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Lows of Wear Process of the Friction Pair "0.45% Carbon Steel—Polytetrafluoroethylene" during Sliding from the Position of Fracture Mechanics

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Abstract

The results of the tests for a friction pair "a cylindrical specimen made of 0.45% carbon steel—a counter specimen-liner made of polytetrafluoroethyleneF4-B" during sliding friction are presented. The test results at different levels of contact load are analyzed using the Archard's equation and are presented as a friction fatigue curve. The concept of the frictional stress intensity factor during sliding friction is introduced, and an expression that relates the wear rate to this factor and is close in shape to the Paris equation in fracture mechanics is proposed.

Keywords

Sliding Friction, Contact Pressure, Wear, Frictional Stress Intensity Factor, Polymer

1. Introduction

Among various theories of mechanical wear of solids in recent decades, fatigue theory has been widely recognized [1] [2] [3]. It turns out to be true if the contact load is relatively small, and the deformation of the friction surface is predominantly elastic.

One characteristic of this type of wear is the material damage under the repetitive action of compressive, tensile and shear deformations during cyclic loading caused by the interaction of the polymer with the hard and blunt projections on the rough surface during sliding, which gives rise to the generation and development of cracks, and which can be assisted by the presence of defects [4]. Some authors modify the term fatigue wear to frictional or rolling wear if the polymer presents a low tearing strength and slides on smooth counterfaces with high fric-

tion coefficient, causing roll formation at the sliding interface and tearing of the rolled fragment [5].

According to several studies, the interaction of the abrasive particles with the polymer produces deformation and tensile, compressive and shear stresses in the worn surface layer, forming in it fatigue cracks due to the repetitive action of these interactions [6]. Other investigations indicate that the largest shear stress takes place at a certain depth under the surface, this point being nearer to the surface as the friction force increases [7] [8]. On the other hand, the deformation of the material is greatest at the surface, which is propitious to the formation of cracks, but at the same time the compressive stress is also at its greatest in this area and restrains crack formation. With the increase of distance to the worn surface, the compressive stress decays faster than the strain, so that at some depth in the worn surface layer, the stress is almost pure shear stress and cracks are able to form more easily [9].

As known within the models based on contact mechanics, a model of particular relevance and broadly used is that proposed by Archard [10] [11], which is commonly expressed as:

$$W = \frac{k}{H} F_N \gamma \,, \tag{1}$$

where W [mm^3] is the worn volume, F_N [N] the applied normal load, γ [m] the sliding distance, k the non-dimensional wear coefficient particular to the contact pair characteristics and H [N/mm^2] the material hardness. When interpreting experimental situations, the hardness of the uppermost layer of material in the contact may not be known with any certainty and consequently a rather more useful quantity than the value of k alone is the ratio k/H [mm^3·N^{-1}·m^{-1}], named hereinafter as K and which is known as the dimensional wear coefficient or specific wear rate [9].

According to **Figure 1**, three different stages are accepted for describing a typical wear process: a first running-in stage in which the wear uniformity in the contact pair is being set up by elimination of the micro-asperities of the surfaces, a second stationary stage where a constant wear rate has been attained and the surface or surfaces are worn in a steady and uniform way, and a third accelerated stage where the wear rate increases in an exponential way and leads to catastrophic failure.

Archard's law referred to in Equation (1) is usually applied to the stationary stage. With the rest of the variables of the equation well known and without variation, the constant K can be considered as the characteristic wear coefficient of the wear process under study.

In this paper, the kinetic process of wear of the steel-polymer mechanical system is analyzed using fatigue fracture mechanics approaches.

2. Sliding Friction Tests and Their Results

Tests on sliding friction of the metal-polymer friction pair were carried out ac-

cording to the shaft-liner scheme. The sample-shaft 1 made of 0.45% carbon steel with 10 mm diameter of working part was cantilevered in the spindle 2 of the upgraded testing machine UKI-6000-2 and rotated at a frequency of 3000 min^{-1} (Figure 2). The counter specimen-liner 3 which is a 10 × 10 × 10 mm cube made of polymer polytetrafluoroethylene (PTFE) F4-B was pressed to the dangerous section of the specimen 1 with a contact load F_N, the value of which was set using a special tool and kept constant during the test of each pair of specimen—liner.

In the process of testing, a drip supply of a lubricant—Universal All-Seasonal Engine Oil "Lukoil Super 15W-40"—was provided and the measurement the linear wear of the friction pair using an indicator head with an accuracy of 2 μm was performed. Since the steel sample in the test pair did not wear out, all wear was obtained by a polymer liner. The liner wear equal to i_{lim} = 1000 μm was taken as the limit state.

The test results of the friction pair with the contact load F_N equal to 150, 180, 280, 350 and 450 N are shown in the form of kinetic graphs of the dependence of wear i [μm] on the number N of rotates [cycles] in Figure 3. These graphs correspond to the stationary stage of the wear process. Therefore, the Archard's equation can be applied to the above test results.

On the other hand, based on the fatigue theory of mechanical wear, the results of the tests can be represented as fatigue (Weller) curves in the coordinates of the contact load F_N—the number N of cycles before the limit state (for i_{lim} = 1000 μm) of the polymer liner (Figure 4). As can be seen from Figure 4, the fatigue

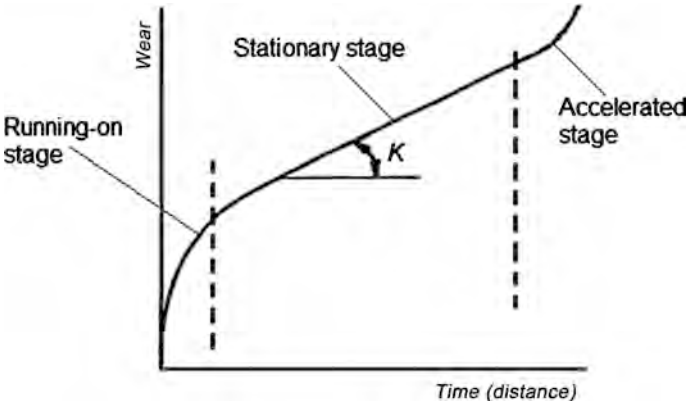

Figure 1. Typical wear curve in a tribological system [2].

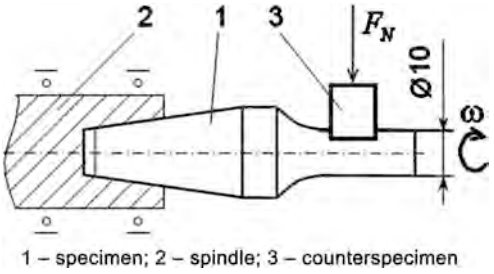

Figure 2. Sliding friction test scheme.

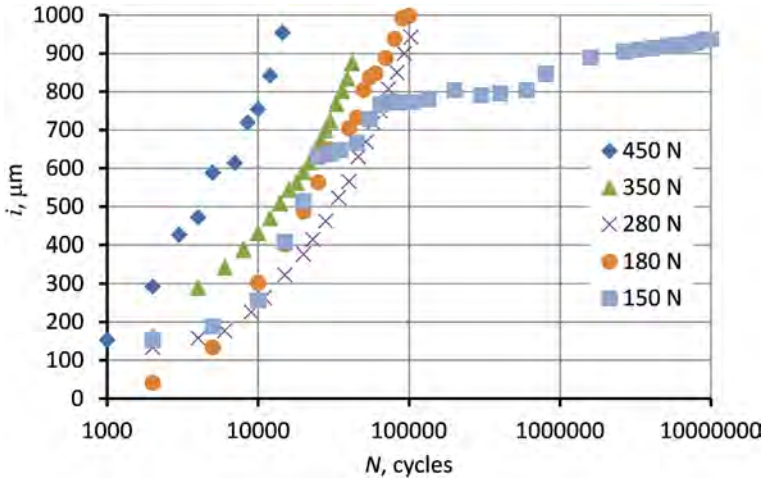

Figure 3. Kinetic curves of wear of the polymer liner during the sliding friction test with contact load 450, 350, 280, 180 and 150 N.

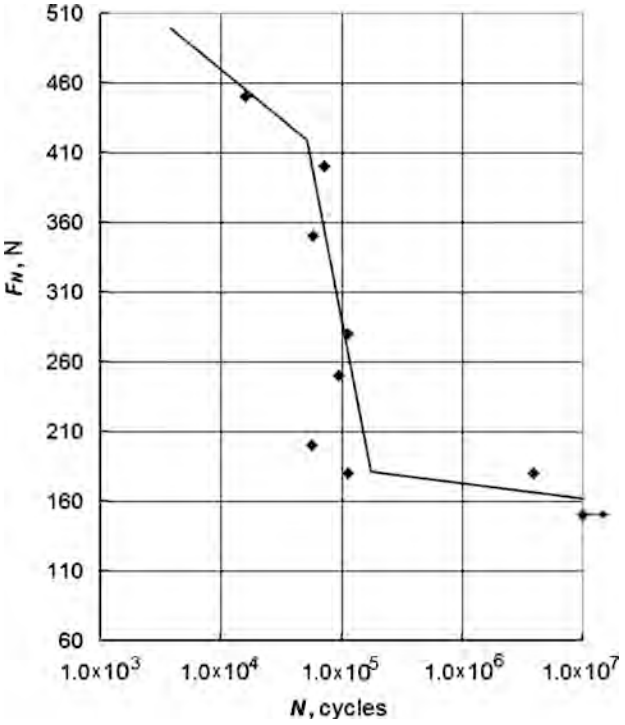

Figure 4. Friction fatigue curve of a polymer liner.

curve consists of three branches: the left branch with a slope (this is a region of quasi-static fracture to approximately $N = 7.2 \times 10^4$ cycles, $F_N = 400$ - 450 N), an average line located almost vertically (this is the area of low-cycle destruction $N = 7.2 \times 10^4$ - 9×10^4 cycles, $F_N = 165$ - 400 N), and the right one with a large slope (this is the area of multi-cycle destruction $N > 1 \times 10^5$ cycles, $F_N < 165$ N).

Let's try to describe the test results using the Archard's equation. In this case, in Equation (1) we write linear wear i instead of volume wear W, since they are proportional to each other. The sliding distance γ is replaced by the number N of loading cycles (these quantities are also proportional to each other). In Figure 5

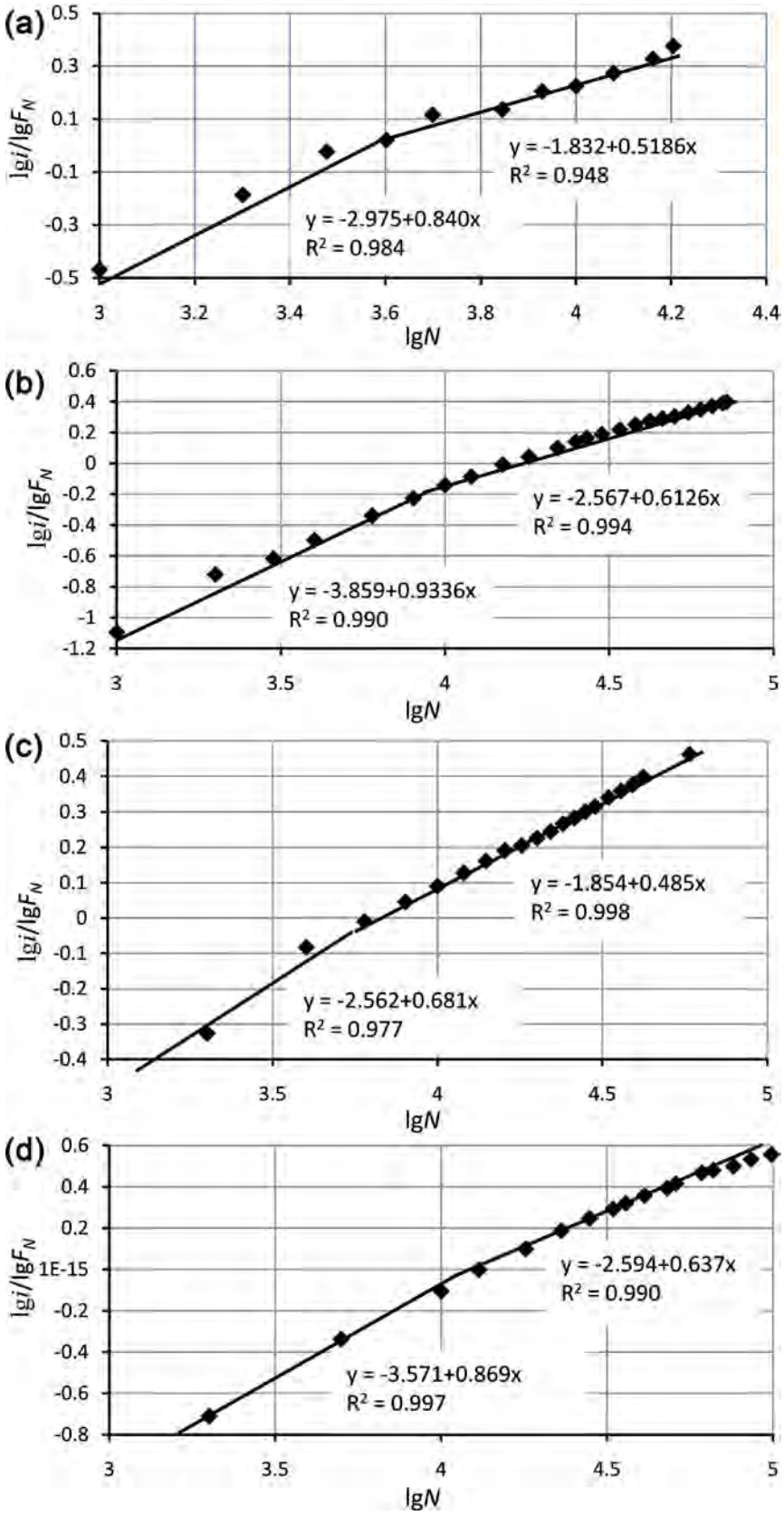

Figure 5. Graphs of the relationship of the ratio i/F_N on the number N of loading cycles of the polymer liner under contact load 450 (a), 400 (b), 350 (c) and 250 (d) N.

shows graphs of the ratio i/F_N against the number N of loading cycles, plotted in logarithmic coordinates according to the test results of the friction pair under study.

As can be seen from Figure 5 the experimental points can be quite satisfactorily described by a linear equation of the form $y = ax + b$. At the same time, in the studied range of i/F_N versus N, each graph can be represented as consisting of two linear dependencies, the values of the parameters a and b of the equations of which are shown in Figure 5. Apparently, the left part of the dependences in Figure 5 corresponds to the stage of steady wear, and the right-hand side to the stage of accelerated wear in accordance with the typical wear curve in Figure 1.

Analysis of graphs in Figure 5 shows that using the Archard's equation it is not possible to describe all the test results of the material under study at different values of the contact load.

3. Wear Process of Polymer from the Position of Fracture Mechanics

Some authors, such as Martinez *et al.* [9], Thomas *et al.* [12], Cho and Lee [13], have carried out investigations into polymers relating the mechanism of wear by abrasion and the mechanical fatigue process of crack growth theories. For the same material, they have observed that within the ranges of stable crack growth rate in fatigue and uniform debris detachment in wear, the slope of the abrasion rate in the wear process is similar to that of the crack growth rate in the fatigue mechanism, suggesting that both phenomena are related, the abrasion of the material occurring as a result of repeated crack propagation on a small scale.

Regarding the fatigue crack process, Figure 6 shows the different zones in

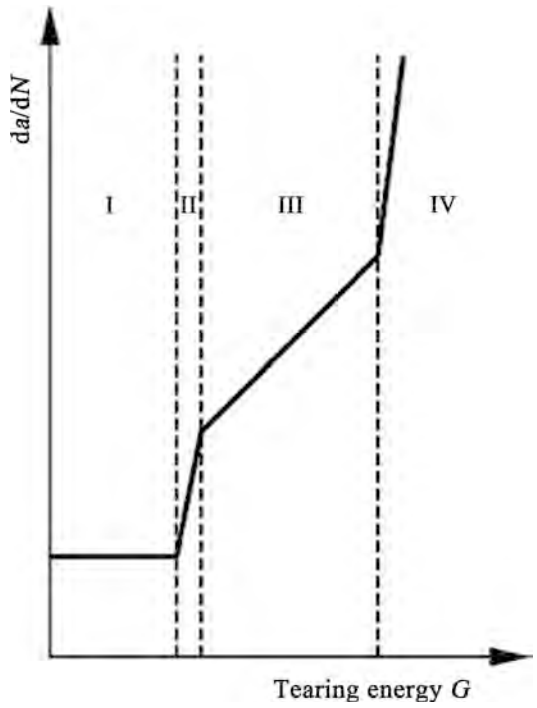

Figure 6. Crack growth characteristics for polymer [9].

which the fatigue crack growth behavior is divided for a polymer [9] [14]. This is known as the crack growth characteristic and is divided into four regions. In region I, the strain energy release rate or tearing energy G, defined as the partial derivative of the total elastic strain energy stored in an article containing a crack by the area of one fracture surface of the crack, is less than the threshold tear energy G_0, hence no mechanical crack growth occurs. In region II, the region of slow crack growth, the crack growth is dependent on both ozone and mechanical factors in an additive way. In region III, a power law dependency between the crack growth rate and the tearing energy is found as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = BG^{\beta}, \tag{2}$$

where a [mm] is the crack length, N is the number of cycles, B and β are material constants. Depending on the polymer type, the value of β lies between 1.5 and 6; in this region, stable crack growth takes place. Region IV corresponds to a rapid and unstable crack growth and therefore to the region of catastrophic failure.

In addition to the energy G, the stress intensity factor proportional to the value of G is often used as a control parameter for crack growth in fracture mechanics. As known, for a sample with limited dimensions the crack growth under the action of shear stresses τ is controlled by the shear stress intensity factor

$$K_{\tau} = Y\tau\sqrt{\pi a}, \tag{3}$$

where Y is the correction function that takes into account the geometry of the sample and its loading circuit.

In the case of volumetric damage during mechanical fatigue the crack size a characterizes the degree of material damage, while the surface damage caused by sliding friction is characterized by the value i of wear. Instead of tangential shear stress τ under friction, we can apply the so-called specific friction force or friction stress τ_w equal to [3]

$$\tau_w = fp_a = f\frac{F_N}{A_a}, \tag{4}$$

where f is the friction coefficient; p_a is the average contact pressure; A_a is the nominal contact area. Consequently, with reference to sliding friction, taking into account the assumptions made and (4), expression (3) can be written as

$$K_{\tau_w} \sim \tau_w\sqrt{i} \sim p_a\sqrt{i} \sim \frac{F_N}{A_a}\sqrt{i}. \tag{5}$$

Thus, using expression (5), it is possible to estimate the frictional stresses intensity factor under sliding friction. Obviously, the damage rate $\Delta a/\Delta N$ with the growth of fatigue cracks can be matched to the wear rate $\Delta i/\Delta N$ (in discrete form). Then for the wear rate during sliding friction, we obtain an expression close in form to (2):

$$\frac{\Delta i}{\Delta N} = B^* K_{\tau_w}^{\beta^*} \sim B^*\left(\frac{F_N}{A_a}\sqrt{i}\right)^{\beta^*}, \tag{6}$$

where B^* and β^* are material constants characterizing the steady-state stage of the wear process.

The analysis of experimental data on the expression (6) showed their satisfactory compliance. In Figure 7 as an example the graph $\lg(\Delta i/\Delta N) - \lg K_{\tau w}$ for the contact load 280 N is plotted.

The generalized graph $\lg(\Delta i/\Delta N) - \lg K_{\tau w}$ for the test results for all levels of contact load is presented in Figure 8. It completely corresponds to the classical *S*-shaped curve of the dependence of the fatigue crack growth rate on the stress intensity factor known in fracture mechanics. If we compare the obtained graph with a typical dependence of the crack growth rate on tearing energy for polymers

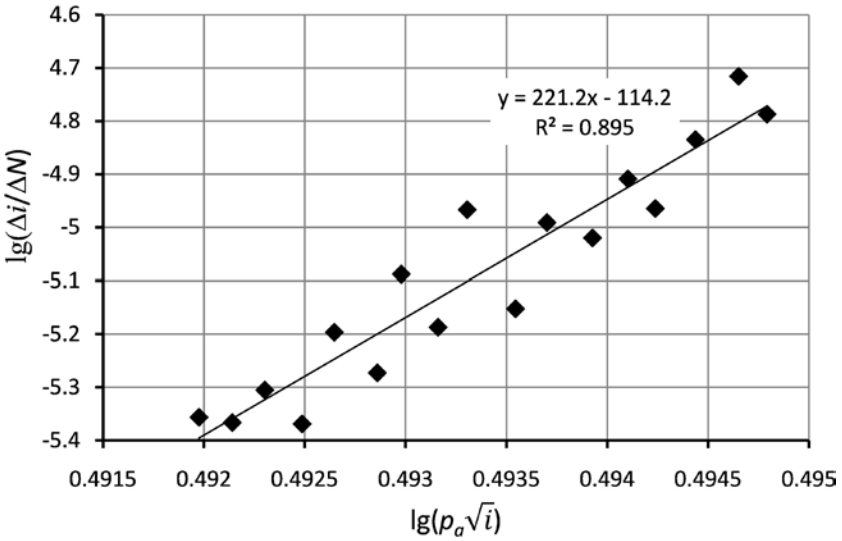

Figure 7. Graph of i/F_N ratio as a function of $K_{\tau w}$ for a PTFE liner with a contact load of 280 N in logarithmic coordinates.

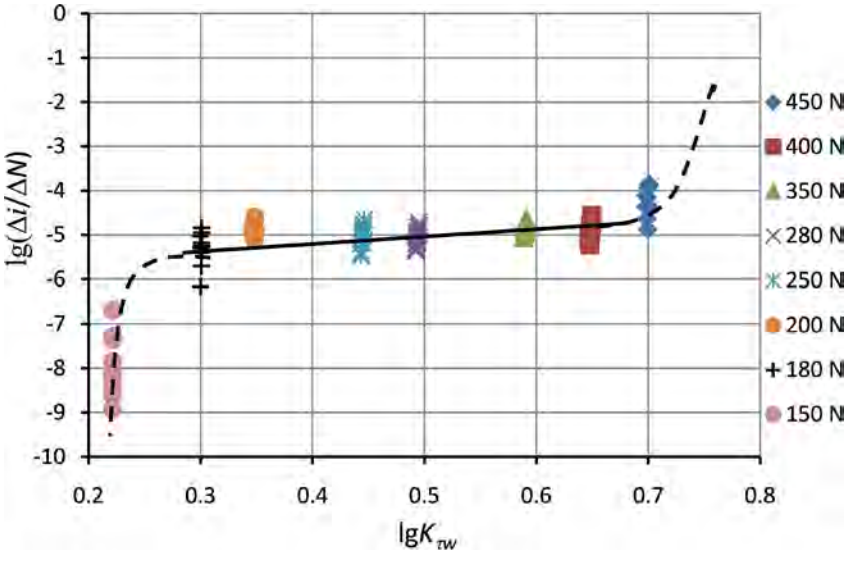

Figure 8. A generalized graph of the dependence of the i/F_N ratio on the $K_{\tau w}$ value for a PTFE liner with a contact load of 450, 400, 350, 280, 250, 200, 180 and 150 N in logarithmic coordinates.

(Figure 6), then we can see that in the contact load range from 180 to 400 N we have a steady wear stage (it corresponds to section III in Figure 6), which satisfies the Equation (6) with the parameters $B^* = 1.122 \times 10^{-6}$, $\beta^* = 2.21$. Note that for a number of polymers $\beta = 1.5 \cdots 3.0$ [9] [14] was set.

Obviously with contact loads smaller than 180 N, we will have a stage of low wear rates (Figure 8 shows a curve going down), corresponding to section II of low growth rates of polymer cracks (Figure 6). With contact loads exceeding 400 - 450 N, we obtain a stage of high wear rates (Figure 8 shows a dashed curve going up) corresponding to section IV of high crack growth rates (Figure 6). It should be noted that stage IV in Figure 8 is not obvious as no data point is plotted in the corresponding range. In the same way, the transition stage between I and II is not obvious.

4. Conclusions

Therefore, and according to the expressions stated in Equations (4)-(6), a clear analogy between the wear and the crack growth phenomena can be established, obtaining similar wear and crack growth rates, respectively. This is true for the friction pair studied as applied to the specified test conditions.

However, it is necessary to conduct additional experiments with other contact loads, other test conditions and other materials of a friction pair in order to assess the validity of the proposed approach to the description of wear kinetics during sliding friction. In addition, it is necessary to give a clear physical meaning to the parameters B^* and β^* of Equation (6).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Mathematical Modeling of the Transfer of Energy Forces from the Engine through Hydro Transmission and Hydro Differential to Executive Bodies

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Abstract

Mathematically simulated energy transfers from the energy source to the chassis through hydro transmissions and hydro differential. The developed unified mathematical model of a dynamic system allows, at the design stage, of many branched drive mechanisms, including transmission hydraulic and hydraulic differential actuators, to explore dynamic processes and select rational parameters.

Keywords

Energy Forces, Dynamic System, Many-Branched Mechanism, Engine, Hydro Transmission, Hydro Differential

1. Introduction

The results of experimental studies of experimental structures indicate a high dynamic load during transient processes of moving off, shifting gears and locking hydraulic differentiation, as well as in steady-state motion modes of mobile machines, which limits the durability of the elements of power mechanisms [1]. This determines the need for in-depth research aimed at reducing dynamic loading.

At present, the available theoretical and experimental data for previously de-

signed machines do not allow taking into account the potential properties of the designed machines, features of new design solutions, operating conditions, etc. Analytical methods for predicting durability and reliability based on the works of scientists and specialists were created for machines with low power at steady motion, for which the probability of moving at high speeds and high-loaded modes is not high. The mode of operation of many branched mechanisms of promising machines has not been scientifically studied.

Known mathematical models do not allow sufficiently taking into account the real design features, conditions and modes of motion control of machines, their interaction of parallel-sequentially installed driving mechanisms.

2. Problem Definition

One of the promising areas for improving the designs of wheeled vehicles and creating high-performance equipment is the use of volumetric hydraulic drives as a drive for the drive wheels of the undercarriage system. At present, industrial and agricultural mobile energy equipment (wheeled and tracked tractors) has been improved and has reached the level of mobile power facilities (MPF). MPF universal and unified tractor. It can simultaneously hang and fasten several different many operating machines in front, side and rear. In this case, there will be a lot of branching of the transmission of power from the engine to the actuators. In particular, energy is transmitted to the undercarriage, working actuators—to the drive shaft, a number of active working bodies. To describe these phenomena by mathematical expression is considered an important task.

Taking into account the influencing factors in the design of many branched mechanisms, including the flexibility of the working fluid and hydraulic drive elements, allows the design stage to provide a high technical level, reduce the amount of testing by increasing the reliability of calculations.

The creation of many branched mechanisms of a high technical level is hampered by the fact that the magnitudes of the influence of the flexibility of the working fluid, the elements of the hydraulic drive and the links of the mechanisms on its dynamic loading are insufficiently investigated. The influence of the flexibility of the working fluid and hydraulic drive elements on the optimal values of the parameters of the boom lifting mechanisms and the handle drive has not been taken into account.

The development of more accurate mathematical models of working processes of hydraulic mechanisms taking into account leaks in the hydraulic system, operation of safety systems, pliability of the working fluid and hydraulic drive elements makes it possible to fully and objectively determine the loads overcome by a many-branched mechanism during operation, and therefore evaluate the pressure state of the links, including number and in transient conditions.

The laboratory field tests of experimental tractors of the Research and Development Institute of Automobile and Tractor of Russia and the Institute of Agri-

cultural Machinery (England) showed increased slipping of tractor wheels with hydro-transmission in comparison with mechanical transmission. At the same time, the Institute of Agricultural Machinery in Leipzig (Germany) and MTZ specialists note a reduction in slipping, which confirms the opinion of the influence of the type and parameters of the transmission, hydraulic machines and characteristics of working fluids on the traction properties of self-propelled machines.

From the side of Panasenko S.M. it was determined that the hydraulic drive of propulsion with high-torque hydraulic motors with kinematic perturbation or uneven rotation in the entire load range exceeds the skidding of the tractor with a mechanical transmission; Installing low-torque hydro motors helps reduce slippage when exposed to forced vibrations. The most rational volume hydraulic actuators of the tractor chassis are characterized by the use of high-speed hydraulic motors followed by a manual gearbox and torque reduction to the propulsion units. Such a scheme has higher dissipation properties, which exclude the sources of oscillations that tell the effect on the driving movement of the thrusters.

Usually, traditionally, after the transmission gearbox, the differential, semi-axle and front-wheel reducer is installed. All these mechanisms serve to reduce the power during transmission from the hydraulic motor to the wheel.

In the thesis [2], the hypothesis about the destruction of metal-ceramic disks of friction elements, the control system of hydromechanical transmission due to the occurrence of resonant modes, caused by high-frequency disturbances generated by the torque converter, is put forward and substantiated. Based on the results of the study, an improved method of friction elements is presented.

Energy transfer previously performed on all mathematical models is described in parts and in a vague form. The transmissions of generated power or power from the engine by a mathematical expression are not fully recorded in any scientific work. The creation of a multi-operational and multi-functional mobile power and super-power engine makes a single transfer model of the power from the engine through a lot of branched transmission mechanisms to the executive bodies written. Unity is a unified system of equations, which are written by the expression the generated energy of an engine in parallel-sequential order transmitting to the executive bodies. Basically, MPF uses mechanical, hydraulic and electric drive transmissions. In this paper, we model the mechanical and hydraulic drive mechanisms.

To write a single mathematical model, we accept the following assumptions: oscillation phenomena in all mechanisms and nodes are not taken into account; sustainability is not considered; not studied external perturbing all sorts of phenomena.

3. Results and Discussion

3.1. Draw an Equivalent Design Scheme (Figure 1)

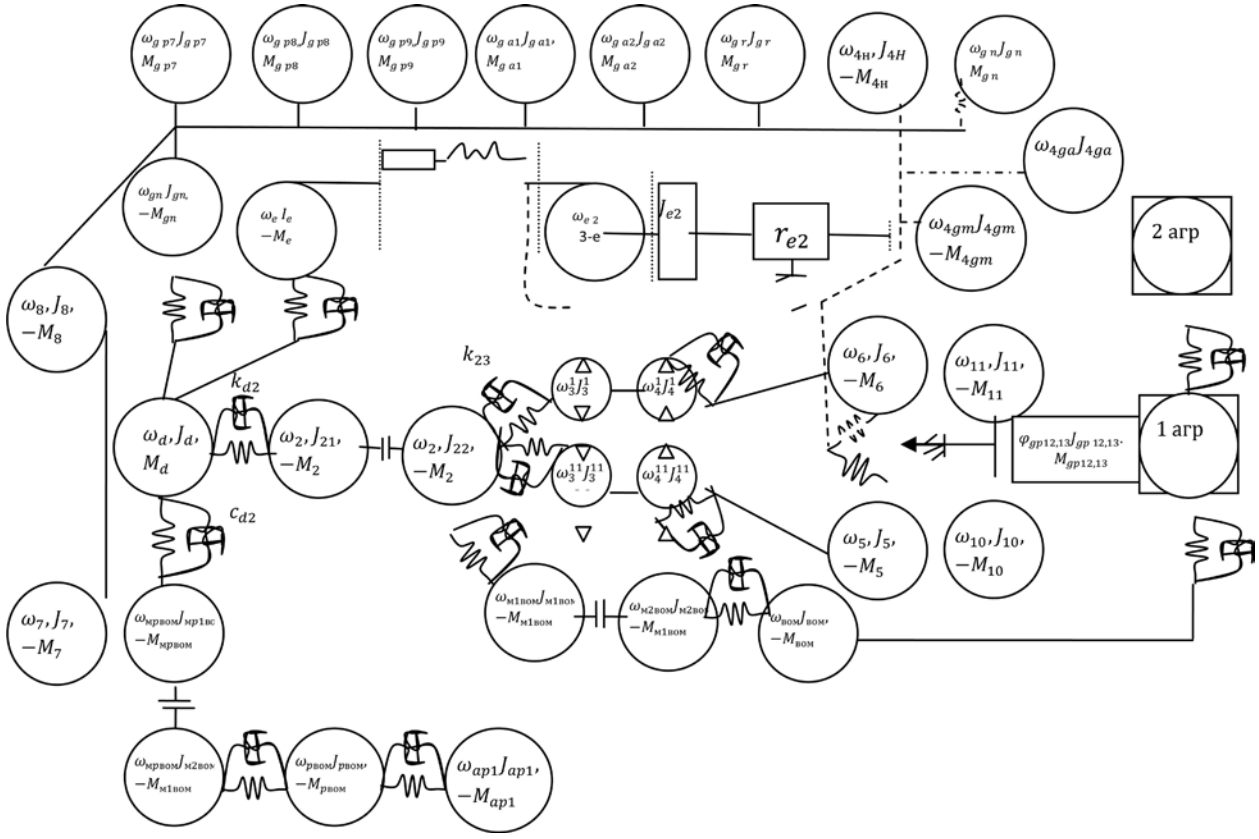

Figure 1. Design diagram of an equivalent integrated dynamic model of MPF with a technological module, rear and front units with hydraulic transmission and hydraulic differential.

3.2. Describe the Transfer of Energy from the Engine to Branched Mechanisms Involving Hydraulic Transmission

Equations describing the rotational motion of mechanisms from the engine

$$\left(J_d + J_{21}\right)\ddot{\varphi}_d + k_{d2}\left(\dot{\varphi}_d - \dot{\varphi}_2\right) + c_{d2}\left(\varphi_d + \varphi_2\right) = M_d - \frac{M_2}{i_2}, \varphi_d = \varphi_2$$

$$\left(J_{21} + J_{22}\right)\ddot{\varphi}_2 + c_2\left(\varphi_d + \varphi_2\right) = -M_2 sign\left(\dot{\varphi}_2\right) i_2,$$

$$\left(J_{\text{мп1вом}} + J_{\text{мп2вом}}\right)\ddot{\varphi}_{\text{мпвом}} + c_{\text{мпвом}}\left(\varphi_d + \varphi_{\text{мпвом}}\right) = -M_{\text{мпвом}} sign\left(\dot{\varphi}_{\text{мпвом}}\right) x$$

$$x i_{\text{мпвом}},$$

PTO n-clutch

$$J_{\text{вом}}\ddot{\varphi}_{\text{вом}} + k_{\text{вом}}\left(\dot{\varphi}_2 - \dot{\varphi}_{\text{вом}}\right) + c_{\text{вом}}\left(\varphi_2 + \varphi_{\text{вом}}\right) = -\frac{M_{\text{вом}} sign\left(\dot{\varphi}_{\text{вом}}\right)}{i_{\text{вом}}},$$

$$J_{\text{пвом}}\ddot{\varphi}_{\text{пвом}} + k_{\text{пвом}}\left(\dot{\varphi}_{\text{мпвом}} - \dot{\varphi}_{\text{пвом}}\right) + c_{\text{пвом}}\left(\varphi_{\text{мпвом}} + \varphi_{\text{пвом}}\right) = -\frac{M_{\text{пвом}} sign\left(\dot{\varphi}_{\text{пвом}}\right)}{i_{\text{пвом}}}$$

n-PTO

Describes the transfer of energy through an extensive mechanism involving hydraulic transmission

$$J_3^1\ddot{\varphi}_3^1 - k_{23}^1\left(\dot{\varphi}_2 - \dot{\varphi}_3^1\right) - e_{23}^1\left(\varphi_2 - \varphi_3^1\right) = -p_1 Rf\sin\gamma\sum_{k=0}^{n-1}\sin\left(\varphi_3^1 - k\beta\right)\Big/\eta_{HM},$$

$$J_3^{11}\ddot{\varphi}_3^{11} - k_{23}^{11}\left(\dot{\varphi}_2 - \dot{\varphi}_3^{11}\right) - e_{23}^{11}\left(\varphi_2 - \varphi_3^{11}\right) = -p_2 Rf\sin\gamma\sum_{k=0}^{n-1}\sin\left(\varphi_3^{11} - k\beta\right)\Big/\eta_{HM},$$

$$J_3^1\ddot{\varphi}_3^1 Rf_{gm\text{эк}}\sin\gamma\sum_{k=0}^{n-1}\sin\left(\varphi_3^1 + k\beta\right) = \varphi_4^1 F\sum_0^{\theta} v_\varphi + p_1 r + \dot{p}_1 V_1/E,$$

$$J_3^{11}\ddot{\varphi}_3^{11} Rf_{gm\text{эк}}\sin\gamma\sum_{k=0}^{n-1}\sin\left(\varphi_3^{11} + k\beta\right) = \varphi_4^{11} F\sum_0^{\theta} v_\varphi + p_2 r + \dot{p}_2 V_2/E,$$

$$J_4^1\ddot{\varphi}_4^1 + k_{45}\left(\dot{\varphi}_4^1 - \dot{\varphi}_5^1\right) + e_{45}\left(\varphi_4^1 - \varphi_5^1\right) = p_1 F\eta_{MM}\sum_0^{\theta} v_\varphi,$$

$$J_4^{11}\ddot{\varphi}_4^{11} + k_{45}\left(\dot{\varphi}_4^{11} - \dot{\varphi}_5^{11}\right) + e_{45}\left(\varphi_4^{11} - \varphi_5^{11}\right) = p_2 F\eta_{MM}\sum_0^{\theta} v_\varphi,$$

$$J_5^1\ddot{\varphi}_5^1 - k_{45}\left(\dot{\varphi}_4^1 - \dot{\varphi}_5^1\right) + k_{56}\left(\dot{\varphi}_5^1 - \dot{\varphi}_6\right) - e_{45}\left(\varphi_4^1 - \varphi_5^1\right) + c_{56}\left(\varphi_5^1 - \varphi_6\right) = -0.5M_\varphi$$

$$J_5^{11}\ddot{\varphi}_5^{11} - k_{45}\left(\dot{\varphi}_4^{11} - \dot{\varphi}_5^{11}\right) + k_{56}\left(\dot{\varphi}_5^{11} - \dot{\varphi}_6\right) - e_{45}\left(\varphi_4^{11} - \varphi_5^{11}\right) + c_{56}\left(\varphi_5^{11} - \varphi_6\right) = -0.5M_\varphi$$

And consider hydraulic transmissions from hydraulic leaks and leakages.

$$k_n\dot{\varphi}_3^1\gamma - c_n\left(p_1 - p_2\right) - c_y p_1 - q_{gm}\dot{\varphi}_4 - 2e_{23}\dot{p}_1 = \begin{cases} 0, & \text{при } p_1 > p_{nkl} \\ r_{nkl}p_1 - Q_{nn} & \text{при } p_1 \le p_{nkl} \end{cases};$$

$$k_n\dot{\varphi}_3^1\gamma - c_n\left(p_1 - p_2\right) - c_y p_2 - q_{gm}\dot{\varphi}_4 - 2e_{34}\dot{p}_2 = \begin{cases} 0, & \text{при } p_2 > p_{nkl} \\ r_{nkl}p_2 - Q_{nn} & \text{при } p_2 \le p_{nkl} \end{cases};$$

$$M_{gm} = q_{gm}\left(p_1 - p_2\right) - f_{gm\text{эк}}\dot{\varphi}_4,$$

$$M_n = k_n\gamma\left(p_1 - p_2\right) - f_{n\text{эк}}\dot{\varphi}_3,$$

where, $J_3^1, J_4^1, J_3^{11}, J_4^{11}, J_5^1, J_5^{11}$—are the reduced moments of inertia of the concentrated masses of the right and left pumps, right and left hydromators, and the leading right and left wheels; φ_2, φ_6—angular movement of the clutch and an active executive body; $\dot{\varphi}_2, \dot{\varphi}_6$—angular speed of the clutch and an active executive body; k_{23}^1, k_{23}^{11}—damping factor of the pump shaft, e_{23}^1, e_{23}^{11}—hydraulic compliance pressure pump parts; R—created efforts RJ; γ—angle of rotation of the pump control device; r—leakage ratio of RJ; V_1, V_2—volume in pressure and drain cavities; E—volume modulus of elasticity RJ; k_{45}—damping coefficient of hydraulic motor shaft; e_{45}—hydraulic compliance of the working part of the motor; e_{23}, e_{34}—hydraulic compliance of pressure and drain lines between the pump and the hydraulic unit; k_{56}—shaft damping ratio of the active executive body; c_{56}—circumferential rigidity of the shaft of the active executive body; φ_3^1, φ_4^1—angular displacement of the shaft in the right pump 3 and the right hydraulic motor 4; $\varphi_3^{11}, \varphi_4^{11}$—the angular displacement of the shaft in the left pump 3 and in the left hydraulic motor 4; $\varphi_5^1, \varphi_5^{11}$—are the angular displacements of the shaft of the link 5 from the side of the respective hydraulic motors; F—cross-sectional area; η_{HM}, η_{MM}—efficiency of the pump and hydraulic motor; v_φ—reduced speed of a high-speed hydraulic motor; c_n, c_y—hydraulic leakage and leakage rates; p_1, p_2—pressure in the pressure and discharge lines; $\dot{p}_1, \dot{p}_2$—time derivatives of pressure in the pressure and discharge lines; p_{nkl}—pressure setting of the make-up valve; r_{nkl}—specific flow through the return pick valve; k_n—coefficient specific feed pump; q_{gm}—specific consumption of

the hydraulic motor; $f_{gm\text{эк}}$, $f_{n\text{эк}}$ —are the coefficients of the generalized equivalent damping of the pump and the hydraulic motor; M_{gm}, M_n —the moment created by the hydraulic motor and the pump; M_φ —the moment of engagement of thrusters with the ground.

The resulting components of the differential equations for the hydraulic pump NAR-53 in the form $\sum_{k=0}^{n-1}\sin(\varphi+k\beta)=0.5\sin\varphi+2.83\cos\varphi$, $-20^\circ\le\varphi\le 20^\circ$, φ_3^1,φ_4^1 —for angular displacement is written accordingly; hydromotor MG-265T $\sum_0^\theta v_\varphi = a+\text{в}\varphi = 18.55+3.044\varphi$.

Moment of adhesion of propulsion with the ground is determined taking into account vibrodynamic effects of disturbing loads and variable wheel speed according to the formula [3]

$$M_\varphi=\left[mq\varphi_p+(1-m)(c+qtg\varphi)\right]F_0r_k\sqrt[\alpha]{\frac{\delta}{\delta_{\max}}}\sum_{i=1}^{n}\sqrt[\alpha]{i}\exp\left[-\alpha_\tau\left(\left|\ddot{\varphi}_5-\ddot{\varphi}_c\right|r_k\right)\right]$$

here, m—saturation coefficient tire tread; q—normal tire pressure on the ground; φ_p —the walking angle; c—connectedness of the soil; φ —the angle of internal friction of the soil; F_0 —contact area; r_k —the radius of the wheel; α —an indicator depending on the type of soil; $\delta,\delta_{\max}$ —shear characteristics; α_τ —a constant coefficient characterizing the physical and mechanical properties of soil; $\ddot{\varphi}_c$ —acceleration of the load on the soil; i—quantitatively lugs that are engaged.

3.3. Describes Energy Transfer in Hydro Differential

According to the calculation scheme (4*H*, 4*gm* links) (**Figure 1**) and accepted assumptions, the mathematical model of the hydro differential action will be the system of equations [4]

$$\begin{cases} J_{4H}\ddot{\varphi}_{4H}=M_{4H}-M_p-\beta_{4H}\dot{\varphi}_{4H} \\ J_{4gm}\ddot{\varphi}_{4gm}=M_p+M_{4gm} \\ \left(V_{4H}/2\pi\right)\dot{\varphi}_{4H}-\left(V_{4gm}/2\pi\right)\dot{\varphi}_{4gm}=e\dot{p}+k_0p+f_0\dot{x}_0 \\ p-p_0=0.5k^{-2}\rho\dot{x}_0\left|\dot{x}_0\right| \\ f_0\dot{x}_0p_0^2=V_3p_3\dot{p}_0 \end{cases} \tag{1}$$

$M_p=\left(V_{4H}/2\pi\right)p$ —moment of pressure created RJ, $M_{4gm}=M_n-M_i$, Initial conditions $\dot{\varphi}_{4H}=150c^{-1}$, $\dot{\varphi}_{4H}=150c^{-1}$, $p=0$, $p_0=p_3$, $\dot{x}_0=0$.

Here, J_{4H} —moment of inertia reduced to the pump axis; J_{4gm} —moment of inertia of the hydraulic motor brought to the axis; M_{4H} —torque axis of the pump; M_{4gm} —torque of the motor axis; M_n —moment of resistance to the hydraulic motor axes; M_i —moment of the i-th link located in turn; V_{4H}, V_{4gm} —volumes of the pump and hydraulic motor; V_3, p_3, p_0 —volume, pre-charge pressure and pressure in the accumulator; p—pressure in the pressure line; β_{4H} —damping coefficient of the pump; $\varphi_{4H}, \varphi_{4gm}$ —angular displacements of the axis of the pump and the hydraulic motor; k_0, e —coefficient of volumetric losses in the hydraulic drive and the flexibility of the pressure line; $f_0, \dot{x}_0$ —cross-section and velocity of the RJ at the inlet of the hydro accumulator; k, ρ

—flow coefficient and the density of the RJ.

In the system of Equation (1), the first two equations reflect the rotation of the pump and the hydraulic motor, the third one—the flow rate of the fluid, the fourth one—the fluid flow in the throttle of the accumulator, the fifth—the change in pressure in the hydro accumulator.

In the absence of a hydro accumulator, the system of Equation (1) is simplified to three equations

$$\begin{cases} J_{4H}\ddot{\varphi}_{4H} = M_{4H} - M_p \\ J_{4gm}\ddot{\varphi}_{4gm} = M_p + M_{4gm} \\ \left(V_{4H}/2\pi\right)\dot{\varphi}_{4H} - \left(V_{4gm}/2\pi\right)\dot{\varphi}_{4gm} = e\dot{p} + k_0 p \end{cases} \tag{2}$$

Initial conditions $\dot{\varphi}_{4H} = 150c^{-1}, \varphi_{4gm} = 0, p = 0$.

In relative coordinate $\theta = \varphi_{4H} - 5\varphi_{4gm}$, two equations of the system are distinguished (2)

$$\begin{cases} \ddot{\theta} + \left(V_0/2\pi\right)p = \varepsilon_0 \\ -\left(V_0/2\pi\right)\dot{\theta} + e\dot{p} + k_0 p = 0 \end{cases} \tag{3}$$

System of Equation (2) with = 0, $p_1 = -p$ take the form

$$\begin{cases} J_{4gm}\ddot{\varphi}_{4gm} - \left(V_{4gm}/2\pi\right)p = M_{4gm} \\ e\dot{p} + k_0 p + \left(V_{4gm}/2\pi\right)\dot{\varphi}_{4gm} = 0 \end{cases}$$

As a result of operational calculus, the angle, time and ways of braking are determined.

$$\varphi_T = \dot{\varphi}_0^2/\varepsilon_T \;;\; t_T = \varphi^2/\varepsilon_T \;;\; \varepsilon_T = V_{4gm}p_0 .$$

Load maxima on the motor axis

$$p_{4gm} \approx p_0 + J_{4gm}\dot{\varphi}_0^2\left(2p_0 e\right)^{-1}; M_{4gm} \approx \left(V_{4gm}/2\pi\right)p_{4gm}. \tag{4}$$

For the functioning of the hydro differential with the connection with the drive wheels, we supplement the system of equations (1) and (2) corresponding to the transfer of power

$$\begin{cases} J_{4H}\ddot{\varphi}_{4H} = M_{4H} - M_p - \beta_{4H}\dot{\varphi}_{4H} \\ J_{4gm}\ddot{\varphi}_{4gm} = M_p + M_{4gm} \\ \left(V_{4H}/2\pi\right)\dot{\varphi}_{4H} - \left(V_{4gm}/2\pi\right)\dot{\varphi}_{4gm} = e\dot{p} + k_0 p + f_0\dot{x}_0 \\ p - p_0 = 0.5k^{-2}\rho\dot{x}_0\left|\dot{x}_0\right| \\ f_0\dot{x}_0 p_0^2 = V_3 p_3 \dot{p}_0 \\ J_{4gm}\ddot{\varphi}_{4gm} + k_{4gm}\left(\dot{\varphi}_{4gm} - \dot{\varphi}_4\right) + e_{4gm}\left(\varphi_{4gm} + \varphi_4\right) = p_{24}F_4\eta_{4gm}\sum_0^\theta v_{\varphi 4} \\ J_5\ddot{\varphi}_5 + k_{45}\left(\dot{\varphi}_4 - \dot{\varphi}_5\right) + c_{45}\left(\varphi_4 + \varphi_5\right) = -M_5 sign\left(\dot{\varphi}_5\right)i_5 \end{cases} \tag{5}$$

$$\begin{cases} J_{4H}\ddot{\varphi}_{4H} = M_{4H} - M_p \\ J_{4gm}\ddot{\varphi}_{4gm} = M_p + M_{4gm} \\ \left(V_{4H}/2\pi\right)\dot{\varphi}_{4H} - \left(V_{4gm}/2\pi\right)\dot{\varphi}_{4gm} = e\dot{p} + k_0 p \\ J_{4gm}\ddot{\varphi}_{4gm} + k_{4gm}\left(\dot{\varphi}_{4gm} - \dot{\varphi}_4\right) + e_{4gm}\left(\varphi_{4gm} + \varphi_4\right) = p_{24}F_4\eta_{4gm}\sum_0^\theta v_{\varphi 4} \\ J_5\ddot{\varphi}_5 + k_{45}\left(\dot{\varphi}_4 - \dot{\varphi}_5\right) + c_{45}\left(\varphi_4 + \varphi_5\right) = -M_5 sign\left(\dot{\varphi}_5\right)i_5 \end{cases} \tag{6}$$

here, k_{4gm}, φ_{4gm}—coefficient of damping and angular displacement and shafting between the hydraulic motor and link 4; e_{4gm}—hydro hydraulics compliance; p_{24}—RJ pressure in the side of the link 4; F_4—sectional area; η_{4gm}—hydro motor efficiency; $v_{\varphi 4}$—reduced speed high-speed hydraulic motors of hydro differential.

The systems of Equations (5) and (6) are subject to the mechanisms under the condition $M_{\varphi 5} = 0$, $M_6 \neq 0$, and vice versa with $M_6 = 0$, $M_{\varphi 5} \neq 0$.

The system of Equation (5) reflects the hydromechanical network with a hydro accumulator and (6) without it.

4. Conclusions

A single mathematical model of the transfer of energy from the engine through a multi-branched transmission mechanism to the executive bodies is described by the expression of the transfer of energy from the joint work of mechanical and hydraulic actuators. The developed mathematical model of a dynamic system allows, at the design stage, of many branched drive mechanisms, including transmission hydraulic and hydraulic differential actuators, to explore dynamic processes and choose rational parameters of hydraulic drive compliance, time and damping coefficient, which are variable by selecting kinematic dynamic parameters, hydraulic motors working volume, the volume of the injection line and the reduced mass of inertia.

A unified mathematical model of the transfer of energy from the engine through many branched transmission mechanisms to the executive bodies take into account the hydraulic transmission and hydraulic differential.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Circular Scale of Time as a Guide for the Schrödinger Perturbation Process of a Quantum-Mechanical System

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Abstract

We point out that a suitable scale of time for the Schrödinger perturbation process is a closed line having rather a circular and not a conventional straight-linear character. A circular nature of the scale concerns especially the time associated with a particular order N of the perturbation energy which provides us with a full number of the perturbation terms predicted by Huby and Tong. On the other hand, a change of the order N—connected with an increased number of the special time points considered on the scale—requires a progressive character of time. A classification of the perturbation terms is done with the aid of the time-point contractions present on a scale characteristic for each N. This selection of terms can be simplified by a partition procedure of the integer numbers representing $N-1$. The detailed calculations are performed for the perturbation energy of orders $N=7$ and $N=8$.

Keywords

Quantum Mechanics, Schrödinger's Perturbation Process, Accuracy of a Circular Scale of Time in the Perturbation Calculations

1. Introduction

The scale of time, which is well known in everyday life and in science, too, is a product of a long experience. As far as we can distinguish the later events from the earlier ones, we organize the idea of time as a parameter which allows us to get an insight into the degree of the past, or future, connected with our observations.

In effect a tool to classify the events, and the time distances between them, is established. Conventionally this is done with the aid of an infinite scale extended between an infinite past—say representing the negative coordinates—and a sim-

ilar scale—say having the positive coordinates—representing the future:

$$-\infty < t < \infty. \tag{1}$$

The distances between the time points on the scale can be measured with a smaller or larger accuracy. These distances provide us with separations between different time points.

In practice the Schrödinger's quantum mechanics—developed in course of 1920's [1] [2] [3] [4]—has not much to do with the intervals of time. Its main idea was rather to distinguish between the stationary states of the chosen pieces of matter. Such pieces are described with the aid of the stationary eigenenergies and eigenfunctions, both kinds of parameters being independent of time. Concretely the classical Hamiltonian function of a chosen object is transformed into its operator form, and the integration of the classical Hamilton equations is replaced by a study of a differential eigenequation of the form

$$\hat{H}\psi = E\psi. \tag{2}$$

Here $\hat{H}$ is the Hamiltonian operator represented by a sum of the kinetic and potential operators

$$\hat{H} = \hat{E}_{\text{kin}} + \hat{E}_{\text{pot}}, \tag{3}$$

so—for a single particle system—

$$\hat{E}_{\text{kin}} = \frac{1}{2m}\left(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2\right), \tag{4}$$

$$\hat{E}_{\text{pot}} = \hat{V}(\boldsymbol{r}) = V(\boldsymbol{r}), \tag{5}$$

ψ is the eigenfunction called the wave function of an object, say a particle submitted to an external field having the potential V, symbol $\boldsymbol{r}$ is the position vector, E is the energy eigenvalue.

Because of

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}, \quad \hat{p}_y = -i\hbar\frac{\partial}{\partial y}, \quad \hat{p}_z = -i\hbar\frac{\partial}{\partial z}, \tag{6}$$

the momentum operator in (4) is of a differential character, whereas (5) represented by a function of the particle (object) position $\boldsymbol{r}$, is of a multiplicative nature.

The problem is that even in relatively simple physical cases the eigenequation (2) is difficult to solve. By solution we understand a set of the eigenenergies

$$E = E_1, E_2, E_3, \cdots \tag{7}$$

and eigenfunctions

$$\psi = \psi_1, \psi_2, \psi_3, \cdots \tag{8}$$

which satisfy (2). Only in very few physical cases equation (2) can provide us with simple solutions (7) and (8). The (7) are considered to be real energies of the system's quantum states, the (8) are the wave functions suitable in calculating other physical observables than energy.

In general the case when all eigenvalues on the right of (7) are different is classified as a non-degenerate problem, an opposite case is called to be a dege-

nerate one.

Schrödinger was certainly aware about the difficulties connected with the solution of his Equation (2); see [3]. His proposal became to calculate the solutions of a rather complicated (2) with the aid of solutions of a less complicated equation

$$\hat{H}^{(0)}\psi^{(0)} = E^{(0)}\psi^{(0)}, \tag{2a}$$

having the potential $V^0(\boldsymbol{r})$ more simple than $V(\boldsymbol{r})$ in (2). The potentials difference

$$\hat{V}^{\text{per}} = V^{\text{per}} = V(\boldsymbol{r}) - V^{(0)}(\boldsymbol{r}) \tag{9}$$

is called the perturbation potential, or simply a perturbation. In order to obtain possibly accurate results Schrödinger developed a formalism in which the solutions of (2) can be expressed with the aid of solutions of (2a). In this process—beyond of the solutions of (2a)—the matrix elements of the kind

$$\left\langle \psi_{\alpha}^{(0)} \middle| V^{\text{per}} \middle| \psi_{\beta}^{(0)} \right\rangle \tag{10}$$

are also involved.

A more easy treatment of the perturbation does concern the calculation of the energies of Equation (2) with the aid of solutions of Equation (2a) obtained in case of a non-degenerate case. Nevertheless an accurate calculation of these energies requires a complicated superposition of the solutions of (2a), as well as calculation of the matrix elements in (10). In principle these calculations were performed with no reference to the parameter of time; see Sec. 2.

The aim of the present paper is to point out that an introduction of the time scale—which has, however, a nature different than the well-known scale characterized by the formula (1)—provides us with a rather spectacular simplification of the original Schrödinger's perturbation scheme.

2. Outline of the Time-Independent Perturbation Theory of a Non-Degenerate Quantum State

A characteristic point is that Schrödinger obtained the solution of his perturbed equation without any reference to time [3]. An outline of a more modern time-independent perturbation theory is given, for example, in [5]. In the case of a non-degenerate quantum system let the unperturbed eigenequation

$$\hat{H}^{(0)}\psi_n^{(0)} = E_n^{(0)}\psi_n^{(0)} \tag{11}$$

be considered as solved. In principle we have an infinite set of the quantum numbers n for wnich the eigenequation (11) does hold. The number of the eigenfunctions and eigenenergies of the perturbed eigenequation

$$\hat{H}(\lambda)\psi = \left(\hat{H}_0 + \lambda V^{\text{per}}\right)\psi = E\psi \tag{12}$$

let be also infinite.

For $\lambda = 0$ we obtain the unperturbed problem equivalent to (11), whereas for $\lambda = 1$ we have a full perturbation problem. In principle we assume as valid

the following series expansions

$$\psi_n(\lambda) = \psi_n^{(0)} + \lambda\psi_n^{(1)} + \lambda^2\psi_n^{(2)} + \cdots \tag{13}$$

and

$$E_n(\lambda) = E_n^{(0)} + \lambda\Delta E_n^{(1)} + \lambda^2\Delta E_n^{(2)} + \cdots \tag{14}$$

and look for the solution of (12) in terms of the functions

$$\psi_n^{(0)}, \psi_n^{(1)}, \psi_n^{(2)}, \cdots \tag{15}$$

and numbers

$$E_n^{(0)}, \Delta E_n^{(1)}, \Delta E_n^{(2)}, \cdots \tag{16}$$

which make (12) valid for any λ from the interval

$$0 < \lambda < 1.$$

The function combined on the right of (13), *viz.*

$$\psi_n = \psi_n^{(0)} + \psi_n^{(1)} + \psi_n^{(2)} + \cdots + \psi_n^{(N)} \tag{17}$$

is called the perturbed wave function of state n presented with the accuracy to the perturbation order N, whereas the numbers entering on the right of (14), *viz.*

$$E_n = E_n^{(0)} + \Delta E_n^{(1)} + \Delta E_n^{(2)} + \cdots + \Delta E_n^{(N)} \tag{18}$$

give the perturbed energy of state n also with the accuracy of the perturbation order N.

By assuming the convergence of the series in (13) and (14), an increase the order number N applied in the sequence

$$1, 2, 3, \cdots, N \tag{19}$$

improves the accuracy of solutions presented in (13) and (14).

Physically as a more easy accessible and more interesting parameter, is considered the perturbed energy (14). Huby and Tong presented the number of the kinds of terms necessary to obtain the successive components

$$\Delta E^{(1)}, \Delta E^{(2)}, \Delta E^{(3)}, \cdots \Delta E^{(N)}, \tag{20}$$

entering the Schrödinger series for the energy perturbation of any non-degenerate state n; see [6] [7]. This number is expressed as a function of N by the formula

$$S_N = \frac{(2N-2)!}{N!(N-1)!}. \tag{21}$$

For low N the numbers S_N are also rather small, for example

$$S_1 = 1, S_2 = 1, S_3 = 2, S_4 = 5, S_5 = 14, S_6 = 42, \cdots \tag{22}$$

It should be noted that the kinds of the perturbation terms entering the set of S_N do not depend explicitly on state n, but they depend solely on N. Any kind of terms is, in its turn, a combination of the matrix elements of the perturbation potential with the unperturbed wave functions $\psi_\alpha^{(0)}$ and $\psi_\beta^{(0)}$, given in (10).

Another dependence of the terms is due to the differences of the unperturbed energy $E_n^{(0)}$ of state n submitted to perturbation and similar unperturbed ener-

gies $E_{\alpha}^{(0)}$, $E_{\beta}^{(0)}$, $E_{\gamma}^{(0)}$, *i.e.*

$$E_n^{(0)} - E_{\alpha}^{(0)}, E_n^{(0)} - E_{\beta}^{(0)}, E_n^{(0)} - E_{\gamma}^{(0)}, \cdots \tag{23}$$

As a rule the differences (23) enter the denominators of the perturbation terms, so there should be satisfied the relations

$$\alpha \neq n, \beta \neq n, \gamma \neq n, \cdots \tag{24}$$

etc.; see e.g. [8] for further details.

For $N > 2$ numerous terms entering S_N composed of (10), (23) and (24) can be submitted to infinite summations over the states indicated on the left of (24).

In practice the way of deriving the sets of S_N terms necessary for the Schrödinger perturbation formalism indicated above becomes a complicated task. Concurrent methods, obtained mainly without inclusion of the time parameter, are given in [9]-[17]. The computational applications performed with the aid of these methods seem to not provide us with a complete formalism suitable for a large perturbation order *N*. One of the by-products of the present paper is to make the perturbation method for large *N* to be more simple than before.

3. Feynman's Time-Dependent Formalism Referred to the Schrödinger Perturbation Theory

Feynman diagrams including the time variable became a well-known tool in representing the quantum phenomena of different kind [18] [19] [20]. They could be applied also in the case of the Schrödinger perturbation calculation. A fundamental difficulty of such a treatment comes from an enormous inflation of the number of diagrams which had to be considered in case of a large perturbation order *N*. For, according to the Feynman formalism, we should calculate and combine the results of

$$P_N = (N-1)! \tag{25}$$

diagrams in order to obtain the energy expression equivalent to the S_N terms entering the Schrödinger theory.

It is evident that

$$P_N = S_N \tag{26}$$

for $N = 1, 2$, and 3, but already for $N = 4$ we have

$$P_4 = 6 > S_4 = 5. \tag{27}$$

It is easy to check that for $N \gg 3$ we have $P_N \gg S_N$. For example for $N = 20$ we obtain

$$P_{20} = 19! \cong 1.23 \times 10^{17} \tag{28}$$

and

$$S_{20} = 1.77 \times 10^{9}. \tag{29}$$

Evidently the ratio

$$P_N / S_N \tag{30}$$

increases systematically with N tending to a huge number.

But the Feynman theory was based on a linear time scale represented by the interval given in formula (1). We demonstrate—in the remainder part of the paper—that a different kind of the time scale, namely that having a circular-like character, can lead precisely to the diagrams and, in consequence, the energy terms dictated by the Schrödinger perturbation calculus.

4. Scale of Time Suitable for the Schrödinger Perturbation Formalism, Its Contraction Points and Side Loops

Our idea is to replace a tedious calculation of the perturbation energy attained with the aid of solving the perturbed Schrödinger eigenequation by an immediate production of the perturbed energy terms due to an application of a suitable scale of time; see **Figures 1-4**.

According to Leibniz [21] [22] [23] time is an ordering parameter for the events occurring in the nature. A reference to the Leibniz concept of time as a merely successive order of things can be done also in connection of a discussion of the Mach's principle and the structure of dynamical theories [24] [25] [26].

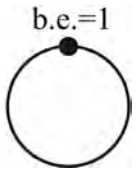

Figure 1. Time scale for the perturbation order $N=1$. The beginning-end point is 1.

Figure 2. Time scale for the perturbation order $N=2$. Beyond of the beginning-end point 2 there exists also point 1 on the scale. No contraction between 1 and 2 is admissible.

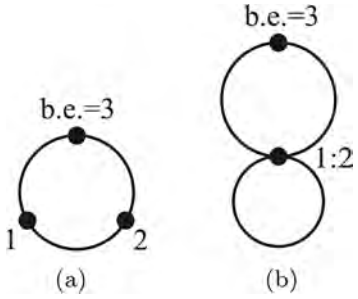

Figure 3. Time scale for the perturbation order $N=3$. Beyond of the beginning-end point 3 there exist also points 1 and 2. They can remain either free [diagram (a)] or are contracted [diagram (b)] giving a side loop.

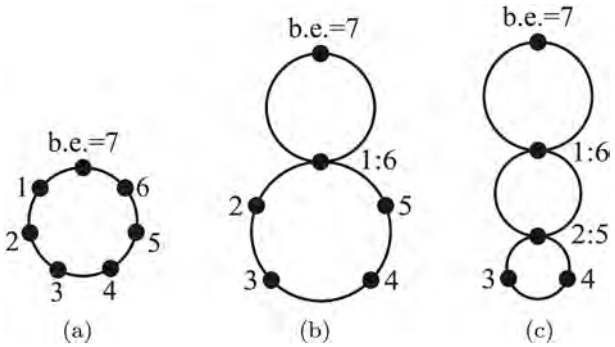

Figure 4. Time scale for the perturbation order $N=7$. Beyond of the beginning-end point 7 there exist also points 1, 2, 3, 4, 5 and 6. They are free on the diagram (a), but can form—for example—a maximal side loop for $N=7$ due to contraction $1:6$ [diagram (b)] or a cascade of loops [diagram (c)] due to contractions $1:6$ and $2:5$.

In case of the perturbation calculation, the Leibniz idea suggests to choose an appropriate scale of time, so it will be helpful to represent the results of the perturbation process. A necessary scale for any perturbation order N occurs to be a circular-like scale. This implies that N points of time—representing N successive collisions of the quantum system with the perturbation potential (9)—are present on a topological circle. One of these points, say the Nth point, let be the beginning-end point of the scale, called henceforth the main scale, or loop, of time. The remainder $N-1$ time points on the scale can be left either free, or submitted to contractions.

The contractions of the time points done on the main scale lead to the side loop, or loops, of time. Since it occurs that the Nth point should be excluded from contractions, a maximal size of the loop created from the main loop of the N points of time is given by the contraction between the time points 1 and $N-1$. This contraction is labelled by

$$1:N-1. \tag{31}$$

Beyond of the maximal loop of (31), a set of the minimal loops due to contractions

$$1:2,\ 2:3,\ 3:4,\cdots,\ N-2:N-1 \tag{32}$$

can be also created. We can have still the intermediate side loops like

$$1:3,\ 2:4,\ 3:5,\cdots,\ N-3:N-1, \tag{33}$$

or other loops larger than those due to contractions in (33).

Beyond of single contractions listed in (31)-(33), also multiple contractions of the time points like

$$1:2:3,\ 2:4:5,\ 2:4:7, \tag{34}$$

or

$$2:3:5:6,\ 2:4:6:7, \tag{34a}$$

or those composed of a still larger number of the time points, should be taken into account: the size of the admitted contractions depends on the size of the

considered N. Moreover, the combined contractions of the time points like

$$1:2 \bigcap 4:5 \tag{35}$$

may come also into play. A general rule is that the time loops due to the acceptable contractions should not cross. This means that, for example, the combined contraction due to the pair

$$1:4 \bigcap 2:5 \tag{36}$$

is not admissible.

A fundamental effect is that a full set of acceptable contractions for a considered order N gives precisely the number S_N of the Schrödinger perturbation terms predicted by the formula (21) for that N; no superflous neither lacking terms do occur. This is checked for the orders between $N=1$ and $N=7$ in the earlier papers by the author [25-34]. A full set of diagrams necessary for $N=6$ is given in [25], a similar set for $N=7$ enters [34]. In the present paper the perturbation energy of the order $N=8$ is also examined from the same point of view giving a similar agreement of the results; see Sec. 9.

5. Concentrations of the Contraction Points and Their Use

The concentration of a contraction point is equal to the number of the loops of time which meet together in that point. Evidently, if the contraction point is located on the main loop of time, one of the loops met in that point is the main loop itself. The other loops created by the time contractions on the main loop are called the side loops. An advantage to operate with the concentrations of loops is that they allow us to express the perturbation results in a more compact form than could be expected before.

This is so because the concentrations which are characteristic for a given N can be referred directly to partitions of the number $N-1$. In the next step the knowledge of partitions does lead to the number of the perturbation terms and the formulae for these terms. The effect of partitions and their connections with the contraction points will become evident in the computational practice giving the S_N Schrödinger terms for $N=7$ and $N=8$; see Sec. 9.

The next advantage of the time-point formalism is that any admissible contraction of the time points gives a correct contribution to the perturbation energy belonging to some N. The details of contributions can be easily derived by the analysis of contractions. The notation of the energy terms—see Sec. 6—can be simplified, because several terms of a given N can combine into the expressions representing the perturbation energies belonging to $N' < N$ —the point usually neglected in the former perturbation calculations done by many authors. The way of calculation of particular energy expressions and examination of their properties are presented in Sec. 9, where the examples of $N=7$ and 8 are studied in detail. An evident effect due to such study is that the energy contribution given by any of the time-point contractions can be obtained. Details concerning such contributions and notation suitable to represent the perturbation terms are

presented both in Sec. 6 and Sec. 9.

6. Notation Applied to Represent the Energy Perturbation Terms

Only for the perturbation orders $N=1$ and $N=2$ the side loops for the main loop of time do not exist. But any S_N term for $N>2$ is a product of energy contributions due to the main loop of time and those due to the side loops, respectively.

The contributions due to the side loops are easy to access from contractions of the time points and will be discussed first. Any contraction

$$\alpha:\beta \tag{37}$$

where as a rule we have

$$\alpha<\beta \tag{37a}$$

provide us with the energy multiplier equal to the energy correction

$$\Delta E_{\beta-\alpha}. \tag{38}$$

The difference

$$\beta-\alpha \tag{39}$$

indicates the perturbation order of energy contributed by the side loop represented by ΔE. In result, when the difference indicated in (39) is larger than 2, we have more than one Schrödinger perturbation term represented by the side loop, for

$$S_{\beta-\alpha}>1. \tag{40}$$

The contribution to energy due to the main loop of time depends on the number and situation of the time points present on that loop. When no contractions are present for the time points on the loop, the loop has N time points on it and gives the energy term in the form

$$\langle VPVPVPVP\cdots PV\rangle. \tag{41}$$

Such loop carries N symbols V and $N-1$ symbols P.

Evidently for $N=1$ no P symbol enters (41) and we obtain a single term for the perturbation energy equal to

$$\langle V\rangle=\langle n|V^{\text{per}}|n\rangle=\Delta E_1. \tag{42}$$

For the order $N=2$ we have no side loops and the perturbation energy is represented by the formula

$$\langle VPV\rangle=\Delta E_2. \tag{43}$$

The symbol P within the brackets on the left of (43) represents a reciprocal value of the energy difference, *viz.*

$$P=\frac{1}{E_n^{(0)}-E_p^{(0)}}, \tag{44}$$

situated between two matrix elements of V^{per}, *viz.*

$$\langle n|V^{\text{per}}|p\rangle,\ \langle p|V^{\text{per}}|n\rangle, \tag{44a}$$

and submitted to summation process over the dummy state index p. In effect

$$\langle VPV\rangle = \sum_{p\neq n}\frac{\langle n|V^{\text{per}}|p\rangle\langle p|V^{\text{per}}|n\rangle}{E_n^{(0)}-E_p^{(0)}}. \tag{45}$$

The meaning similar to the term (45) does prolongate to any perturbation term given by the main loop of time carrying no contraction points. For example for $N=3$ such term is represented by

$$\langle VPVPV\rangle = \sum_{p\neq n}\sum_{q\neq n}\frac{\langle n|V^{\text{per}}|p\rangle\langle p|V^{\text{per}}|q\rangle\langle q|V^{\text{per}}|n\rangle}{\left[E_n^{(0)}-E_p^{(0)}\right]\left[E_n^{(0)}-E_q^{(0)}\right]}. \tag{46}$$

This formula has two P and two dummy indices (p and q) for summation over the quantum states with exclusion of state n which is submitted to perturbation. It is easy to extend (46) to an arbitrary order N.

More complicated contributions to energy due to the main loop occur in case when the side loops are also present. For $N=3$ the only possible contraction of the time points is

$$1:2. \tag{47}$$

Evidently the side loop created by (47) does provide us with the term

$$\langle V\rangle = \Delta E_1, \tag{47a}$$

however our task is to present also a contribution due to the main loop of time. In this case contraction (47) transforms the term (46)—having no contractions—to the formula

$$\langle VP^2V\rangle = \sum_{p\neq n}\frac{\langle n|V^{\text{per}}|p\rangle\langle p|V^{\text{per}}|n\rangle}{\left[E_n^{(0)}-E_p^{(0)}\right]^2}. \tag{48}$$

The whole perturbation energy due to contraction (47) is represented by the product of (47a) and (48) taken with a minus sign:

$$1:2\rightarrow -\langle VP^2V\rangle\langle V\rangle \tag{49}$$

because we have an even number of terms entering the product in (49); an odd number of terms would give a positive sign. A characteristic point is that the total number of P and V entering the term in (49) remains the same as it does exist in the term (46): there are two P and three V together.

Another situation can be when the non-neighbouring time points, say 1 and 3, enter contraction

$$1:3. \tag{50}$$

This may occur for the perturbation order equal at least to $N=4$, so the last time point 4 is the beginning-end point on the scale and does not enter into contractions.

The energy term for the main loop of time having no contraction points becomes —in this case—a triple sum of terms

$$\langle VPVPVPV\rangle = \sum_{p\neq n}\sum_{q\neq n}\sum_{r\neq n}\frac{\langle n|V^{\text{per}}|p\rangle\langle p|V^{\text{per}}|q\rangle\langle q|V^{\text{per}}|r\rangle\langle r|V^{\text{per}}|n\rangle}{\left[E_n^{(0)}-E_p^{(0)}\right]\left[E_n^{(0)}-E_q^{(0)}\right]\left[E_n^{(0)}-E_r^{(0)}\right]} \tag{51}$$

whereas the contraction (50) implies the side loop having point 2 as free on it. This makes the energy contribution due to the side loop equal to

$$\langle VPV\rangle = \Delta E_2 \tag{52}$$

But the main loop of time having a contraction point (50) on it changes its contribution to the perturbation energy. Together with the beginning-end point of time the loop becomes similar to that representing the term (52), however the presence of the contraction point (50) implies the loop contribution to energy equal to

$$\langle VP^2V\rangle = \sum_{p\neq n}\frac{\langle n|V^{\text{per}}|p\rangle\langle p|V^{\text{per}}|n\rangle}{\left[E_n^{(0)}-E_p^{(0)}\right]^2}. \tag{53}$$

In effect the perturbation term due to contraction (50) is equal to product of (52) and (53):

$$-\Delta E_2\langle VP^2V\rangle. \tag{54}$$

The minus sign in (54) is dictated by the presence of an even number of terms entering the final product.

The notation procedure indicated above can be extended to any perturbation order N.

7. Time-Point Contractions on a Circular Scale and a Check of Validity of the Energy Terms Contributed by the Side-Loops of Time

Let us begin with a maximal side loop presented by the time point contraction in (31). Because the number of free points of time present on the side loop in (31) is

$$N' = N-1-1 = N-2, \tag{55}$$

the energy contributed by the side loop due to (31) is equal to

$$\Delta E_{N-2}. \tag{56}$$

This energy has to be joined with the energy contribution given by the main loop of time which—due to contraction (31)—possess only two points of time: the beginning-end point and the contraction point (31). It should be noted that the presence of the beginning-end point does not give any contribution to the perturbation energy, for such contribution can be given only by a loop of time.

In effect of the contraction point of two loops, they are joined together. This implies that the energy term of the main loop should have the term

$$P^2 \tag{57}$$

and not P alone; see (53). But beyond of (57) we note that the main loop becomes similar to the time loop characteristic for the second-order perturbation

term; see (52). In effect the main loop makes the whole contribution of the contraction (31) to the perturbation energy equal to

$$-\left\langle VP^2V\right\rangle \Delta E_{N-2}. \tag{58}$$

A formal check of validity of the energy expression given in (58) is simple: since the perturbation energy concerns order *N*, it should have the total number of *P* in the perturbation expression equal to $N-1$ and the number *V* is equal to *N*. Respectively, the perturbation energy in (51) contains the number of *P* equal to $N-3$ and that of *V* equal to $N-2$. The multiplier

$$\left\langle VP^2V\right\rangle \tag{59}$$

present in (58) supplies the lacking number of *P* and *V* in the term ΔE_{N-2} to the required number of *P* and *V* in an energy term belonging to ΔE_N.

The same reasoning can be applied to any contraction of the time points

$$1,2,3,\cdots,N-1 \tag{60}$$

entering the time scale useful for calculating the perturbation energy of the order *N*.

Examples of such calculations are presented in the earlier papers; see e.g. [33] [34].

The number of the time points which can be submitted to contractions for a given *N* is $N-1$; evidently different contractions can give different concentrations at the contraction points. The point present on the scale having no contractions has concentration 1, a maximal concentration of the $N-1$ time points is evidently $N-1$.

The same number $N-1$ is equal to the number of *P*'s present in any term of the Schrödinger perturbation energy; evidently for $N=1$ we have no *P* present in the perturbation term.

8. Systematic Time-Dependent Approach to the Schrödinger Perturbation Method. Partitions of the Number *N* – 1 and the Time Point Contractions

A fundametnal process of quantum mechanics is a change of a given system upon the action of a perturbation which—in its character—can be independent of time. To calculate the result of such a change on a non-degenerate system the Schrödinger perturbation formalism—represented by the sets of terms labelled by their order numbers *N*—is usually applied.

In principle there exist several ways according to which the necessary sets of terms can be obtained. In many cases no time approach should be used to this purpose. We are guided, however, by the Leibniz idea that a suitable arrangement of the perturbation events along a time scale can be helpful in an analysis of the expected change of a quantum system, including that due to the perturbation effect. Consequently the change of a system upon the action of a time-independent perturbation implies the importance of time. In fact we find that a circular scale of time—supplemented by the necessary time-point contractions on it—can represent with a perfect accuracy the perturbation terms of energy obtained in

an almost automatic way.

All possible partitions of the number $N-1$ lead to respective time-point contractions necessary for calculating the perturbation terms belonging to that *N*. Certainly the time points entering contractions are dependent on the position of a given partition number in the sum equal to $N-1$. In this way we obtain a full set of necessary contractions for a given number $N-1$.

For example for $N=3$ we have $N-1=2$ and the only set of the acceptable contractions is reduced to a single contraction

$$1:2=N-2:N-1 \tag{61}$$

represented by a partition number of $N-1$ equal to 2. But there exists also the partition

$$1 \quad 1 \tag{61a}$$

without contractions.

For $N=4$ we have the contractions $1:2$ and $2:3$ represented by partitions

$$\begin{array}{c} 2 \quad 1 \\ 1 \quad 2. \end{array} \tag{62}$$

Here the time point 3 and time point 1 remain free in the first and second row of (62), respectively. A full set of partitions for $N-1=3$ becomes

$$\begin{array}{c} 1 \quad 1 \quad 1 \\ 2 \quad 1 \\ 1 \quad 2 \\ 3; \end{array} \tag{63}$$

the partition 3 does represent the contractions $1:2:3$ and $1:3$.

For $N=5$ we obtain the partitions

$$1\ 1\ 1\ 1 \rightarrow \text{no contractions } (\text{points } 1,2,3,4 \text{ free}); S_1^4=1 \tag{64}$$

$$2\ 1\ 1 \rightarrow 1:2, \text{ points } 3,4 \text{ free}; S_2S_1^2=1 \tag{65}$$

$$1\ 2\ 1 \rightarrow 2:3, \text{ points } 1,4 \text{ free}; S_1S_2S_1=1 \tag{66}$$

$$1\ 1\ 2 \rightarrow 3:4, \text{ points } 1,2 \text{ free}; S_1^2S_2=1 \tag{67}$$

$$2\ 2 \rightarrow 1:2, 3:4; S_1S_2=1 \tag{68}$$

$$3\ 1 \rightarrow 1:3, 1:2:3; \text{ point } 4 \text{ free}; S_3S_1=2 \tag{69}$$

$$1\ 3 \rightarrow 2:4, 2:3:4; \text{ point } 1 \text{ free}; S_1S_3=2 \tag{70}$$

$$4 \rightarrow 1:4, 1:2:4, 1:3:4, 1:2:3:4, 1:4 \bigcap 2:3; S_4=5. \tag{71}$$

A characteristic point is that the *S*-like results are equal to the number of contractions; an exception is the first term [see (64)] where the absence of contractions is associated with all partitions equal to 1. A total value of the sum of the *S*-products on the left of (64)-(71) is equal to:

$$\begin{array}{l} S_1^4+S_2S_1^2+S_1S_2S_1+S_1^2S_2+S_2^2+S_3S_1+S_1S_3+S_4 \\ =1+1+1+1+1+2+2+5=14=S_5. \end{array} \tag{72}$$

Therefore a set of partitions of $N-1=4$ gives the S_5 perturbation terms.

A similar situation does repeat for $N=6$ which gives $N-1=5$; the partitions are

$$1\ 1\ 1\ 1\ 1\ (\text{points } 1,2,3,4,5 \text{ free});\ S_1^5=1 \tag{73}$$

$$2\ 1\ 1\ 1\to 1:2,\ (3,4,5 \text{ free});\ S_2S_1^3=1 \tag{74}$$

$$1\ 2\ 1\ 1\to 2:3,\ (1,4,5 \text{ free});\ S_1S_2S_1=1 \tag{75}$$

$$1\ 1\ 2\ 1\to 3:4,\ (1,2,5 \text{ free});\ S_1^2S_2S_1=1 \tag{76}$$

$$1\ 1\ 1\ 2\to 4:5,\ (1,2,3 \text{ free});\ S_1^3S_2=1 \tag{77}$$

$$1\ 2\ 2\to 2:3\bigcap 4:5;\ S_1S_2^2=1 \tag{78}$$

$$2\ 1\ 2\to 1:2\bigcap 4:5;\ S_2S_1S_2=1 \tag{79}$$

$$2\ 2\ 1\to 1:2\bigcap 3:4;\ S_2^2S_1=1 \tag{80}$$

$$1\ 1\ 3\to 3:5,\ 3:4:5;\ S_1^2S_3=2 \tag{81}$$

$$1\ 3\ 1\to 2:4,\ 2:3:4;\ S_1S_3S_1=2 \tag{82}$$

$$3\ 1\ 1\to 1:3,\ 1:2:3;\ S_3S_1^2=2 \tag{83}$$

$$4\ 1\to 1:4,\ 1:2:4,\ 1:3:4,\ 1:2:3:4,\ 1:4\bigcap 2:3;\ S_4S_1=5 \tag{84}$$

$$1\ 4\to 2:5,\ 2:3:5,\ 2:4:5,\ 2:3:4:5,\ 2:5\bigcap 3:4;\ S_1S_4=5 \tag{85}$$

$$5\to 1:5,\ 1:2:5,\ 1:3:5,\ 1:4:5,\ 1:2:3:5,\ 1:3:4:5,\ 1:2:3:4:5;\ S_5=14; \tag{86}$$

The total sum obtained from the 14 S like terms on the right is equal to

$$\begin{aligned} &S_1^5+S_2S_1^3+S_1S_2S_1^2+S_1^2S_2S_1+S_1^3S_2+S_1S_2^2+S_2S_1S_2+S_2^2S_1\\ &+S_1^2S_3+S_1S_3S_1+S_3S_1^2+S_2S_3+S_3S_2+S_1S_4+S_4S_1+S_5\\ &=1+1+1+1+1+1+1+1+2+2+2+2+2+5+5+14=42=S_6. \end{aligned} \tag{87}$$

Having the contraction data in (73)-(86) it becomes easy to construct the perturbation terms belonging to $N=6$. These terms are respectively:

$$1\ 1\ 1\ 1\ 1\to\langle VPVPVPVPVPV\rangle, \tag{88}$$

$$2\ 1\ 1\ 1\to-\langle VP^2VPVPVPV\rangle\Delta E_1, \tag{89}$$

$$1\ 2\ 1\ 1\to-\langle VPVP^2VPVPV\rangle\Delta E_1, \tag{90}$$

$$1\ 1\ 2\ 1\to-\langle VPVPVP^2VPV\rangle\Delta E_1, \tag{91}$$

$$1\ 1\ 1\ 2\to-\langle VPVPVPVP^2V\rangle\Delta E_1, \tag{92}$$

$$1\ 2\ 2\to\langle VPVP^2VP^2V\rangle(\Delta E_1)^2, \tag{93}$$

$$2\ 1\ 2\to\langle VP^2VPVP^2V\rangle(\Delta E_1)^2, \tag{94}$$

$$2\ 2\ 1\to\langle VP^2VP^2VPV\rangle(\Delta E_1)^2 \tag{95}$$

$$\begin{aligned} 1\ 1\ 3\to&-\langle VPVPVP^2V\rangle\Delta E_2,\\ &\langle VPVPVP^3V\rangle(\Delta E_1)^2, \end{aligned} \tag{96}$$

$$1\ 3\ 1 \rightarrow -\left\langle VPVP^2VPV\right\rangle \Delta E_2, \quad \left\langle VPVP^3VPV\right\rangle \left(\Delta E_1\right)^2, \tag{97}$$

$$3\ 1\ 1 \rightarrow -\left\langle VP^2VPVPV\right\rangle \Delta E_2, \quad \left\langle VP^3VPVPV\right\rangle \left(\Delta E_1\right)^2, \tag{98}$$

$$3\ 2 \rightarrow \left\langle VP^2VP^2V\right\rangle \Delta E_2 \Delta E_1, \quad -\left\langle VP^3VP^2V\right\rangle \left(\Delta E_1\right)^2 \Delta E_1, \tag{99}$$

$$2\ 3 \rightarrow \left\langle VP^2VP^2V\right\rangle \Delta E_1 \Delta E_2, \quad -\left\langle VP^2VP^3V\right\rangle \Delta E_1 \left(\Delta E_1\right)^2, \tag{100}$$

$$\begin{aligned} 4\ 1 \rightarrow & -\left\langle VP^2VPV\right\rangle \Delta E_3 \quad (2) \\ & \left\langle VP^3VPV\right\rangle \Delta E_2 \Delta E_1 \quad (1) \\ & \left\langle VP^3VPV\right\rangle \Delta E_1 \Delta E_2 \quad (1) \\ & -\left\langle VP^4VPV\right\rangle \left(\Delta E_1\right)^3 \quad (1) \end{aligned} \tag{101}$$

$$\begin{aligned} 1\ 4 \rightarrow & -\left\langle VPVP^2V\right\rangle \Delta E_3 \quad (2) \\ & \left\langle VPVP^3V\right\rangle \Delta E_2 \Delta E_1 \quad (1) \\ & \left\langle VPVP^3V\right\rangle \Delta E_1 \Delta E_2 \quad (1) \\ & -\left\langle VPVP^4V\right\rangle \left(\Delta E_1\right)^3 \quad (1) \end{aligned} \tag{102}$$

$$\begin{aligned} 5 \rightarrow & -\left\langle VP^2V\right\rangle \Delta E_4 \quad (5) \\ & \left\langle VP^3V\right\rangle \Delta E_1 \Delta E_3 \quad (2) \\ & \left\langle VP^3V\right\rangle \left(\Delta E_2\right)^2 \quad (1) \\ & \left\langle VP^3V\right\rangle \Delta E_3 \Delta E_1 \quad (2) \\ & -\left\langle VP^4V\right\rangle \left(\Delta E_1\right)^2 \Delta E_2 \quad (1) \\ & -\left\langle VP^4V\right\rangle \Delta E_1 \Delta E_2 \Delta E_1 \quad (1) \\ & -\left\langle VP^4V\right\rangle \Delta E_2 \left(\Delta E_1\right)^2 \quad (1) \\ & \left\langle VP^5V\right\rangle \left(\Delta E_1\right)^4 \quad (1) \end{aligned} \tag{103}$$

The numbers in brackets represent the quantity of the perturbation terms in a given rows.

In a similar way the results for the perturbation terms belonging to $N=7$ and $N=8$ are obtained; the terms are represented in Tables 1-6.

9. Comparison of the Present Method with an Earlier Recurrent Approach to the Perturbation Energy [34]

In [34] we presented a formalism which makes a recurrent calculation of the Schrödinger perturbation energy possible for an arbitrary order N. The method—outlined in the present paper—is based on partitions of the number $N-1$. It seems to be more transparent and systematical than that given in [34].

The present Section compares the both approaches—that of [34] and that of

Table 1. $N=7$. Perturbation terms based on the smaller size of partitions of the number $N-1=6$. Total number of the perturbation terms in the Table: $(6)+(7)+(8)+(12)+(4)=(37)$.

1 1 1 1 1 1	$S_1^6=1\to\langle VPVPVPVPVPVPV\rangle$	(1)
2 1 1 1 1	$S_2S_1^4=1\to-\langle VP^2VPVPVPVPV\rangle\Delta E_1$	(1)
1 2 1 1 1	$S_2S_1^4=1\to-\langle VPVP^2VPVPVPV\rangle\Delta E_1$	(1)
1 1 2 1 1	$S_2S_1^4=1\to-\langle VPVPVP^2VPVPV\rangle\Delta E_1$	(1)
1 1 1 2 1	$S_2S_1^4=1\to-\langle VPVPVPVP^2VPV\rangle\Delta E_1$	(1)
1 1 1 1 2	$S_2S_1^4=1\to-\langle VPVPVPVPVP^2V\rangle\Delta E_1$	(1)
2 2 1 1	$S_2^2S_1^2=1\to\langle VP^2VP^2VPVPV\rangle(\Delta E_1)^2$	(1)
2 1 2 1	$S_2^2S_1^2=1\to\langle VP^2VPVP^2VPV\rangle(\Delta E_1)^2$	(1)
2 1 1 2	$S_2^2S_1^2=1\to\langle VP^2VPVPVP^2V\rangle(\Delta E_1)^2$	(1)
1 2 2 1	$S_2^2S_1^2=1\to\langle VPVP^2VP^2VPV\rangle(\Delta E_1)^2$	(1)
1 2 1 2	$S_2^2S_1^2=1\to\langle VPVP^2VPVP^2V\rangle(\Delta E_1)^2$	(1)
1 1 2 2	$S_2^2S_1^2=1\to\langle VPVPVP^2VP^2V\rangle(\Delta E_1)^2$	(1)
2 2 2	$S_2^3=1\to-\langle VP^2VP^2VP^2V\rangle(\Delta E_1)^3$	(1)
3 1 1 1	$S_3S_1^3=2\to-\langle VP^2VPVPVPV\rangle\Delta E_2$	(1)
	$\to\langle VP^3VPVPVPV\rangle(\Delta E_1)^2$	(1)
1 3 1 1	$S_3S_1^3=2\to-\langle VPVP^2VPVPV\rangle\Delta E_2$	(1)
	$\to\langle VPVP^3VPVPV\rangle(\Delta E_1)^2$	(1)
1 1 3 1	$S_1^3S_3=2\to-\langle VPVPVP^2VPV\rangle\Delta E_2$	(1)
	$\to\langle VPVPVP^3VPV\rangle(\Delta E_1)^2$	(1)
1 1 1 3	$S_1^3S_3=2\to-\langle VPVPVPVP^2V\rangle\Delta E_2$	(1)
	$\to\langle VPVPVPVP^3V\rangle(\Delta E_1)^2$	(1)
3 2 1	$S_3S_2S_1=2\to\langle VP^2VP^2VPV\rangle\Delta E_2\Delta E_1$	(1)
	$\to\langle -VP^3VP^2VPV\rangle(\Delta E_1)^3$	(1)
3 1 2	$S_3S_1S_2=2\to\langle VP^2VPVP^2V\rangle\Delta E_2\Delta E_1$	(1)
	$\to-\langle VP^3VPVP^2V\rangle(\Delta E_1)^3$	(1)
2 3 1	$S_2S_3S_1=2\to\langle VP^2VP^2VPV\rangle\Delta E_1\Delta E_2$	(1)
	$\to-\langle VP^2VP^3VPV\rangle(\Delta E_1)^3$	(1)
2 1 3	$S_2S_1S_3=2\to\langle VP^2VPVP^2V\rangle\Delta E_1\Delta E_2$	(1)
	$\to-\langle VPVPVP^3V\rangle(\Delta E_1)^3$	(1)

Continued

1 3 2	$S_1S_3S_2=2\to\langle VPVP^2VP^2V\rangle\Delta E_2\Delta E_1$	(1)
	$\to-\langle VPVP^3VP^2V\rangle(\Delta E_1)^3$	(1)
1 2 3	$S_1S_2S_3=2\to\langle VPVP^2VP^2V\rangle\Delta E_1\Delta E_2$	(1)
	$\to-\langle VPVP^2VP^3V\rangle(\Delta E_1)^3$	(1)
3 3	$S_3S_3=4\to\langle VP^2VP^2V\rangle(\Delta E_2)^2$	(1)
	$\to-\langle VP^3VP^2V\rangle(\Delta E_1)^2\Delta E_2$	(1)
	$\to-\langle VP^2VP^3V\rangle\Delta E_2(\Delta E_1)^2$	(1)
	$\to\langle VP^3VP^3V\rangle(\Delta E_1)^4$	(1)

Table 2. $N=7$. Perturbation terms based on the intermediate size of partitions of the number $N-1=6$. Total number of the perturbation terms: $(5)+(5)+(5)+(14)+(14)=(53)$.

4 1 1	$S_4S_1^2=5\to-\langle VP^2VPVPV\rangle\Delta E_3$	(2)
	$\to\langle VP^3VPVPV\rangle\Delta E_1\Delta E_2$	(1)
	$\to\langle VP^3VPVPV\rangle\Delta E_2\Delta E_1$	(1)
	$\to-\langle VP^4VPVPV\rangle(\Delta E_1)^3$	(1)
1 4 1	$S_1^2S_4=5\to-\langle VPVP^2VPV\rangle\Delta E_3$	(2)
	$\to\langle VPVP^3VPV\rangle\Delta E_1\Delta E_2$	(1)
	$\to\langle VPVP^3VPV\rangle\Delta E_2\Delta E_1$	(1)
	$\to-\langle VPVP^4VPV\rangle(\Delta E_1)^3$	(1)
1 1 4	$S_1^2S_4=5\to-\langle VPVPVP^2V\rangle\Delta E_3$	(2)
	$\to\langle VPVPVP^3V\rangle\Delta E_1\Delta E_2$	(1)
	$\to\langle VPVPVP^3V\rangle\Delta E_2\Delta E_1$	(1)
	$\to-\langle VPVPVP^4V\rangle(\Delta E_1)^3$	(1)
4 2	$S_4S_2=5\to\langle VP^2VP^2V\rangle\Delta E_3\Delta E_1$	(2)
	$\to-\langle VP^3VP^2V\rangle(\Delta E_1)^2\Delta E_2$	(1)
	$\to-\langle VP^3VP^2V\rangle\Delta E_1\Delta E_2\Delta E_1$	(1)
	$\to\langle VP^4VP^2V\rangle(\Delta E_1)^4$	(1)
2 4	$S_2S_4=5\to\langle VP^2VP^2V\rangle\Delta E_1\Delta E_3$	(2)
	$\to-\langle VP^2VP^3V\rangle\Delta E_1\Delta E_2\Delta E_3$	(1)
	$\to-\langle VP^2VP^3V\rangle\Delta E_2(\Delta E_1)^2$	(1)

Continued

	$\to \langle VP^2VP^4V\rangle (\Delta E_1)^4$	(1)
5 1	$S_5S_1 = 14 \to -\langle VP^2VPV\rangle \Delta E_4$	(5)
	$\to \langle VP^3VPV\rangle \Delta E_3\Delta E_1$	(2)
	$\to \langle VP^3VPV\rangle (\Delta E_2)^2$	(1)
	$\to \langle VP^3VPV\rangle \Delta E_1\Delta E_3$	(2)
	$\to -\langle VP^4VPV\rangle (\Delta E_1)^2 \Delta E_2$	(1)
	$\to -\langle VP^4VPV\rangle \Delta E_1\Delta E_2\Delta E_1$	(1)
	$\to -\langle VP^4VPV\rangle \Delta E_2 (\Delta E_1)^2$	(1)
	$\to \langle VP^5VPV\rangle (\Delta E_1)^4$	(1)
1 5	$S_1S_5 = 14 \to -\langle VPVP^2V\rangle \Delta E_4$	(5)
	$\to \langle VPVP^3V\rangle \Delta E_3\Delta E_1$	(2)
	$\to \langle VPVP^3V\rangle (\Delta E_2)^2$	(1)
	$\to \langle VPVP^3V\rangle \Delta E_1\Delta E_3$	(2)
	$\to -\langle VPVP^4V\rangle (\Delta E_1)^2 \Delta E_2$	(1)
	$\to -\langle VPVP^4V\rangle \Delta E_1\Delta E_2\Delta E_1$	(1)
	$\to -\langle VPVP^4V\rangle \Delta E_2 (\Delta E_1)^2$	(1)
	$\to \langle VPVP^5V\rangle (\Delta E_1)^4$	(1)

Table 3. $N = 7$. The 42 energy perturbation terms belonging to partition $6 = N - 1$. The time-point contractions applied in the Table are presented. The number in brackets at the end of each row indicates the number of the perturbation terms due to that row. Total number of terms in **Tables 1-3**: $(37) + (53) + (42) = 132 = S_7$.

1:6	$\to -\langle VP^2V\rangle \Delta E_5$	$\to S_5 = (14)$
1:2:6	$\to \langle VP^3V\rangle \Delta E_1\Delta E_4$	$\to S_1S_4 = (5)$
1:3:6	$\to \langle VP^3V\rangle \Delta E_2\Delta E_3$	$\to S_2S_3 = (2)$
1:4:6	$\to \langle VP^3V\rangle \Delta E_3\Delta E_2$	$\to S_3S_2 = (2)$
1:5:6	$\to \langle VP^3V\rangle \Delta E_4\Delta E_1$	$\to S_4S_1 = (5)$
1:2:3:6	$\to -\langle VP^4V\rangle (\Delta E_1)^2 \Delta E_3$	$\to S_1^2S_3 = (2)$
1:2:4:6	$\to -\langle VP^4V\rangle \Delta E_1 (\Delta E_2)^2$	$\to S_1S_2^2 = (1)$
1:2:5:6	$\to -\langle VP^4V\rangle \Delta E_1\Delta E_3\Delta E_1$	$\to S_1^2S_3^2 = (2)$
1:3:4:6	$\to -\langle VP^4V\rangle \Delta E_2\Delta E_1\Delta E_2$	$\to S_2^2S_1^2 = (1)$

Continued

1:3:5:6	$\to -\langle VP^4V\rangle(\Delta E_2)^2\Delta E_1$	$\to S_2^2S_1^2=(1)$
1:4:5:6	$\to -\langle VP^4V\rangle\Delta E_3(\Delta E_1)^2$	$\to S_3S_1^2=(2)$
1:2:3:4:6	$\to \langle VP^5V\rangle(\Delta E_1)^3\Delta E_2$	$\to S_1^3S_2=(1)$
1:2:3:5:6	$\to \langle VP^5V\rangle(\Delta E_1)^2\Delta E_2\Delta E_1$	$\to S_1^3S_2=(1)$
1:2:4:5:6	$\to \langle VP^5V\rangle\Delta E_1\Delta E_2(\Delta E_1)^2$	$\to S_1^3S_2=(1)$
1:3:4:5:6	$\to \langle VP^5V\rangle\Delta E_2(\Delta E_1)^3$	$\to S_1^3S_2=(1)$
1:2:3:4:5:6	$\to -\langle VP^6V\rangle(\Delta E_1)^5$	$\to S_1^5=(1)$

Table 4. $N=8$. Perturbation terms based on the lower-size partitions of the number $N-1=7$. Total number of the perturbation terms represented in Table 4: $(7)+(10)+(4)+(10)+(24)+(6)+(12)+4\times(5)=(93)$.

1 1 1 1 1 1 1	$S_1^7=1\to\langle VPVPVPVPVPVPV\rangle$	(1)
2 1 1 1 1 1	$S_2S_1^5=1\to-\langle VP^2VPVPVPVPV\rangle\Delta E_1$	(1)
1 2 1 1 1 1	$S_2S_1^5=1\to-\langle VPVP^2VPVPVPVPV\rangle\Delta E_1$	(1)
1 1 2 1 1 1	$S_2S_1^5=1\to-\langle VPVPVP^2VPVPVPV\rangle\Delta E_1$	(1)
1 1 1 2 1 1	$S_2S_1^5=1\to-\langle VPVPVPVP^2VPVPV\rangle\Delta E_1$	(1)
1 1 1 1 2 1	$S_2S_1^5=1\to-\langle VPVPVPVPVP^2VPV\rangle\Delta E_1$	(1)
1 1 1 1 1 2	$S_2S_1^5=1\to-\langle VPVPVPVPVPVP^2V\rangle\Delta E_1$	(1)
2 2 1 1 1	$S_2^2S_1^3=1\to\langle VP^2VP^2VPVPVPV\rangle(\Delta E_1)^2$	(1)
1 2 1 1 2	$S_1^3S_2^2=1\to\langle VPVP^2VPVPVP^2V\rangle(\Delta E_1)^2$	(1)
1 1 2 1 2	$S_1^3S_2^2=1\to\langle VPVPVP^2VPVP^2V\rangle(\Delta E_1)^2$	(1)
1 1 1 2 2	$S_1^3S_2^2=1\to\langle VPVPVPVP^2VP^2V\rangle(\Delta E_1)^2$	(1)
2 1 1 1 2	$S_1^3S_2^2=1\to\langle VP^2VPVPVPVP^2V\rangle(\Delta E_1)^2$	(1)
2 1 2 1 1	$S_2^2S_1^3=1\to\langle VP^2VPVP^2VPVPV\rangle(\Delta E_1)^2$	(1)
2 1 1 2 1	$S_2^2S_1^3=1\to\langle VP^2VPVPVP^2VPV\rangle(\Delta E_1)^2$	(1)
1 2 2 1 1	$S_2^2S_1^3=1\to\langle VPVP^2VP^2VPVPV\rangle(\Delta E_1)^2$	(1)
1 1 2 2 1	$S_1^3S_2^2=1\to\langle VPVPVP^2VP^2VPV\rangle(\Delta E_1)^2$	(1)
1 2 1 2 1	$S_1^3S_2^2=1\to\langle VPVP^2VPVP^2VPV\rangle(\Delta E_1)^2$	(1)
2 2 2 1	$S_2^3S_1=1\to-\langle VP^2VP^2VP^2VPV\rangle(\Delta E_1)^3$	(1)
2 2 1 2	$S_2^3S_1=1\to-\langle VP^2VP^2VPVP^2V\rangle(\Delta E_1)^3$	(1)
2 1 2 2	$S_2^3S_1=1\to-\langle VP^2VPVP^2VP^2V\rangle(\Delta E_1)^3$	(1)

Continued

1 2 2 2	$S_1S_2^3=1\rightarrow-\langle VPVP^2VP^2VP^2V\rangle(\Delta E_1)^3$	(1)
3 1 1 1 1	$S_3S_1^4=2\rightarrow-\langle VP^2VPVPVPVPV\rangle\Delta E_2$	(2)
	$\rightarrow\langle VP^3VPVPVPVPV\rangle(\Delta E_1)^2$	
1 3 1 1 1	$S_1^4S_3=2\rightarrow-\langle VPVP^2VPVPVPV\rangle\Delta E_2$	(2)
	$\rightarrow\langle VPVP^3VPVPVPV\rangle(\Delta E_1)^2$	
1 1 3 1 1	$S_1^4S_3=2\rightarrow-\langle VPVPVP^2VPVPV\rangle\Delta E_2$	(2)
	$\rightarrow\langle VPVPVP^3VPVPV\rangle(\Delta E_1)^2$	
1 1 1 3 1	$S_1^4S_3=2\rightarrow-\langle VPVPVPVP^2VPV\rangle\Delta E_2$	(2)
	$\rightarrow\langle VPVPVPVP^3VPV\rangle(\Delta E_1)^2$	
1 1 1 1 3	$S_1^4S_3=2\rightarrow-\langle VPVPVPVPVP^2V\rangle\Delta E_2$	(2)
	$\rightarrow\langle VPVPVPVPVP^3V\rangle(\Delta E_1)^2$	
3 2 1 1	$S_3S_2S_1^2=2\rightarrow\langle VP^2VP^2VPVPV\rangle\Delta E_2\Delta E_1$	(2)
	$\rightarrow-\langle VP^3VP^2VPVPV\rangle(\Delta E_1)^3$	
1 3 2 1	$S_1^2S_3S_2=2\rightarrow\langle VPVP^2VP^2VPV\rangle\Delta E_2\Delta E_1$	(2)
	$\rightarrow-\langle VPVP^3VP^2VPV\rangle(\Delta E_1)^3$	
2 1 3 1	$S_2S_1^2S_3=2\rightarrow\langle VP^2VPVP^2VPV\rangle\Delta E_1\Delta E_2$	(2)
	$\rightarrow-\langle VP^2VPVP^3VPV\rangle(\Delta E_1)^3$	
3 1 2 1	$S_3S_1^2S_2=2\rightarrow\langle VP^2VPVP^2VPV\rangle\Delta E_2\Delta E_1$	(2)
	$\rightarrow-\langle VP^3VPVP^2VPV\rangle(\Delta E_1)^3$	
2 3 1 1	$S_2S_3S_1^2=2\rightarrow\langle VP^2VP^2VPVPV\rangle\Delta E_1\Delta E_2$	(2)
	$\rightarrow-\langle VP^2VP^3VPVPV\rangle(\Delta E_1)^3$	
1 2 3 1	$S_1^2S_2S_3=2\rightarrow\langle VPVP^2VP^2VPV\rangle\Delta E_1\Delta E_2$	(2)
	$\rightarrow-\langle VPVP^2VP^3VPV\rangle(\Delta E_1)^3$	
1 1 2 3	$S_1^2S_2S_3=2\rightarrow\langle VPVPVP^2VP^2V\rangle\Delta E_1\Delta E_2$	(2)
	$\rightarrow-\langle VPVPVP^2VP^3V\rangle(\Delta E_1)^3$	
1 1 3 2	$S_1^2S_3S_2=2\rightarrow\langle VPVPVP^2VP^2V\rangle\Delta E_2\Delta E_1$	(2)
	$\rightarrow-\langle VPVPVP^3VP^2V\rangle(\Delta E_1)^3$	
1 2 1 3	$S_1^2S_2S_3=2\rightarrow\langle VPVP^2VPVP^2V\rangle\Delta E_1\Delta E_2$	(2)
	$\rightarrow-\langle VPVP^2VPVP^3V\rangle(\Delta E_1)^3$	

Continued

1 3 1 2	$S_1^2 S_3 S_2 = 2 \to \langle VPVP^2VPVP^2V \rangle \Delta E_1 \Delta E_2$	(2)
	$\to -\langle VPVP^3VPVP^2V \rangle (\Delta E_1)^3$	
2 1 1 3	$S_2 S_1^2 S_3 = 2 \to \langle VP^2VPVPVP^2V \rangle \Delta E_1 \Delta E_2$	(2)
	$\to -\langle VP^2VPVPVP^3V \rangle (\Delta E_1)^3$	
3 1 1 2	$S_3 S_1^2 S_2 = 2 \to \langle VP^2VPVPVP^2V \rangle \Delta E_2 \Delta E_1$	(2)
	$\to -\langle VP^3VPVPVP^2V \rangle (\Delta E_1)^3$	
3 2 2	$S_3 S_2^2 = 2 \to -\langle VP^2VP^2VP^2V \rangle \Delta E_2 (\Delta E_1)^2$	(2)
	$\to \langle VP^3VP^2VP^2V \rangle (\Delta E_1)^4$	
2 3 2	$S_2^2 S_3 = 2 \to -\langle VP^2VP^2VP^2V \rangle \Delta E_1 \Delta E_2 \Delta E_1$	(2)
	$\to \langle VP^2VP^3VP^2V \rangle (\Delta E_1)^4$	
2 2 3	$S_2^2 S_3 = 2 \to -\langle VP^2VP^2VP^2V \rangle (\Delta E_1)^2 \Delta E_2$	(2)
	$\to \langle VP^2VP^2VP^3V \rangle (\Delta E_1)^4$	
3 3 1	$S_3^2 S_1 = 4 \to \langle VP^2VP^2VPV \rangle (\Delta E_2)^2$	(4)
	$\to -\langle VP^3VP^2VPV \rangle (\Delta E_1)^2 \Delta E_2$	
	$\to -\langle VP^2VP^3VPV \rangle \Delta E_2 (\Delta E_1)^2$	
	$\to \langle VP^3VP^3VPV \rangle (\Delta E_1)^4$	
3 1 3	$S_3^2 S_1 = 4 \to \langle VP^2VPVP^2V \rangle (\Delta E_2)^2$	(4)
	$\to -\langle VP^3VPVP^2V \rangle (\Delta E_1)^2 \Delta E_2$	
	$\to \langle -VP^2VPVP^3V \rangle \Delta E_2 (\Delta E_1)^2$	
	$\to \langle VP^3VPVP^3V \rangle (\Delta E_1)^4$	
1 3 3	$S_1 S_3^2 = 4 \to \langle VPVP^2VP^2V \rangle (\Delta E_2)^2$	(4)
	$\to -\langle VPVP^3VP^2V \rangle (\Delta E_1)^2 \Delta E_2$	
	$\to -\langle VPVP^2VP^3V \rangle \Delta E_2 (\Delta E_1)^2$	
	$\to \langle VPVP^3VP^3V \rangle (\Delta E_1)^4$	
4 1 1 1	$S_4 S_1^3 = 5 \to -\langle VP^2VPVPVPV \rangle \Delta E_3$	(2)
	$\to \langle VP^3VPVPVPV \rangle \Delta E_1 \Delta E_2$	(1)
	$\to \langle VP^3VPVPVPV \rangle \Delta E_2 \Delta E_1$	(1)
	$\to -\langle VP^4VPVPVPV \rangle (\Delta E_1)^3$	(1)
1 4 1 1	$S_1^3 S_4 = 5 \to -\langle VPVP^2VPVPV \rangle \Delta E_3$	(2)
	$\to \langle VPVP^3VPVPV \rangle \Delta E_1 \Delta E_2$	(1)

Continued

	$\rightarrow \langle VPVP^3VPVPV \rangle \Delta E_2 \Delta E_1$	(1)
	$\rightarrow -\langle VPVP^4VPVPV \rangle (\Delta E_1)^3$	(1)
1 1 4 1	$S_1^3 S_4 = 5 \rightarrow -\langle VPVPVP^2VPV \rangle \Delta E_3$	(2)
	$\rightarrow \langle VPVPVP^3VPV \rangle \Delta E_1 \Delta E_2$	(1)
	$\rightarrow \langle VPVPVP^3VPV \rangle \Delta E_2 \Delta E_1$	(1)
	$\rightarrow -\langle VPVPVP^4VPV \rangle (\Delta E_1)^3$	(1)
1 1 1 4	$S_1^3 S_4 = 5 \rightarrow -\langle VPVPVPVP^2V \rangle \Delta E_3$	(2)
	$\rightarrow \langle VPVPVPVP^3V \rangle \Delta E_1 \Delta E_2$	(1)
	$\rightarrow \langle VPVPVPVP^3V \rangle \Delta E_2 \Delta E_1$	(1)
	$\rightarrow -\langle VPVPVPVP^4V \rangle (\Delta E_1)^3$	(1)

Table 5. $N = 8$. Perturbation terms based on the higher-size partitions of the number $N-1=7$. Total number of the perturbation terms:
$6\times(5)+2\times(10)+2\times(14)+3\times(14)+2\times(42)=(204)$.

4 2 1	$S_4 S_2 S_1 = 5 \rightarrow \langle VP^2VP^2VPV \rangle \Delta E_3 \Delta E_1$	(2)
	$\rightarrow -\langle VP^3VP^2VPV \rangle \Delta E_2 (\Delta E_1)^2$	(1)
	$\rightarrow -\langle VP^3VP^2VPV \rangle \Delta E_1 \Delta E_2 \Delta E_1$	(1)
	$\rightarrow \langle VP^4VP^2VPV \rangle (\Delta E_1)^2$	(1)
4 1 2	$S_4 S_1 S_2 = 5 \rightarrow \langle VP^2VPVP^2V \rangle \Delta E_3 \Delta E_1$	(2)
	$\rightarrow -\langle VP^3VPVP^2V \rangle \Delta E_2 (\Delta E_1)^2$	(1)
	$\rightarrow -\langle VP^3VPVP^2V \rangle \Delta E_1 \Delta E_2 \Delta E_1$	(1)
	$\rightarrow \langle VP^4VPVP^2V \rangle (\Delta E_1)^4$	(1)
2 4 1	$S_2 S_4 S_1 = 5 \rightarrow \langle VP^2VP^2VPV \rangle \Delta E_1 \Delta E_3$	(2)
	$\rightarrow -\langle VP^2VP^3VPV \rangle \Delta E_1 \Delta E_2 \Delta E_1$	(1)
	$\rightarrow -\langle VP^2VP^3VPV \rangle (\Delta E_1)^2 \Delta E_2$	(1)
	$\rightarrow \langle VP^2VP^4VPV \rangle (\Delta E_1)^4$	(1)
2 1 4	$S_2 S_1 S_4 = 5 \rightarrow \langle VP^2VPVP^2V \rangle \Delta E_1 \Delta E_3$	(2)
	$\rightarrow -\langle VP^2VPVP^3V \rangle (\Delta E_1)^2 \Delta E_2$	(1)
	$\rightarrow -\langle VP^2VPVP^3V \rangle \Delta E_1 \Delta E_2 \Delta E_1$	(1)
	$\rightarrow \langle VP^2VPVP^4V \rangle (\Delta E_1)^4$	(1)
1 2 4	$S_1 S_2 S_4 = 5 \rightarrow \langle VPVP^2VP^2V \rangle \Delta E_1 \Delta E_3$	(2)

Continued

	$\rightarrow -\langle VPVP^2VP^3V\rangle(\Delta E_1)^2\,\Delta E_2$	(1)
	$\rightarrow -\langle VPVP^2VP^3V\rangle\Delta E_1\Delta E_2\Delta E_1$	(1)
	$\rightarrow \langle VPVP^2VP^4V\rangle(\Delta E_1)^4$	(1)
1 4 2	$S_1S_5S_2=5\rightarrow\langle VPVP^2VP^2V\rangle\Delta E_3\Delta E_1$	(2)
	$\rightarrow -\langle VPVP^3VP^2V\rangle\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow -\langle VPVP^3VP^2V\rangle\Delta E_1\Delta E_2\Delta E_1$	(1)
	$\rightarrow \langle VPVP^4VP^2V\rangle(\Delta E_1)^4$	(1)
4 3	$S_4S_3=5\times2=10\rightarrow\langle VP^2VP^2V\rangle\Delta E_3\Delta E_2$	(2)
	$\rightarrow -\langle VP^3VP^2V\rangle\Delta E_1(\Delta E_2)^2$	(1)
	$\rightarrow -\langle VP^3VP^2V\rangle\Delta E_2\Delta E_1\Delta E_2$	(1)
	$\rightarrow \langle VP^4VP^2V\rangle(\Delta E_1)^3\,\Delta E_2$	(1)
	$\rightarrow -\langle VP^2VP^3V\rangle\Delta E_3(\Delta E_1)^2$	(1)
	$\rightarrow \langle VP^3VP^3V\rangle\Delta E_2\Delta E_1(\Delta E_1)^2$	(1)
	$\rightarrow \langle VP^3VP^3V\rangle\Delta E_1\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow -\langle VP^4VP^3V\rangle(\Delta E_1)^5$	(1)
3 4	$S_3S_4=2\times5=10\rightarrow\langle VP^2VP^2V\rangle\Delta E_2\Delta E_3$	(2)
	$\rightarrow -\langle VP^2VP^3V\rangle\Delta E_2\Delta E_1\Delta E_2$	(1)
	$\rightarrow -\langle VP^2VP^3V\rangle\Delta E_2\Delta E_2\Delta E_1$	(1)
	$\rightarrow \langle VP^2VP^4V\rangle\Delta E_2(\Delta E_1)^3$	(1)
	$\rightarrow -\langle VP^3VP^2V\rangle(\Delta E_1)^2\,\Delta E_3$	(2)
	$\rightarrow \langle VP^3VP^3V\rangle(\Delta E_1)^2\,\Delta E_1\Delta E_2$	(1)
	$\rightarrow \langle VP^3VP^3V\rangle(\Delta E_1)^2\,\Delta E_2\Delta E_1$	(1)
	$\rightarrow -\langle VP^3VP^4V\rangle(\Delta E_1)^5$	(1)
5 2	$S_5S_2=14\rightarrow\langle VP^2VP^2V\rangle\Delta E_4\Delta E_1$	(5)
	$\rightarrow -\langle VP^3VP^2V\rangle\Delta E_3(\Delta E_1)^2$	(2)
	$\rightarrow -\langle VP^3VP^2V\rangle(\Delta E_2)^2\,\Delta E_1$	(1)
	$\rightarrow -\langle VP^3VP^2V\rangle\Delta E_3(\Delta E_1)^2$	(2)
	$\rightarrow \langle VP^4VP^2V\rangle\Delta E_2(\Delta E_1)^3$	(1)
	$\rightarrow \langle VP^4VP^2V\rangle\Delta E_1\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow \langle VP^4VP^2V\rangle(\Delta E_1)^2\,\Delta E_2\Delta E_1$	(1)

Continued

	$\rightarrow -\langle VP^5VP^2V\rangle(\Delta E_1)^5$	(1)
2 5	$S_2S_5=14\rightarrow\langle VP^2VP^2V\rangle\Delta E_1\Delta E_4$	(5)
	$\rightarrow -\langle VP^2VP^3V\rangle(\Delta E_1)^2\Delta E_3$	(2)
	$\rightarrow -\langle VP^2VP^3V\rangle\Delta E_1(\Delta E_2)^3$	(1)
	$\rightarrow -\langle VP^2VP^3V\rangle\Delta E_1\Delta E_3\Delta E_1$	(2)
	$\rightarrow \langle VP^2VP^4V\rangle(\Delta E_1)^3\Delta E_2$	(1)
	$\rightarrow \langle VP^2VP^4V\rangle(\Delta E_1)^2\Delta E_2\Delta E_1$	(1)
	$\rightarrow \langle VP^2VP^4V\rangle\Delta E_1\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow -\langle VP^2VP^5V\rangle(\Delta E_1)^5$	(1)
5 1 1	$S_5S_1^2=14\rightarrow -\langle VP^2VPVPV\rangle\Delta E_4$	(5)
	$\rightarrow \langle VP^3VPVPV\rangle\Delta E_1\Delta E_3$	(2)
	$\rightarrow \langle VP^3VPVPV\rangle(\Delta E_2)^2$	(1)
	$\rightarrow \langle VP^3VPVPV\rangle\Delta E_3\Delta E_1$	(2)
	$\rightarrow -\langle VP^4VPVPV\rangle(\Delta E_1)^2\Delta E_2$	(1)
	$\rightarrow -\langle VP^4VPVPV\rangle\Delta E_1\Delta E_2\Delta E_1$	(1)
	$\rightarrow -\langle VP^4VPVPV\rangle\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow \langle VP^5VPVPV\rangle(\Delta E_1)^4$	(1)
1 5 1	$S_1^2S_5=14\rightarrow -\langle VPVP^2VPV\rangle\Delta E_4$	(5)
	$\rightarrow \langle VPVP^3VPV\rangle\Delta E_1\Delta E_3$	(2)
	$\rightarrow \langle VPVP^3VPV\rangle(\Delta E_2)^2$	(1)
	$\rightarrow \langle VPVP^3VPV\rangle\Delta E_3\Delta E_1$	(2)
	$\rightarrow -\langle VPVP^4VPV\rangle(\Delta E_1)^2\Delta E_2$	(1)
	$\rightarrow -\langle VPVP^4VPV\rangle\Delta E_1\Delta E_2\Delta E_1$	(1)
	$\rightarrow -\langle VPVP^4VPV\rangle\Delta E_2(\Delta E_1)^2$	(1)
	$\rightarrow \langle VPVP^5VPV\rangle(\Delta E_1)^4$	(1)
1 1 5	$S_1^2S_5=14\rightarrow -\langle VPVPVP^2V\rangle\Delta E_4$	(5)
	$\rightarrow \langle VPVPVP^3V\rangle\Delta E_1\Delta E_3$	(2)
	$\rightarrow \langle VPVPVP^3V\rangle(\Delta E_2)^2$	(1)
	$\rightarrow \langle VPVPVP^3V\rangle\Delta E_3\Delta E_1$	(2)
	$\rightarrow -\langle VPVPVP^4V\rangle(\Delta E_1)^2\Delta E_2$	(1)

Continued

	$\to -\langle VPVPVP^4V\rangle \Delta E_1\Delta E_2\Delta E_1$	(1)
	$\to -\langle VPVPVP^4V\rangle \Delta E_2 (\Delta E_1)^2$	(1)
	$\to \langle VPVPVP^5V\rangle (\Delta E_1)^4$	(1)
6 1	$S_6S_1 = 42 \to -\langle VP^2VPV\rangle \Delta E_5$	(14)
	$\to \langle VP^3VPV\rangle \Delta E_4\Delta E_1$	(5)
	$\to \langle VP^3VPV\rangle \Delta E_3\Delta E_2$	(2)
	$\to \langle VP^3VPV\rangle \Delta E_2\Delta E_3$	(2)
	$\to \langle VP^3VPV\rangle \Delta E_1\Delta E_4$	(5)
	$\to -\langle VP^4VPV\rangle \Delta E_3 (\Delta E_1)^2$	(2)
	$\to -\langle VP^4VPV\rangle (\Delta E_2)^2 \Delta E_1$	(1)
	$\to -\langle VP^4VPV\rangle (\Delta E_1)^2 \Delta E_3$	(2)
	$\to -\langle VP^4VPV\rangle \Delta E_1 (\Delta E_2)^2$	(1)
	$\to -\langle VP^4VPV\rangle \Delta E_1 (\Delta E_2)^2$	(1)
	$\to -\langle VP^4VPV\rangle (\Delta E_1)^2 \Delta E_3$	(2)
	$\to \langle VP^5VPV\rangle \Delta E_2 (\Delta E_1)^3$	(1)
	$\to \langle VP^5VPV\rangle \Delta E_2 (\Delta E_1)^3$	(1)
	$\to \langle VP^5VPV\rangle (\Delta E_1)^3 \Delta E_2$	(1)
	$\to \langle VP^5VPV\rangle (\Delta E_1)^3 \Delta E_2$	(1)
	$\to -\langle VP^6VPV\rangle (\Delta E_1)^5$	(1)
1 6	$S_1S_6 = 42 \to -\langle VPVP^2V\rangle \Delta E_5$	(14)
	$\to \langle VPVP^3V\rangle \Delta E_1\Delta E_4$	(5)
	$\to \langle VPVP^3V\rangle \Delta E_2\Delta E_3$	(2)
	$\to \langle VPVP^3V\rangle \Delta E_3\Delta E_2$	(2)
	$\to \langle VPVP^3V\rangle \Delta E_4\Delta E_1$	(5)
	$\to -\langle VPVP^4V\rangle (\Delta E_1)^2 \Delta E_3$	(2)
	$\to -\langle VPVP^4V\rangle \Delta E_1 (\Delta E_2)^2$	(1)
	$\to -\langle VPVP^4V\rangle (\Delta E_1)^2 \Delta E_3$	(2)
	$\to -\langle VPVP^4V\rangle (\Delta E_2)^2 \Delta E_1$	(1)
	$\to -\langle VPVP^4V\rangle (\Delta E_2)^2 \Delta E_1$	(1)
	$\to -\langle VPVP^4V\rangle \Delta E_3 (\Delta E_1)^2$	(2)

Continued

$\to \langle VPVP^5V\rangle (\Delta E_1)^3 \Delta E_2$	(1)
$\to \langle VPVP^5V\rangle (\Delta E_1)^3 \Delta E_2$	(1)
$\to \langle VPVP^5V\rangle \Delta E_2 (\Delta E_1)^3$	(1)
$\to \langle VPVP^5V\rangle \Delta E_2 (\Delta E_1)^3$	(1)
$\to -\langle VPVP^6V\rangle (\Delta E_1)^5$	(1)

Table 6. $N=8$. Perturbation terms due to the highest partition number of $N-1=7$. Total number of the perturbation terms obtained in the present Table: $(42)+3\times(14)+(5)+(14)+(9)+(5)+(5)+(4)+(6)=(132)=S_7$. Total number of the perturbation terms in **Tables 4-6**: $(93)+(204)+132=(429)=S_8$.

1:7	$\to -\langle VP^2V\rangle \Delta E_6$	$\to S_6=(42)$
1:2:7	$\to \langle VP^3V\rangle \Delta E_1\Delta E_5$	$\to S_1S_5=(14)$
1:3:7	$\to \langle VP^3V\rangle \Delta E_2\Delta E_4$	$\to S_2S_4=(5)$
1:4:7	$\to \langle VP^3V\rangle \Delta E_3\Delta E_3$	$\to S_3S_3=(4)$
1:5:7	$\to \langle VP^3V\rangle \Delta E_4\Delta E_2$	$\to S_4S_2=(5)$
1:6:7	$\to \langle VP^3V\rangle \Delta E_5\Delta E_1$	$\to S_5S_1=(14)$
1:2:3:7	$\to -\langle VP^4V\rangle (\Delta E_1)^2 \Delta E_4$	$\to S_1^2S_4=(5)$
1:2:4:7	$\to -\langle VP^4V\rangle \Delta E_1\Delta E_2\Delta E_3$	$\to S_1S_2S_3=(2)$
1:2:5:7	$\to -\langle VP^4V\rangle \Delta E_1\Delta E_3\Delta E_2$	$\to S_1S_3S_2=(2)$
1:2:6:7	$\to -\langle VP^4V\rangle \Delta E_1\Delta E_4\Delta E_1$	$\to S_1S_4S_1=(5)$
1:3:4:7	$\to -\langle VP^4V\rangle \Delta E_1\Delta E_2\Delta E_3$	$\to S_1S_2S_3=(2)$
1:3:5:7	$\to -\langle VP^4V\rangle \Delta E_2\Delta E_2\Delta E_2$	$\to S_2^3=(1)$
1:3:6:7	$\to -\langle VP^4V\rangle \Delta E_2\Delta E_3\Delta E_1$	$\to S_2S_3S_1=(2)$
1:4:5:7	$\to -\langle VP^4V\rangle \Delta E_3\Delta E_1\Delta E_2$	$\to S_3S_1S_2=(2)$
1:4:6:7	$\to -\langle VP^4V\rangle \Delta E_3\Delta E_2\Delta E_1$	$\to S_3S_2S_1=(2)$
1:5:6:7	$\to -\langle VP^4V\rangle \Delta E_4 (\Delta E_1)^2$	$\to S_4S_1^2=(5)$
1:2:3:4:7	$\to \langle VP^5V\rangle (\Delta E_1)^3 \Delta E_3$	$\to S_1^3S_3=(2)$
1:2:3:5:7	$\to \langle VP^5V\rangle (\Delta E_1)^2 (\Delta E_2)^2$	$\to S_1^2S_2^2=(1)$
1:2:3:6:7	$\to \langle VP^5V\rangle (\Delta E_1)^2 \Delta E_2\Delta E_1$	$\to S_1^3S_3=(2)$
1:2:4:5:7	$\to \langle VP^5V\rangle \Delta E_1\Delta E_2\Delta E_1\Delta E_2$	$\to S_1^2S_2^2=(1)$
1:2:4:6:7	$\to \langle VP^5V\rangle \Delta E_1 (\Delta E_2)^2 \Delta E_1$	$\to S_1^2S_2^2=(1)$

Continued

1:2:5:6:7	$\to \langle VP^5V\rangle \Delta E_1 \Delta E_3 (\Delta E_1)^2$	$\to S_1 S_3 S_1^2 = (2)$
1:3:4:5:7	$\to \langle VP^5V\rangle \Delta E_2 (\Delta E_1)^2 \Delta E_2$	$\to S_2 S_1^3 S_2 = (1)$
1:3:4:6:7	$\to \langle VP^5V\rangle \Delta E_2 \Delta E_1 \Delta E_2 \Delta E_1$	$\to S_2 S_1 S_2 S_1 = (1)$
1:3:5:6:7	$\to \langle VP^5V\rangle (\Delta E_2)^2 (\Delta E_1)^2$	$\to S_2^2 S_1^2 = (1)$
1:4:5:6:7	$\to \langle VP^5V\rangle \Delta E_3 (\Delta E_1)^3$	$\to S_3 S_1^3 = (2)$
1:2:3:4:5:7	$\to -\langle VP^6V\rangle (\Delta E_1)^4 \Delta E_2$	$\to S_1^4 S_2 = (1)$
1:2:3:4:6:7	$\to -\langle VP^6V\rangle (\Delta E_1)^3 \Delta E_2 \Delta E_1$	$\to S_1^3 S_2 S_1 = (1)$
1:2:3:5:6:7	$\to -\langle VP^6V\rangle (\Delta E_1)^2 \Delta E_2 (\Delta E_1)^2$	$\to S_1^2 S_2 S_1^2 = (1)$
1:2:4:5:6:7	$\to -\langle VP^6V\rangle \Delta E_1 \Delta E_2 (\Delta E_1)^3$	$\to S_1 S_2 S_1^3 = (1)$
1:3:4:5:6:7	$\to -\langle VP^6V\rangle \Delta E_2 (\Delta E_1)^4$	$\to S_2 S_1^4 = (1)$
1:2:3:4:5:6:7	$\to \langle VP^7V\rangle (\Delta E_1)^6$	$\to S_1^6 = (1)$

actual paper—for an example. To this purpose we choose the calculation of $N = 6$ being the most developed case considered in Appendix of [34]. The mentioned data of Appendix are next compared with the corresponding data due to the present method; see **Table 7**.

On the left-hand side of **Table 7** are presented the symbols of the perturbation terms applied in the partition notation of the present paper, on the right-hand side of **Table 7** the method represented in Appendix of [34] is applied.

There exists a full agreement of the data obtained in the present paper with those taken from Appendix of [34].

10. Summary: General Properties of the Scale of Time Suitable to Calculate the Schrödinger Perturbation Energy

One of the fundamental processes of quantum mechanics is a change of a given system upon the action of some perturbation potential which—in its character—can be independent of time, but is dependent solely on the particle coordinates. To calculate the result of such a change acting on a non-degenerate quantum system, the Schrödinger perturbation formalism—represented by the sets of energy terms labelled by orders *N*—is required. In principle no time approach, or time parameter, should be used to this purpose.

We are guided, however, by the Leibniz idea that a suitable arrangement of the physical events along a time scale can be helpful in an analysis of any system change, including the perturbation effect. Consequently, by assuming that a change of a system—also due to the action of a time-independent perturbation potential—requires some interval of time, a sequence and origin of the time moments entering such interval can be of importance.

In principle there exist many ways according to which the necessary sets of

Table 7. Comparison of the energy terms calculated in Appendix of [34] with those obtained in the present paper: an example giving the terms belonging to the order $N=6$. The first 14 terms presented in the right-hand side column are calculated—according to [34]—automatically on the basis of the results obtained for $N=5$.

Number of the term	Partition symbol applied in the present method	Energy result obtained in [34]
1	1 1 1 1 1	$\langle VPVPVPVPVPV\rangle$
2	2 1 1 1	$-\langle VP^2VPVPVPV\rangle\Delta E_1$
3	3 1 1 (1st term)	$-\langle VP^2VPVPV\rangle\Delta E_2$
4	3 1 1 (2nd term)	$\langle VP^3VPVPV\rangle(\Delta E_1)^2$
5	1 2 1 1	$-\langle VPVP^2VPVPV\rangle\Delta E_1$
6, 7	4 1 (1st term)	$-\langle VP^2VPV\rangle\Delta E_3$
8	4 1 (2nd term)	$\langle VP^3VPV\rangle\Delta E_1\Delta E_2$
9	4 1 (3rd term)	$\langle VP^3VPV\rangle\Delta E_2\Delta E_1$
10	4 1 (4th term)	$-\langle VP^4VPV\rangle(\Delta E_1)^3$
11	1 3 1 (1st term)	$-\langle VPVP^2VPV\rangle\Delta E_2$
12	1 3 1 (2nd term)	$\langle VPVP^3VPV\rangle(\Delta E_1)^2$
13	1 1 2 1	$-\langle VPVPVP^2VPV\rangle\Delta E_1$
14	2 2 1	$\langle VP^2VP^2VPV\rangle\Delta E_1$
15-19	5 (1st term)	$-\langle VP^2V\rangle\Delta E_4$
20, 21	5 (2nd term)	$\langle VP^3V\rangle\Delta E_1\Delta E_3$
22	5 (3rd term)	$\langle VP^3V\rangle(\Delta E_2)^2$
23, 24	5 (4th term)	$\langle VP^3V\rangle\Delta E_3\Delta E_1$
25	5 (5th term)	$-\langle VP^4V\rangle(\Delta E_1)^2\Delta E_2$
26	5 (6th term)	$-\langle VP^4V\rangle\Delta E_1\Delta E_2\Delta E_1$
27	5 (7th term)	$-\langle VP^4V\rangle\Delta E_2(\Delta E_1)^2$
28	5 (8th term)	$\langle VP^5V\rangle(\Delta E_1)^4$
29, 30	1 4 (1st term)	$-\langle VPVP^2V\rangle\Delta E_3$
31	1 4 (2nd term)	$\langle VPVP^3V\rangle\Delta E_1\Delta E_2$
32	1 4 (3rd term)	$\langle VPVP^3V\rangle\Delta E_2\Delta E_1$
33	1 4 (4th term)	$\langle VPVP^4V\rangle(\Delta E_1)^4$
34	1 1 3 (1st term)	$-\langle VPVPVP^2V\rangle\Delta E_2$
35	1 1 3 (2nd term)	$\langle VPVPVP^3V\rangle(\Delta E_1)^2$

Continued

36	2 3 (1st term)	$\langle VP^2VP^2V\rangle \Delta E_1 \Delta E_2$
37	2 3 (2nd term)	$-\langle VP^2VP^3V\rangle (\Delta E_1)^3$
38	1 1 1 2	$-\langle VPVPVPVP^3V\rangle \Delta E_1$
39	2 1 2	$\langle VP^2VPVP^2V\rangle (\Delta E_1)^2$
40	3 2 (1st term)	$\langle VP^2VP^2V\rangle \Delta E_2 \Delta E_1$
41	3 2 (2nd term)	$-\langle VP^3VP^2V\rangle (\Delta E_1)^3$
42	1 2 2	$\langle VPVP^2VP^2V\rangle (\Delta E_1)^2$

terms representing the perturbation effect can be calculated. But we find that a circular scale of time—accompanied by the time-point contractions on it—can represent in full the necessary terms belonging to a particular order of the perturbation energy. These terms become obtainable on the basis of the time scale in an almost automatic way. Only the change of the perturbation order—associated with a change of the number of time points considered on the scale—implies a progressive action attributed to time.

A full presentation of the perturbation terms has been done for orders $N=7$ and $N=8$; the terms of the lower N are accessible in the literature presented before [25]-[34].

At the first sight it seems that the paper has only a purely mathematical background. In fact the aim is to solve a definite Schrödinger differential equation, but the way to do that is to solve first a presumably more simple equation. Next the solutions of that more simple equation should be combined into those belonging to a more complicated problem.

Both equations are assumed to be different by a potential change independent of time. In fact the time parameter neither enters the actual perturbation equation, nor the equation representing a former more simple problem. Nevertheless the change of the potential—equivalent to the change of the Hamiltonian operator between the unperturbed and perturbed equations—occupies some time. We assume the time of the potential change as negligibly small. A much more longer time, therefore of a non-negligible size, is expected to be occupied as an effect of the original potential change. This is so because the perturbed, *i.e.* originally unstable system, should wait to occupy one of its stationary states. In effect the time, required to make the perturbed system equivalent to a stationary object, can be long. An estimate of the size of that interval is beyond of our ability. Mathematically however, the both states, unperturbed and perturbed one, are both accessible and can be defined without any reference to the notion of time. So a question may arise what is the role of time—if any such role does exist—in the perturbation theory?

With the absence of any time intervals in the formalism, the answer is that time is an ordering parameter. Moreover this role is rather of a gradual character

because it does not concern the perturbation process as a whole, but is decisive in the successive steps of that process. In fact the perturbation effect can be separated into parts called the perturbation orders. Any order N is characterized by: 1) a definite number of the Huby-Tong kinds of the perturbation terms specified by the formula (21); 2) a constant number of $N-1$ terms P and number of N terms V entering any perturbation term belonging to the order N.

But beyond of the number of terms characteristic for a given N, an important role plays the sequence of the "collision" events of an unperturbed system with the perturbation potential. This sequence follows a closed scale of time equivalent to a topological circle. Any scale representing some Nth perturbation order has N time points on it. Among these points only one—called the beginning-end point—is free from contractions. Other time points on the scale, being $N-1$ in their number, can be submitted to contractions in a definite way. The contractions give the corresponding (closed) loops of time discussed in Sec. 4 and consequently the expected corrections to energy.

As an effect of contractions variable with N, the number of kinds of the perturbation terms increased by a single term due to an uncontracted loop of time characteristic for a given N, becomes equal to the number S_N presented in the formula (21). The arrangement of the time points in the contractions is providing us with the side loops of time corresponding to contractions. After combining the energy contributions due to the side loops with those given by the main loop of time, we arrive at the proper Schrödinger perturbation terms for energy of a given order N. No supplementary calculations are required to attain that purpose.

In fact the shape of the time scale, together with the time-point contractions done on it, play a decisive role in calculating all kinds of the Schrödinger perturbation terms entering a given order N. This result is not proved in an exact way but obtained in course of the systematic energy calculations belonging to the individual N's.

11. A Philosophical Background Concerning the Present Results

A philosophical background of the results obtained in the paper seems to be twofold. The first aim was to obtain a general look on the shape of the time scale. A principal point becomes here to get a real relevance of the question of the direction of time, or more simply the problem of sequence of the time events, to some physical process [35]. Let us note here an opinion that the theory of the whole world time is a redundant concept—one only needs a knowledge of world's possible configurations [36].

A reply in the present case is that if the scales belonging to individual N are considered, their shape is evidently a closed line. A characteristic circular-like character of the time scale suitable to calculations of the perturbation energy for a given N seems to be not a unique property in physics; see e.g. Rey [37] and Zawirski quoted in [38]. A much discussed reference which can be cited here is

the paper by Gödel [39]. In an analysis of cosmological solutions concerning the Einstein's field equations for gravitation he remarked that: (a) it is not possible to assign a time coordinate t to each space-time point in such a way that t always increases, if one moves in a positive time-like direction, and this holds for both for an open and a closed time coordinate; (b) every world line of matter occurring in the solution is an open line of an infinite length which approaches any of its preceding points again; (c) there also exist closed time-like lines.

In fact a total scale of time applied in the present paper—because of an increasing number of collisions with the potential V^{per} —does increase gradually with N. This makes the second step of the time way, *i.e.* that due to an increase of N, similar to an infinite linear scale referred in (1).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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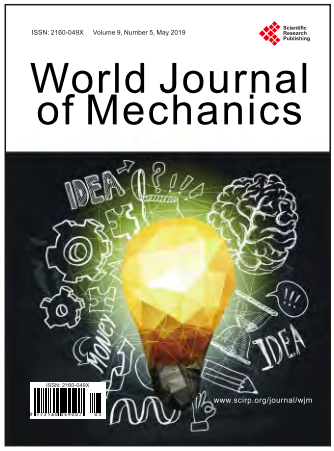

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