of this theory. A fortiori, no solution to this equation is compared with experimental data.

The case of the KG equation is different. Here a differential equation for the quantum particle exists. Unfortunately, there is no experimental confirmation of an *elementary* KG particle. For example, a pion is $\bar{q}q$ bound state and its function takes the form $\Phi(\boldsymbol{r}_1, \boldsymbol{r}_2, t)$. On the other hand, a KG function takes the form $\phi(x)$, where x denotes the four space-time coordinates. The different number of degrees of freedom proves that a pion is not a KG particle. It means that there is no experimental support for the KG theory.

These arguments mean that theories of second-order differential equations fail to satisfy the requirements A-C of subsection 2.3.

5. Conclusions

The Dirac-Pauli differences about the order of the fundamental differential equations of an elementary massive quantum particle are examined. The concept of dimension and the structure of the differential equations are important elements of this examination. The indisputable results that are obtained point out a quite strange situation: the present mainstream textbooks and the prevalent SM theory of elementary particles support Pauli's opinion, where second-order differential equations are quite acceptable. By contrast, the solid mathematical analysis of this work primarily relies on fundamental physical concepts, namely the dimension of the quantum functions and the compatibility of their differential equations. The discussion shows several convincing examples that prove that Dirac was right in this issue. Namely, second-order quantum theories of an elementary massive particle, like the $W^{\pm}$ bosons, are unacceptable simply because due to the dimension of their quantum function, one cannot define consistent expressions for crucial quantities. This result means that the SM theory and its literature require fundamental corrections.

This work relies on constraints that stem from fundamental physical principles, and are imposed on quantum theories. The main result of this work says that a QFT of a massive particle should rely on a first-order differential equation, namely on a Dirac-like equation. It means that the six spin-1/2 leptons—electron, muon, tau, and their associated neutrinos—together with the six spin-1/2 quarks—up, down, strange, charm, bottom and top—are (together with their antiparticles) the building blocks of massive matter.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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