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Helix Angle Effect on the Helical Gear Load Carrying Capacity

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Abstract

The aim of this study is to investigate the helix angle effect on the helical gear load carrying capacity, including the bending and contact load carrying capacity. During the simulation, the transverse contact ratio is calculated with respect to the constant pressure angle. By changing the helix angle, both the overlap contact ratio and total contact ratio are calculated and simulated. The bending stress and contact stress of a helical gear are calculated and simulated with respect to the helix angle. Solid (CAD) modelling of a pinion gear was obtained using SOLIDWORKS software. The analytically obtained results and finite elements method results are compared. It is observed that increasing the helix angle causes an increase of the contact ratio of the helical gear. Furthermore, increasing the contact ratio reduces the bending stress and contact stress of the helical gear. However, with a constant transverse contact ratio, it is possible to improve the total contact ratio depending on the helix angle. It is concluded that a higher helix angle increases the helical gear bending and contact load carrying capacity.

Keywords

Helical Gear, Helix Angle, Contact Ratio, Bending Stress, Contact Stress

1. Introduction

Gears are widely used to mechanically transmit power in automotive transmissions. The aim of the gears is to couple two shafts together; the rotation of the drive-shaft is a function of the rotation of the drive-shaft in the gear mechanism. Therefore, determining the geometric design parameters of gears is crucial.

The contact ratio is an important parameter for successful gear design. The helix angle is considered to be an effective parameter to increase the contact ratio of a helical gear. Thus, it is possible to increase the helical gear load carrying capacity, including the tooth bending stress and tooth contact stress.

One of the disadvantages of increasing the helix angle is the axial forces caused on the helical gear mechanism.

Optimisation of effective design parameters to reduce the tooth bending stress in an automotive transmission gearbox is presented. Therefore, the contact ratio effect on the tooth bending stress by changing the contact ratio with respect to the pressure angle is analysed [1]. It is concluded that a higher contact ratio results in reduced tooth bending stress, while a higher pressure angle and decreased contact ratio caused an increase in tooth bending stress and contact stress [1].

When the helix angle is increased from 15 [˚] to 35 [˚], the corresponding bending stress and compression stress decrease [2].

It was concluded that a helix angle increase had significant effects on the tooth-root bending stress and tooth compressive stress. Moreover, it was observed that when the helix angle increased from 0 [˚] to 22.5 [˚], both the bending stress and compression stress were reduced approximately 10% [3].

For a given number of teeth, a smaller pressure angle may produce an undercut. However, the contact ratio increases, so the load carrying capacity may improve as the load is distributed along a longer line of contact [4].

The contact ratio for a helical gear pair increases with the helix angle, which generates the screwed surface of the tooth face [5].

The aim of this study is to investigate the helix angle effect on the helical gear load carrying capacity, including the bending and contact load carrying capacity. For this aim the analytically obtained results and finite elements method results are compared. It is concluded that a higher helix angle increases the helical gear bending and contact load carrying capacity.

2. Method

2.1. Pinion and Wheel Gears Mechanism

In the proposed pinion and wheel gear mechanism, all pinion and wheel gears are helical and are made of 16MnCr5.

2.1.1. Helix Angle β

The helix angle, β, is the angle between the helix line and horizontal axis, as shown in **Figure 1**.

2.1.2. Contact Ratio

The dimensions of the helical gear are shown in **Figure 2**, and the contact line of the helical gear is shown in **Figure 3**.

A second pair of mating teeth should come into contact before the first pair is out of contact during pinion and wheel gear running [6].

If the gear contact ratio is equal to 1, one tooth is leaving contact just as the next tooth is beginning contact. If the gear contact ratio is larger than 1, load sharing among the teeth is possible during pinion and wheel gear running [7].

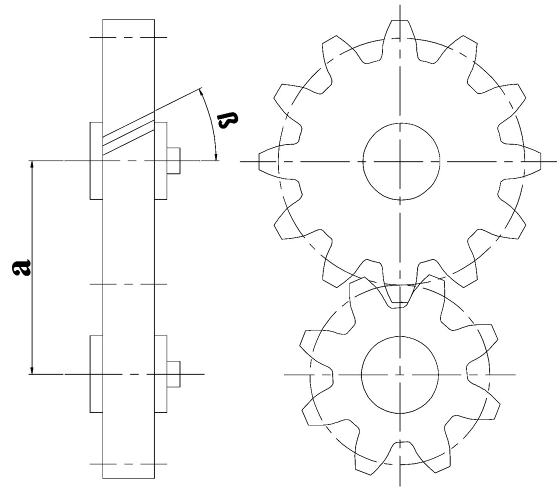

Figure 1. Pinon and wheel gear mechanism.

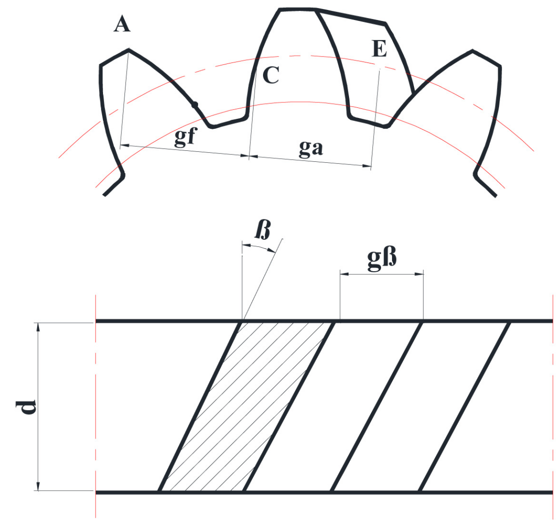

Figure 2. Dimensions of a helical gear.

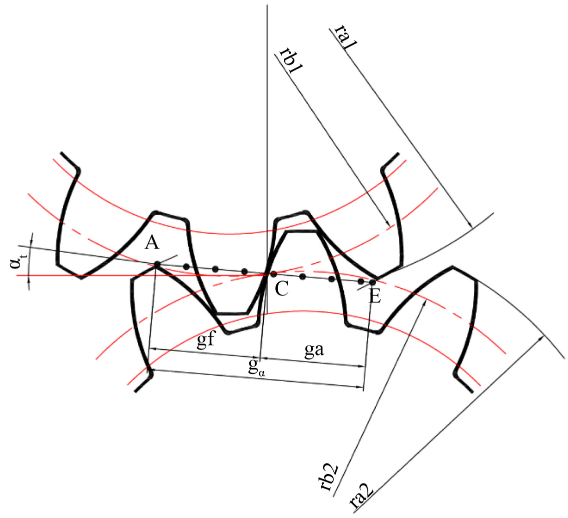

Figure 3. Contact line of a helical gear.

When the contact ratio is equal to 2 or more, at least two pairs of teeth are theoretically in contact [7].

If the gear profile contact ratio is less than 2.0, it is called the Low Contact Ratio (LCR). If he gear profile contact ratio equals 2.0 or greater, it is called the High Contact Ratio (HCR).

The contact ratio consists of two parts, such as the transverse contact ratio, ε_α, and the overlap or face contact ratio, ε_β.

1) *Transverse contact ratio* ε_α

It is well-known that the average number of teeth that are in contact as the gears rotate is the contact ratio (CR). The contact ratio is calculated from the following equation.

The transverse contact ratio, ε_α, is calculated as follows [8] [9] [10] [11].

$$\varepsilon_\alpha = \frac{g_\alpha}{p_{et}} \tag{1}$$

$$\varepsilon_\alpha = \frac{0.5 \cdot \left(\sqrt{d_{a1}^2 - d_{b1}^2} + \sqrt{d_{a2}^2 - d_{b2}^2}\right) - a_d \cdot \sin \alpha_t}{\pi \cdot m_t \cdot \cos \alpha_t} \tag{2}$$

where g_α is the path length of the contact line [mm], p_{et} is the base pitch [mm], d_{a1} is the addendum circle diameter of the pinion gear [mm], d_{b1} is the base circle diameter of the pinion gear [mm], d_{a2} is the addendum circle diameter of the wheel gear [mm], d_{b2} is the base circle diameter of the wheel gear [mm], a_d is the centre distance [mm], α_t is the transverse pressure angle [˚], and m_t is the transverse module [mm].

2) *Overlap ratio* ε_β

The overlap ratio, ε_β, is calculated as follows [8] [9] [10] [11].

$$\varepsilon_\beta = \frac{U}{p_t} \tag{3}$$

$$\varepsilon_\beta = \frac{b \cdot \tan \beta}{p_t} \tag{4}$$

$$\varepsilon_\beta = \frac{b \cdot \sin \beta}{\pi \cdot m_n} \tag{5}$$

where U is the action length [mm], p_t is the transverse pitch [mm], b is the face width [mm], and m_n is the normal module [mm].

3) *Total contact ratio* ε_γ

The total contact ratio, ε_γ is calculated as follows.

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \tag{6}$$

where ε_α is the transverse contact ratio and ε_β is the overlap ratio. Helical gears have higher load carrying capacities than spur gears because their contact ratios are larger than those of spur gears.

2.2. Calculating the Load Carrying Capacity of Helical Gears

2.2.1. Nominal Tangential Load

The nominal tangential load F_t is calculated as follows.

$$F_t = \frac{2 \cdot T_L}{d_1} \tag{7}$$

where T_L is the applied torque [N·mm] and d_1 is the base diameter of the tooth diameter [mm].

2.2.2. Axial Load

Axial load F_a is calculated as follows.

$$F_a = \frac{F_t}{\tan\beta} \tag{8}$$

where β is helix angle [˚].

2.2.3. Tooth Bending Stress

The real tooth-root stress, σ_F is calculated as follows [8] [9] [10] [11] [12]. The bending stress of the tooth-root is shown in **Figure 4**.

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} \tag{9}$$

where F_t is the nominal tangential load [N], b is the face width [mm], m_n is the normal module [mm], Y_F is the form factor [-], Y_s is the stress correction factor [-], Y_ε is the contact ratio factor [-], K_A is the application factor [-], K_V is the internal dynamic factor [-], $K_{F\beta}$ is the face load factor for tooth-root stress [-] and $K_{F\alpha}$ is the transverse load factor for tooth-root stress [-]. The safety factor for bending stress S_F is calculated as follows [8] [9] [10] [11] [12].

$$S_F = \frac{\sigma_{Fp}}{\sigma_F} \tag{10}$$

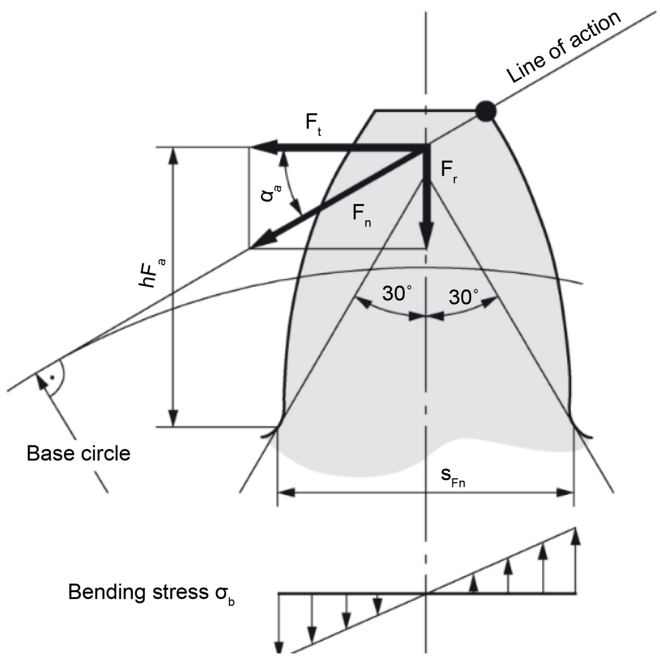

Figure 4. Bending stress at the tooth root.

where σ_{Fp} is the permissible bending stress.

2.2.4. Tooth Contact Stress

The real contact stress, σ_H is calculated as follows [8] [9] [10] [11] [12]. The contact stress at the tooth flank is shown in **Figure 5**.

$$\sigma_H = \sqrt{\frac{F_t}{bm_n}\frac{u+1}{u}} Z_H Z_E Z_\varepsilon Z_\beta \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{11}$$

where u is the gear ratio [-], Z_H is the zone factor [-], Z_E is the elasticity factor [ ], Z_ε is the contact ratio factor [-], Z_β is the helix angle factor [-], $K_{H\beta}$ is the face load factor for contact stress [-] and $K_{H\alpha}$ is the transverse load factor for contact stress [-].

The safety factor for contact stress, S_H is calculated as follows [8] [9] [10] [11] [12].

$$S_H = \frac{\sigma_{Hp}}{\sigma_H} \tag{12}$$

where σ_{Hp} is the permissible contact stress.

2.3. Finite Elements Model

Solid (CAD) modelling of a pinion gear was obtained using SOLIDWORKS software. A solid model is essential for finite element method (FEM) analysis [13] [14]. Solid (CAD) modelling of a pinion gear is shown in **Figure 6**.

Finite Element Analysis

The obtained Solid (CAD) model is used to obtain the finite element method (FEM) model using the SOLIDWORKS finite element tool.

1) *Boundary conditions*

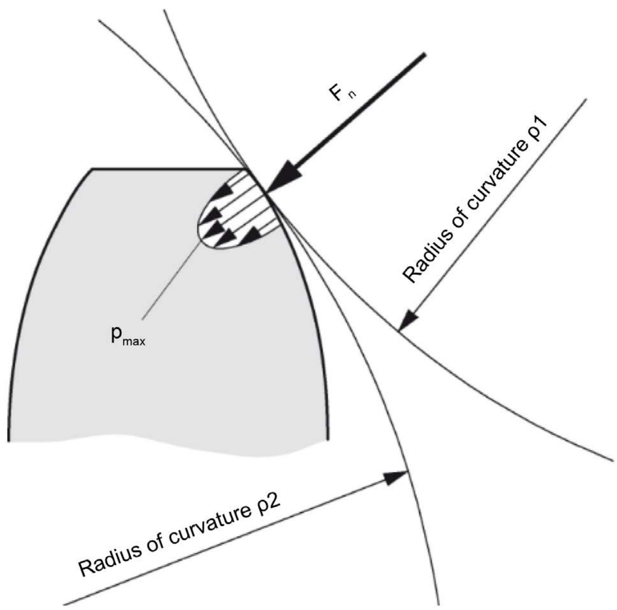

Figure 5. Contact stress at the tooth flank.

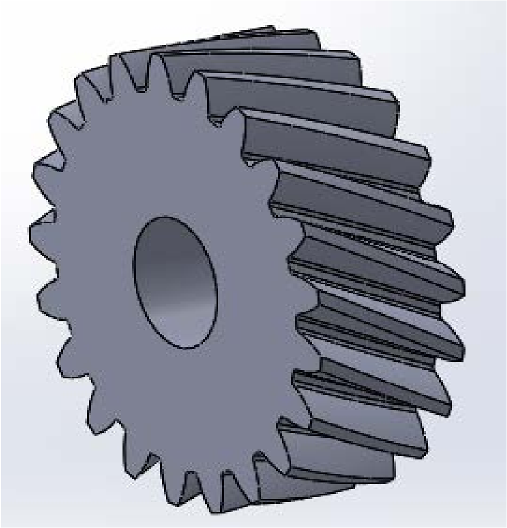

Figure 6. Solid (CAD) modelling of a pinion gear.

To simulate the actual conditions of the pinion gear for analysis, the boundary conditions below were used.

a) The pinion gear was constrained in the centre of the pinion gear.

b) The applied load for bending the pinion gear tooth was considered at the tooth top surface.

3. Numerical Example

During simulation, the tooth bending stress and tooth contact stress were calculated according to ISO 6336. The effects of the helix angle on the tooth bending stress and tooth contact stress are analysed by varying the helix angle. The tooth bending stress and tooth contact stress parameters are shown in Table 1.

The tooth bending stress and tooth contact stress simulation results are shown in Table 2.

3.1. Helix Angle and Overlap Contact Ratio Relation

The helix angle and overlap contact ratio relation are shown in Figure 7. As the helix angle increases from 22 [˚] to 32 [˚], the overlap contact ratio increases from 1.01 [-] to 1.43 [-].

3.2. Helix Angle and Total Contact Ratio Relation

The helix angle and total contact ratio relation are shown in Figure 8. As the helix angle increases from 22 [˚] to 32 [˚], the total contact ratio increases from 2.52 [-] to 2.88 [-].

3.3. Helix Angle and Bending Stress Relation

The helix angle and tooth bending stress relation are shown in Figure 9. As the helix angle decreases from 22 [˚] to 12 [˚], the bending stress is reduced from 365 [N/mm^2] to 233 [N/mm^2].

Table 1. Tooth bending stress and tooth contact stress parameters.

Parameters	Value	Unit
Torque T_L	260×10^3	[N·mm]
Gear ratio u	1.814	[-]
Module m	4	[mm]
Number of teeth z	21	[-]
Face width b	34	[mm]
Form factor Y_F	2.87	[-]
Stress correction factor Y_S	1.6	[-]
Application factor K_A	1.25	[-]
Dynamic factor K_V	1.14	[-]
Transverse load factor $K_{F\alpha}$	1.2	[-]
Nominal stress number σ_{FLim}	350	[N/mm^2]
Life factor Y_N	1	[-]
Relative notch sensitivity factor Y_δ	1	[-]
Relative surface factor Y_R	1	[-]
Size factor Y_X	1	[-]
Zone factor Z_H	2.36	[-]
Elasticity factor Z_E	189.8	[N/mm^2]$^{1/2}$
Transverse load factor $K_{H\alpha}$	1.2	[-]
Allowable stress number σ_{HLim}	1400	[N/mm^2]
Life factor Z_N	1	[-]
Roughness factor Z_R	1	[-]
Work hardening factor Z_W	1	[-]
Size factor Z_X	1	[-]

Table 2. Tooth bending stress and tooth contact stress simulation results.

Helix angle β [°]	Overlap contact ratio ε_β [-]	Total contact ratio ε_γ [-]	Tooth bending stress σ_F [N/mm^2]	Tooth contact stress σ_H [N/mm^2]
22	1.01	2.52	365	1265
24	1.10	2.60	340	1241
26	1.19	2.67	314	1215
28	1.27	2.74	287	1188
30	1.35	2.81	260	1159
32	1.43	2.88	233	1128

3.4. Helix Angle and Tooth Contact Stress Relation

The helix angle and contact stress relation are shown in Figure 10. As the helix angle decreases from 22 [˚] to 12 [˚], the bending stress is reduced from 1265 [N/mm^2] to 1128 [N/mm^2].

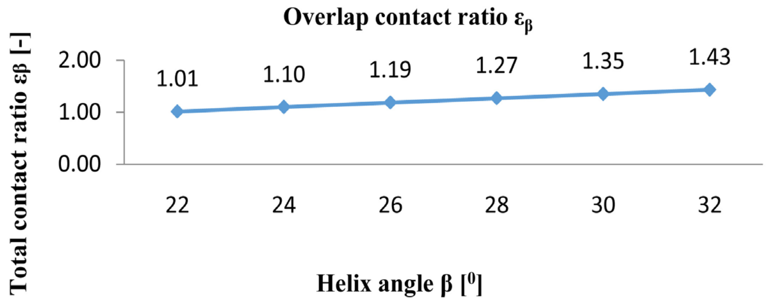

Figure 7. Helix angle and overlap contact ratio relation.

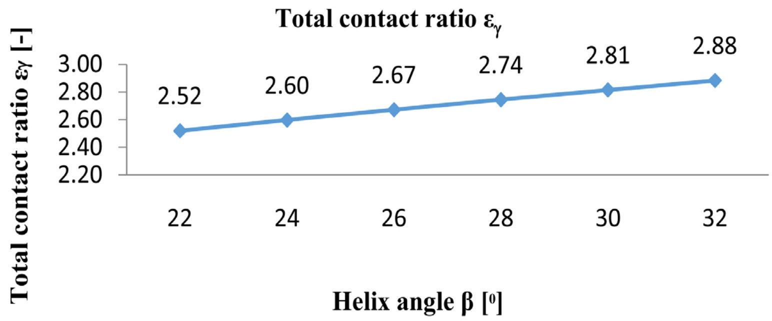

Figure 8. Helix angle and total contact ratio relation.

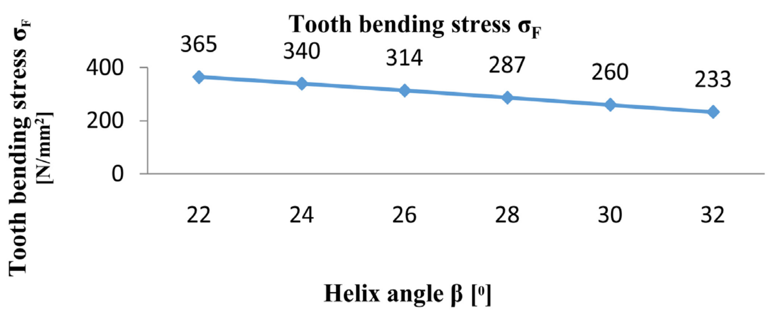

Figure 9. Helix angle and tooth bending stress relation.

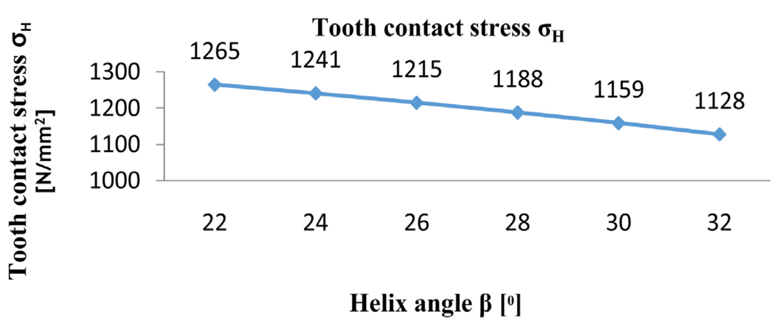

Figure 10. Helix angle and tooth contact stress relation.

3.5. Helix Angle and Axial Force Relation

The helix angle and axial force relation is shown in Figure 11. As the helix angle decreases from 22 [˚] to 12 [˚], the axial force is increased from 2319 [N] to 3280 [N].

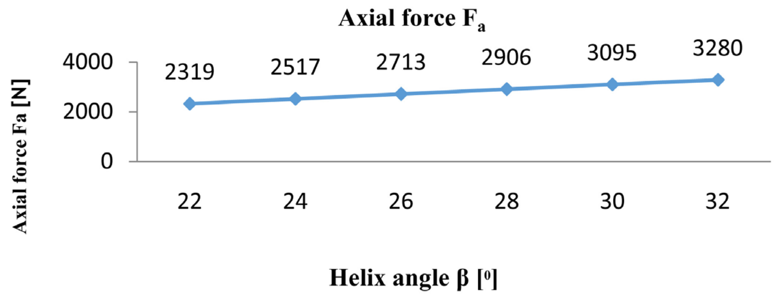

Figure 11. Helix angle and axial force relation.

3.6. Static Structural Analysis with FEM

Static structural analysis of the pinon gear was completed for the applied load considering the Von Mises stress. The Von Mises stress is written as follows.

$$\sigma_{vM} = \sqrt{\sigma^2 + 3\tau^2} \tag{13}$$

Theoretical bending stress is written as follows.

$$\sigma_{FT} = \frac{F_t}{b \cdot m} Y_F \tag{14}$$

3.6.1. Applied Load

The applied load was considered in 6 different pinion gears that had 6 different helix angles. The Von-Mises stresses are shown in Figures 12-17.

3.6.2. Solution

The Von Mises stress obtained by finite elements analyses are shown in Table 3. A comparison between Von Mises Stress with FEM and analytical static stress depending on the helix angle is shown in Figure 12. By observing obtained finite elements method results, it is concluded that 45% increased helix angle result in 6.5% decreased Von Mises stress.

The maximum Von Mises stress of the pinion gear reaches 112, 900 [N/mm^2] on the pinion tooth root for a helix angle β = 22 [˚], as shown in Figure 13.

The maximum Von Mises stress of the pinion gear reaches 112, 340 [N/mm^2] on the pinion tooth root for a helix angle β = 24 [˚], as shown in Figure 14.

The maximum Von Mises stress of the pinion gear reaches 111, 236 [N/mm^2] on the pinion tooth root for a helix angle β = 26 [˚], as shown in Figure 15.

The maximum Von Mises stress of the pinion gear reaches 110, 348 [N/mm^2] on the pinion tooth root for a helix angle β = 28 [˚], as shown in Figure 16.

The maximum Von Mises stress of the pinion gear reaches 108, 242 [N/mm^2] on the pinion tooth root for a helix angle β = 30 [˚], as shown in Figure 17.

The maximum Von Mises stress of the pinion gear reaches 105, 984 [N/mm^2] on the pinion tooth root for a helix angle β = 30 [˚], as shown in Figure 18.

4. Conclusions

During simulation, tooth bending stress and tooth contact stress were calculated according to ISO 6336. Solid (CAD) modelling of a pinion gear was completed

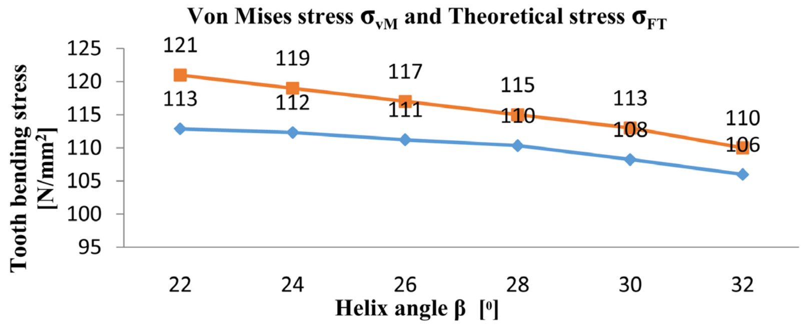

Figure 12. Comparison of Von Mises stress and Theoretical stress.

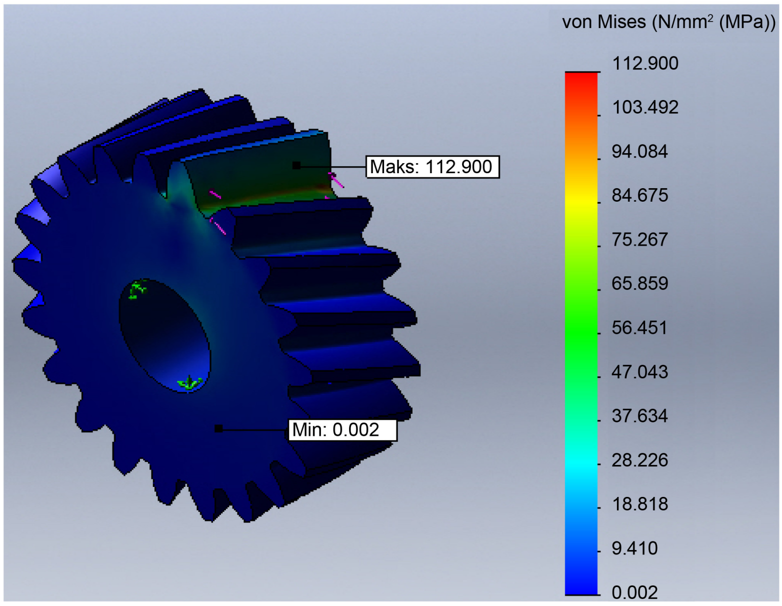

Figure 13. Von Mises stress for a helix angle $\beta = 22$ [°].

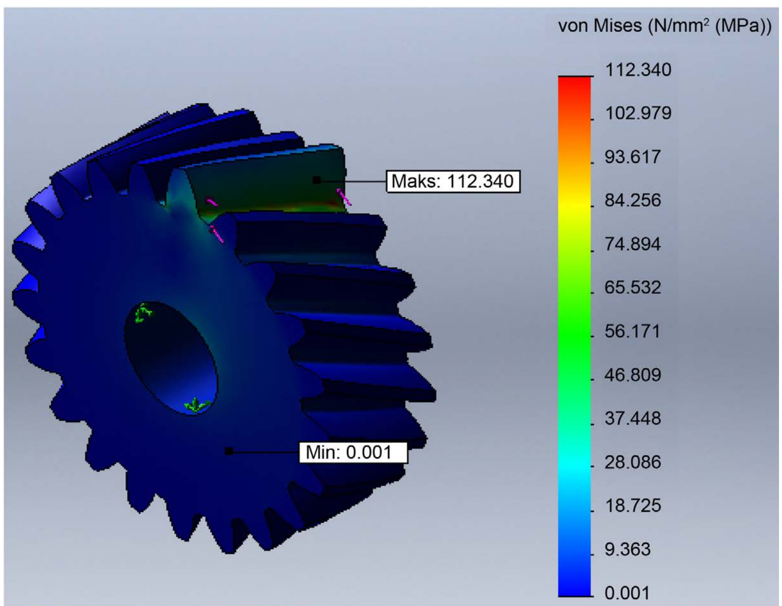

Figure 14. Von Mises stress for a helix angle $\beta = 24$ [°].

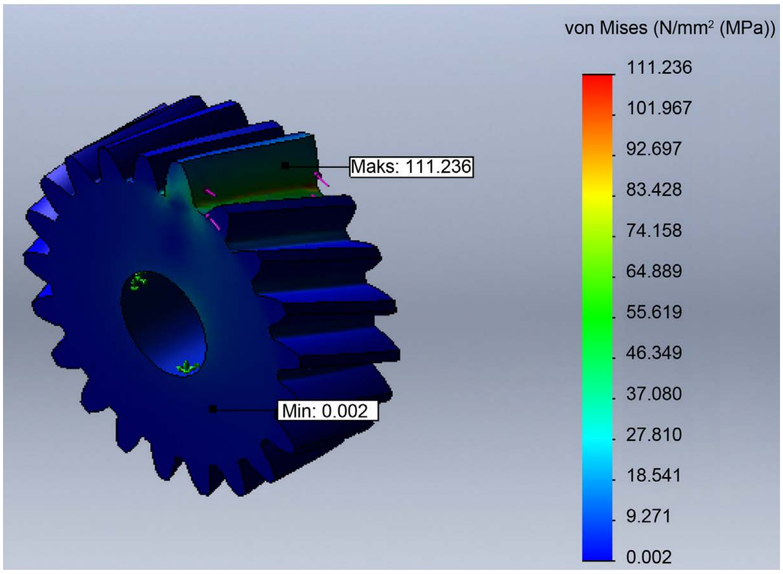

Figure 15. Von Mises stress for a helix angle $\beta = 26$ [˚].

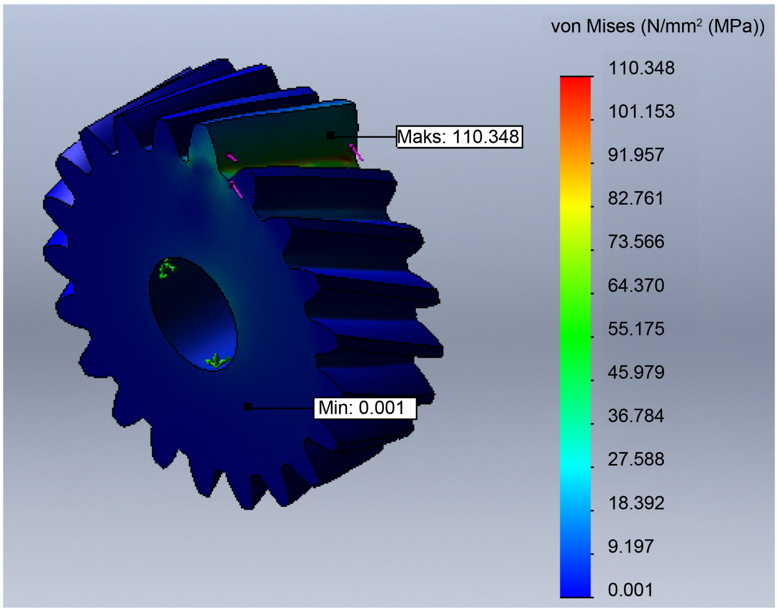

Figure 16. Von Mises stress for a helix angle $\beta = 28$ [˚].

using SOLIDWORKS software. The analytically obtained results and finite element method results were compared.

The effect of the helix angle on the tooth bending stress and tooth contact stress was analysed by varying helix angle, and the following conclusions are drawn.

Increasing the helix angle β, results in increasing the overlap contact ratio ε_β. Thus, increasing the helix angle β results in increasing the total contact ratio ε_γ.

Increasing the helix angle β results in a reduction of the tooth bending stress σ_F and tooth contact stress σ_H.

The analytically obtained results are verified by the finite element method

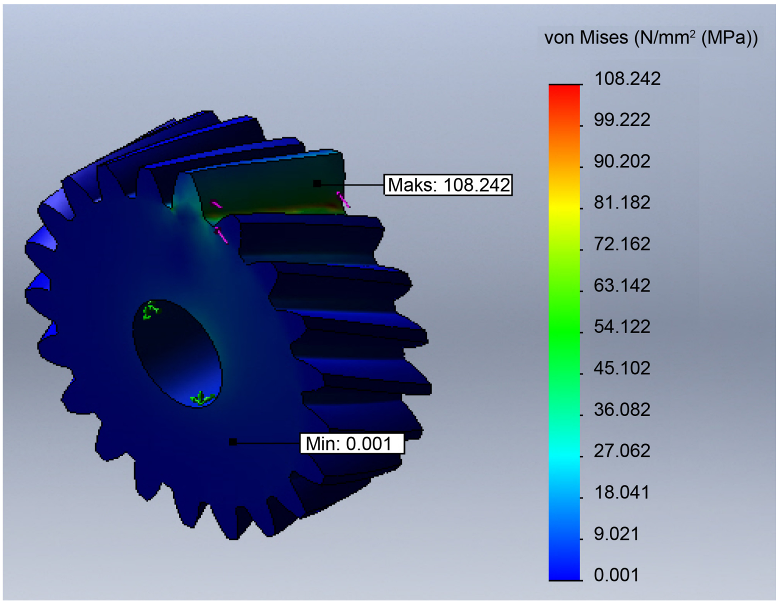

Figure 17. Von Mises stress for a helix angle $\beta = 30$ [˚].

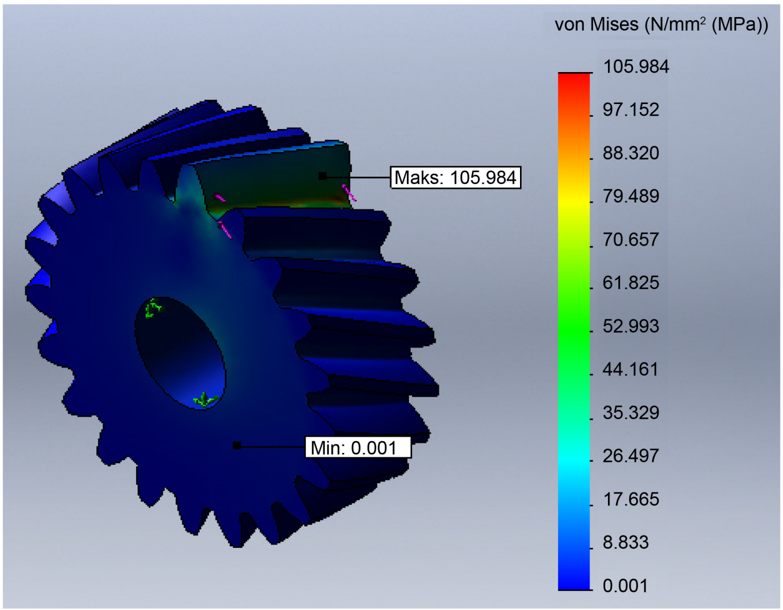

Figure 18. Von Mises stress for a helix angle $\beta = 32$ [˚].

Table 3. Von Mises stresses as determined by finite element analyses.

Helix angle β	Von Mises stress σ_{vM}	Theoretical bending stress σ_{FT}
$\beta = 22$	112,900	121,13
$\beta = 24$	112,340	119,35
$\beta = 26$	111,236	117,41
$\beta = 28$	110,348	115,34
$\beta = 30$	108,242	113,13
$\beta = 32$	105,984	110,79

results. By considering the tooth bending stress, the analytically obtained results and finite element method results only differ by 5%.

Helical gears have higher load carrying capacities than spur gears because their contact ratios are larger than those of spur gears.

Increasing the helix angle β results in an increase in the axial force Fa. Thus, one of the disadvantages of increasing the helix angle is the increase of axial forces on the helical gear mechanism.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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