Capacity Choice in a Quantity-Setting Mixed Duopoly with Network Effects

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ABSTRACT

This paper explores the capacity choice for a public firm that is a social welfare-maximizer and a private firm that is an absolute profit-maximizer in the context of a quantity-setting mixed duopoly with a simple mechanism of network effects where the surplus that a firm’s client gets increases with the number of other clients of the firm. In this paper, we show that the social welfare-maximizing public firm chooses under-capacity irrespective of both the degree of product differentiation and strength of network effects, whereas the absolute profit-maximizing private firm chooses over-capacity irrespective of both the degree of product differentiation and strength of network effects, which is strikingly different from the results on the capacity choice problems for public and private firms obtained in price-setting mixed duopolistic markets in the existing literature.

KEYWORDS

Mixed Duopoly; Quantity Competition; Network Effects; Capacity Choice

1. Introduction

This paper investigates the capacity choice issue for a public firm that is a social welfare-maximizer and a private firm that is an absolute profit-maximizer in the context of a quantity-setting mixed duopoly with network effects where the surplus that a firm’s client gets increases with the number of other clients of the firm. Similar to the works on the capacity selection issues in private oligopolies composed of private firms only, such as the seminal works of Dixit [1] and Brander Spencer [2], studies exist in the context of mixed oligopolistic markets composed of both public and private firms that broadly investigate the capacity choice problems. Most recently, Nakamura [7] investigated the capacity choice problems in a price-setting mixed duopoly with network effects in the fashion of Katz and Shapiro [8] and Hoernig [9], and showed that a public firm chooses under-capacity, whereas the difference between the quantity and capacity levels of a private firm strictly depends on both the degree of product differentiation and strength of network effects. In this paper, in a quantity-setting mixed duopoly with network effects, we confirm the robustness of the results of the difference between the quantity and capacity choice problems of both the public and private firms obtained in price-setting mixed duopolistic markets in the existing literature.
capacity levels of both public and private firms in the case of price-setting mixed duopolies obtained by Nakamura [7]. More precisely, we ascertain whether or not the differences between the output and capacity levels of both the public firm and private firm depend on the strength of network effects à la Katz and Shapiro [8] and Hoernig [9] and the degree of product differentiation in a quantity-setting mixed duopoly.

In this paper, we show that in quantity competition with the network effects à la Katz and Shapiro [8], Hoernig [9], and Nakamura [7], a social welfare-maximizing public firm chooses under-capacity irrespective of the degree of product and strength of network effects, whereas an absolute profit-maximizing private firm chooses over-capacity irrespective of the degree of product differentiation and strength of network effects, which is strikingly different from the results in price competition obtained in Nakamura [7]. The intuition behind the result that the difference between the quantity and capacity levels of a social welfare-maximizing firm is always positive can be explained as follows: In quantity competition with network effects, regardless of the degree of product differentiation and strength of network effects, a public firm attempts to increase the quantity level of the rival private firm in order to enhance social welfare. Then public firm attempts to increase the quantity level of the private firm through the following two effects: 1) the negative association between the quantity of the private firm and its own capacity level and 2) the combination of the strategic substitutability between the capacity levels of both the firms and the positive association between the quantity and capacity levels of the private firms. From the two effects, the public firm has a strong incentive to refrain from increasing its own capacity level, implying that a public firm always chooses under-capacity irrespective of the degree of product differentiation and strength of network effects. On the contrary, in the quantity competition, the private firm has a strong incentive to use its capacity level strategically in order to expand its market share, since it must compete with the social welfare-maximizing public firm that is a strong competitor in the market. More precisely, the private firm attempts to increase its quantity level by raising its own capacity level through the following two effects: 1) the negative association between the quantity level of the public firm and its own capacity level, and 2) the combination of the strategic relation between the capacity levels of both the firms and the positive association between the quantity and capacity levels of the private firm. Consequently, the private firm always chooses over-capacity irrespective of the degree of product differentiation and the strength of network effects.

The remainder of this paper is organized as follows. In Section 2, we formulate a quantity-setting mixed duopolistic model with capacity choice of both the social welfare-maximizing public firm and the absolute profit-maximizing private firm with network effects à la Katz and Shapiro [8] and Hoernig [9]. In Section 3, we consider the difference between the quantity and capacity levels of both the public firm and private firm. Section 4 concludes the paper with several remarks.

2. Model

We formulate a quantity-setting competition model in a mixed duopoly with the capacity choice of both a public firm and a private firm and with an additional term that reflects network effects in the fashion of Katz and Shapiro [8] and Hoernig [9].

We assume that firm 0 is a public firm that is a welfare-maximizer whereas firm 1 is a private firm that is an absolute profit-maximizer. Similar to Hoernig [9], firm $i$ faces a linear demand of the following form:

$$q_i(p_i, p_j, y_i, y_j) = a + ny_i - p_i + bp_j, \quad i = 0, 1; i \neq j,$$

where $a > 0$ and $b \in (0, 1)$ are demand parameters. $n \in [0, 1]$ indicates the strength of network effects, and $y_i$ is the consumers’ expectation on firm $i$’s equilibrium market share. This specification implies the following inverse demand functions for positive demand:

$$p_i(q_i, q_j; y_i, y_j) = \left[ a(1 + b) - q_i - bq_j + ny_i + bny_j \right]/(1 - b^2), \quad i, j = 0, 1; i \neq j.$$

As explained in Hoernig [9], the above demand system can be derived from the following quasi-linear concave utility function of a representative consumer:

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1. In another strand of research on the capacity choice problems of public and private firms in a mixed duopoly, Tomaru et al. [10] analyzed the influence of the separation between ownership and management in the fashion of Fershtman and Judd [11], Sklivas [12], and Vickers [13] on the difference between the output and capacity levels of both the public and the private firms and the changes therein before and after privatization of the public firm. Furthermore, more recently, Nakamura and Saito [14] and Nakamura and Saito [15] investigated the capacity choice of a public firm that is a social welfare-maximizer and a private firm that is a relative profit-maximizer in the context of quantity-setting and price-setting mixed duopolies, respectively; their most important contribution in these two papers is to show that even though the relation between the goods produced by the two firms is restricted to being substitutable, the difference between the output and capacity levels of the private firm can change in accordance with the degree of importance of its relative performance.

2. In the model employed in this paper, the parameters are $a$, $b$, $c$, and $n$. The economic meanings of such parameters are given in the following body of this paper.
where $m$ denotes the income of the representative consumer and $f(\cdot)$ represents some symmetric expectation function. In this paper, as in Hoernig [9], we suppose that $f(y, y_i) = -n(y_0^2/2 + b y_0 y_i + y_i^2/2)/(1 - b^2)^6 .^6$

We further suppose that both firms adopt identical technologies represented by cost function $C(q, x_i)$, where $x_i$ is the capacity level of firm $i$ ($i = 0, 1$). Following Vives [16], Ogawa [5], Barcena-Ruiz and Garzon [6], Tomaru et al. [10], Nakamura and Saito [14], and Nakamura and Saito [15], we assume that the cost function is given by $C(q, x_i) = cq_i + (q_i - x_i).^7$ This cost function implies that if each firm’s output level equals its capacity level, $q_i = x_i$, then the long-run average cost is minimized. The profit of firm $i$ is given by $\Pi_i = p_i q_i - C_i(q_i, x_i)$, ($i = 0, 1$). Consumer surplus as the representative consumer utility is represented as follows: $CS = U(q_0, q_1, y_0, y_1) - p_0 q_0 - p_1 q_1$, whereas producer surplus is given by the sum of the profits of both firms 0 and 1, $\pi_0 + \pi_1$. Finally, we suppose that social welfare in this paper is equal to the sum of consumer surplus and producer surplus.

We investigate the game with the following two stages: In the first stage, firms 0 and 1 simultaneously set their capacity levels. In the second stage, after both the firms observe each other's capacity level, they engage in a quantity-setting competition. In the fashion of Hoernig [9], we consider the subgame perfect Nash equilibrium presented in Katz and Shapiro [8] as our equilibrium concept. Thus, in the equilibrium, we derive the subgame perfect Nash equilibrium under the additional “rational expectations” assumption: $y_0 = q_0$ and $y_i = q_i$.

3. Equilibrium Analysis

We solve the game by backward induction from the second stage to obtain the rational expectations subgame perfect Nash equilibrium. In the second stage, firm 0 maximizes social welfare $W$ with respect to $q_0$, whereas firm 1 maximizes its absolute profit $\pi_1$ with respect to $q_1$. The best-response functions of both firms 0 and 1 in the second stage are given as follows:

$$q_0(q_0, y_0, y_1, x_0) = \frac{a(1 + b) - c + 2 x_0 + n y_0 - b [q_0 - b (c - 2 x_0)] - n y_1}{3 - 2 b^2} \quad (1)$$

$$q_1(q_1, y_0, y_1, x_1) = \frac{a(1 + b) - c + 2 x_1 - b [q_0 - b (c - 2 x_1)] - n y_0 + n y_1}{4 - 2 b^2} \quad (2)$$

From Equations (1) and (2), we find that for any strength of network effects and degree of product differentiation, $n$, $q_i$ is decreasing in $q_j$, and thus the quantity levels of both firms 0 and 1 are strategic substitutes ($i, j = 0, 1; i \neq j$).

Furthermore, we obtain the rational expectations Nash equilibrium of the quantity-setting stage by substituting the two conditions $y_0 = q_0$ and $y_1 = q_1$ into the best-response functions of both firms 0 and 1. Then, we obtain

$$q_0(x_0, x_1) = \frac{(1 + b) [a - (1 - b) c (4 - 2 b^2 - n + b n) + 2 (1 - b^2) (4 - 2 b^2 - n) x_0 - 2 b (1 - b^2) (1 - n) x_1]}{(4 - n) (3 - n) - b^2 [15 - (6 - n) n] + 4 b^4}, \quad (3)$$

$$q_1(x_0, x_1) = \frac{(1 - b^2) [a (3 + 2 b - n) - (1 - b) c (3 + 2 b - n) - 2 b (1 - n) x_0 + 2 (3 - 2 b^2 - n) x_1]}{(4 - n) (3 - n) - b^2 [15 - (6 - n) n] + 4 b^4}. \quad (4)$$

In the first stage, both firms 0 and 1 know that their capacity choice affects their quantity levels in the second stage. Given Equations (3) and (4), firms 0 and 1 simultaneously and independently set their capacity levels with respect to social welfare and own absolute profit, respectively. Thus, by solving the first-order conditions of firms 0 and 1 in the first stage, we have

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6This assumption on the form of $f(\cdot)$ implies that the representative consumer’s utility is the highest with respect to the consumption vector of the goods produced by both the public firm and private firm, $(q_0, q_1)$, when expectations are correct.

7We assume that $a/(1 - b) > c > 0$ in order to ensure the non-negativity of all equilibrium outcomes.

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\[ x_0 = R_0(x_1, b, n) \]
\[ (1 + b) \left[ (a - (1 - b)c) \left[ (4 - n)^2 (3 - n) - b (15 - 23n + 9n^2 - n^3) - b^2 (83 - 51n + 11n^2 - n^3) + b^3 (17 - 23n + 7n^2 - n^3) \right] \right] + 2b^2 \left( 23 - 8n + n^2 \right) - 4b^2 (1 - n) + 8b^3 \] 
\[ = \frac{2b^2 \left( 23 - 8n + n^2 \right) - 4b^2 (1 - n) + 2b \left( 15 - 23n + 9n^2 - n^3 - b (17 - 23n + 7n^2 - n^3) + 4b^4 (1 - n) \right) x_1}{(1 - n) \left[ (4 - n)^2 (3 - n) - 2b^2 \left( 49 - 37n + 10n^2 - n^3 \right) + b^2 \left( 63 - 39n + 9n^2 - n^3 \right) - 4b^3 (3 - n) \right]} \] 

(5)

\[ x_1 = R_1(x_0; b, n) \]
\[ = \frac{2 \left( 2 - 3b^2 + b^3 \right) \left( 3 - 2b^2 - n \right) \left[ a - (1 - b)c \left[ (3 + 2b - n) - 2b (1 - n) x_0 \right] \right]}{(3 - n)^2 (8 - 8n + n^2) - 2b^2 \left( 78 - 125n + 63n^2 - 13n^3 + n^4 \right) + b^4 \left( 109 - 164n + 70n^2 - 12n^3 + n^4 \right) - 8b^6 \left( 3 - 4n + n^2 \right)} \]

(6)

from easy calculations, we obtain the following results on the difference between the output and capacity levels of both firms 0 and 1:

\[ q_0^x - x_0^x = \frac{2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)}{24 - 32n + 11n^2 - n^3 - 2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)} \]

\[ q_1^x - x_1^x = \frac{2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)}{24 - 32n + 11n^2 - n^3 - 2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)} \]

From easy calculations, we obtain the following results on the difference between the output and capacity levels of both firms 0 and 1:

\[ q_0^y - x_0^y = \frac{b \left( 1 - b^2 \right) \left[ a - (1 - b)c \right] \left( 3 - n \right) n + b^2 \left( 1 + 4n^2 \right)}{24 - 32n + 11n^2 - n^3 - 2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)} > 0, \quad \forall b \in (0, 1) \text{ and } \forall n \in [0, 1) \]

\[ q_1^y - x_1^y = \frac{\left( 1 - b^2 \right) \left[ a - (1 - b)c \right] \left( 3 - n \right) n + b^2 \left( 1 + 4n^2 \right)}{24 - 32n + 11n^2 - n^3 - 2b^2 \left( 23 - 31n + 10n^2 - n^3 \right) + b^4 \left( 27 - 35n + 9n^2 - n^3 \right) - 4b^5 (1 - n)} < 0, \quad \forall b \in (0, 1) \text{ and } \forall n \in [0, 1) \]

Thus, we recognize that the social welfare-maximizing public firm 0 chooses under-capacity irrespective of the strength of network effects, \( n \), and demand parameter, \( b \), whereas an absolute profit-maximizing private firm 1 always chooses over-capacity. By summing the above two facts, we obtain the following proposition on the differences between the quantity and capacity levels of both firms 0 and 1.

**Proposition 1** Social welfare-maximizing public firm 0 chooses under-capacity, \( q_0^x > x_0^x \), for any value of the de-
mand parameter \( b \in (0, 1) \) and any strength of network effects \( n \in [0, 1) \). In contrast, absolute profit-maximizing private firm 0 chooses over-capacity, \( q_i^0 < x_i^0 \), for any value of the demand parameter \( b \in (0, 1) \) and any strength of network effects \( n \in (0, 1) \).

Before we state the intuition behind Proposition 1, we need to confirm the strategic relation between the capacity levels of firms 0 and 1. From easy calculations, we obtain the following results:

\[
\begin{align*}
\frac{\partial R_n(x_i^0; b, n)}{\partial x_i^0} &= \frac{2(1-b)b(1+b)[15-8n+n^2-b^2(17-6n+n^2)+4b^4]}{(4-n)^2(3-n)-2b^2(49-37n+10n^2-n^3)+b^4(63-39n+9n^2-n^3)-4b^6(3-n)} < 0, \\
\frac{\partial R_n(x_0; b, n)}{\partial x_0} &= \frac{4b(2-3b^2+b^4)(1-n)(3-2b^2-n)}{(3-n)^2(8-8n+n^2)-2b^2(78-125n+63n^2-13n^3+n^4)+b^4(109-164n+70n^2-12n^3+n^4)-8b^6(3-4n+n^2)} < 0.
\end{align*}
\]

Thus, we find that the capacity levels of firms 0 and 1 are strategic substitutes. First, we state the intuition on why the difference between the quantity and capacity levels of public firm 0 is always positive irrespective of both the strength of network effects \( n \) and degree of product differentiation \( b \). Firm 0 attempts to increase the quantity level of firm 1 in order to raise the equilibrium social welfare. Thus, the less aggressive behavior on the capacity setting of firm 0 is explained by the following two effects: 1) the negative association between the quantity level of firm 1 and the capacity level of firm 0, which is described in Equation (4), and 2) the strategic substitutability between the capacity levels of firms 0 and 1 and the positive association between the quantity and capacity levels of firm 1. More precisely, from 1), firm 0 can directly increase the quantity level of firm 1 by refraining from increasing its capacity level, and from 2), firm 0 can increase the quantity of firm 1 through raising the capacity level of firm 1 by refraining from increasing its capacity level. Consequently, firm 0 always chooses under-capacity irrespective of the degree of product differentiation \( b \) and strength of network effects \( n \), which is strikingly different from the result that a social welfare-maximizing public firm chooses over-capacity in a price-setting mixed duopoly with network effects as considered in Nakamura [7]. In contrast, absolute profit-maximizing private firm 1 attempts to expand its market share by increasing its own capacity level through 1) the positive association between the quantity and capacity levels of firm 1 and 2) the strategic substitutability between the capacity levels of firms 0 and 1 and the negative association between the quantity level of firm 0 and the capacity level of firm 1. This result is also different from that obtained in price competition with network effects, which was investigated in Nakamura [7]. In particular, in the case of price competition with network effects, it was shown that the difference between the quantity and capacity levels of absolute profit-maximizing private firm 1 strictly depends on both the product differentiation, \( b \), and the strength of network effects, \( n \). This indicates that the difference in the quantity and capacity levels of a social welfare-maximizing public firm and an absolute profit-maximizing firm strictly changes on the basis of the competition style in a duopoly with network effects.

In sum, as compared with the results on the difference between the quantity and capacity levels of both the public firm and private firm in price competition with network effects, we find that the differences between their quantity and capacity levels obtained in quantity competition are more simple. In addition, in the case of quantity competition with substitutable goods and network effects, as considered in this paper, the differences between the quantity and capacity levels of a social welfare-maximizing public firm and an absolute profit-maximizing private firm are the same as those obtained in the case of a quantity-setting mixed duopoly without any network effects as explored in Ogawa [5]. Therefore, in quantity competition with substitutable goods, the existence of a network effect à la Katz and Shapiro [8] and Hoernig [9] does not influence the difference between the quantity and capacity levels of both the social welfare-maximizing public firm and the absolute-profit maximizing private firm.

4. Conclusions

This paper explored the capacity choice for a public firm that is a social welfare-maximizer and a private firm that is an absolute profit-maximizer in the context of a quantity-setting mixed duopoly with network effects à la Katz and Shapiro [8] and Hoernig [9]. In price competition with network effects which was investigated in Nakamura [7], the social welfare-maximizing public firm chooses over-capacity irrespective of the strength of network effects and product differentiation and the difference between the quantity and capacity levels of the absolute profit-maximizing private firm strictly depends on both the degree of product differentiation and strength of net-
work effects.

In contrast, in quantity competition with network effects, which we considered in this paper, the social welfare-maximizing public firm chooses under-capacity irrespective of the degree of product differentiation and strength of network effects, whereas the absolute profit-maximizing private firm chooses over-capacity irrespective of the degree of product differentiation and strength of network effects. The intuition behind these results are as follows: On the difference between the quantity and capacity levels of the social welfare-maximizing public firm, the firm attempts to increase the quantity level of the rival private firm in order to enhance social welfare. More concretely, regardless of the degree of product differentiation and strength of network effects, the social welfare-maximizing public firm tries to increase the quantity level of the private firm by refraining from increasing its own capacity level, directly through the negative association between the quantity level of the private firm and its own capacity and indirectly through the strategic substitutability between the capacity levels of the firms and the positive association between the quantity and capacity levels of the private firm. In addition, the private firm tends to increase its capacity level through the positive association between its quantity and capacity levels, and through the combination of the strategic substitutability between the capacity levels of both the firms and the positive association between its own quantity and capacity levels in order to be competitive with the social welfare-maximizing public firm, implying that the private firm chooses over-capacity regardless of the degree of product differentiation and strength of network effects.

Finally, we mention an open problem that we need to tackle in the future. Throughout this paper, we considered the absolute profit as the objective of a private firm. In some sort of new papers, we should explore the results on the capacity choice issues between social welfare-maximizing public firms and absolute profit-maximizing private firms on the basis of the strength of network effects and degree of product differentiation under the assumption that the objective function of the private firms is its relative profit.

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