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Special Relativity with a Preferred Frame and the Relativity Principle

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Abstract

The purpose of the present study is to develop a counterpart of the special relativity theory that is consistent with the existence of a preferred frame but, like the standard relativity theory, is based on the relativity principle and the universality of the (*two-way*) speed of light. The synthesis of such seemingly incompatible concepts as the existence of preferred frame and the relativity principle is possible at the expense of the freedom in assigning the *one-way* speeds of light that exists in special relativity. In the framework developed, a degree of anisotropy of the one-way speed acquires meaning of a characteristic of the really existing anisotropy caused by motion of an inertial frame relative to the preferred frame. The anisotropic special relativity kinematics is developed based on the symmetry principles: 1) Space-time transformations between inertial frames leave the equation of anisotropic light propagation invariant and 2) a set of the transformations possesses a group structure. The Lie group theory apparatus is applied to define groups of transformations between inertial frames. Applying the transformations to the problem of calculating the CMB temperature distribution yields a relation in which the angular dependence coincides with that obtained on the basis of the standard relativity theory but the mean temperature is corrected by the terms second order in the observer velocity.

Keywords

Principle of Relativity, Anisotropy of the One-Way Speed of Light, Lie Groups of Transformations, CMB

1. Introduction

Special relativity underpins nearly all of present day physics. Lorentz invariance is one of the cornerstones of general relativity and other theories of fundamental physics. It is thus very crucial to investigate its fundamentals and its potential violation. It seems evident that, if one wishes to contemplate the possibility of Lorentz symmetry violation within the context of a physical theory, then one will have to abandon the relativity principle which leads to the view that there exists a preferred universal rest frame. Also, the discovery of the cosmic microwave background (CMB) radiation has shown that (at least) cosmologically a preferred frame of reference does exist—it is the frame in which the CMB is isotropic. Acceptance of the view that there exists a preferred frame of reference seems to unambiguously abolish (besides the principle of relativity) another basic principle of the special relativity theory, namely, the principle of universality of the speed of light.

Correspondingly, the modern versions of experimental tests of special relativity and the "test theories" of special relativity (theoretical frameworks for analyzing results of experiments to verify special relativity [1] [2]) presume that a preferred inertial reference frame, identified with the CMB frame, is the only frame in which the *two-way* speed of light (the average speed from source to observer and back) is isotropic while it is anisotropic in relatively moving frames. Furthermore, it seems that accepting the existence of a preferred frame forces one to abandon the group structure for the set of space-time transformations between inertial frames. In the test theories, transformations between "moving" frames are not considered, only the transformation between a preferred "rest" frame and a particular moving frame is postulated.

The purpose of the present study is to develop a counterpart of the special relativity kinematics, that is consistent with the existence of a preferred frame but, like the standard relativity theory, is based on the relativity principle and universality of the (two-way) speed of light, and also preserves the group structure of the set of transformations between inertial frames. The analysis shows that the reconciliation and synthesis of those principles with the existence of a preferred frame is possible, and, what is more, such a possibility is naturally present in the framework of the relativity theory. Because of the freedom in assigning the *one-way* speeds of light (any one-way speeds, consistent with the two-way speed equal to c, are acceptable), a preferred frame can be defined as the frame in which the *one-way* speed of light is isotropic while, in any inertial frame moving with respect to the preferred frame, the one-way speed of light is anisotropic. It is similar to a definition accepted in a number of analyses, in which the existence of a preferred frame is assumed, but an important difference of the present analysis from others is that a degree of anisotropy of the one-way speed of light acquires meaning of a characteristic of the really existing anisotropy caused by motion of an inertial frame relative to the preferred frame. It seems to be contradictory to the common view that, because of the inescapable entanglement between remote clock synchronization and one-way speed of light (see, e.g., [3] [4] [5] [6]), the one-way speed of light is irreducibly conventional. Nevertheless, in the framework developed, the one-way speed of light in a specific inertial farme is a physical quantity determined by a physical law in which the anisotropy parameter depends on the frame velocity with respect to a preferred frame. The entanglement between remote clock synchronization and one-way speed of light, in the case if the remote clocks are set using light signals, only implies that the synchronization procedure is implemented using the one-way speed of light determined by that law. If another method of synchronization, as for example, "external synchronization" [2], is used it changes the form of transformations for the space-time variables but the one-way speed of light is not altered by changing the synchronization method.

The analysis is based on the requirements of invariance of the equation of (anisotropic) light propagation and the group structure of a set of transformations between inertial frames which follow from the principles of special relativity. In those transformations, the anisotropy parameter k for the one-way speed of light is a variable that takes part in the transformations. Therefore the fact, that the one-way speed of light is a physical quantity determined by the frame velocity relative to a preferred frame, does not violate the relativity principle. Nothing distinguishes the frame in which $k = 0$ from others and the transformations from/to that frame are members of a group of transformations that are equivalent to others.

The space-time transformations between inertial frames derived as a result of the analysis differ from the Lorentz transformations. Since the theory is based on the special relativity principles, it means that the Lorentz invariance is violated without violation of the relativistic invariance. The theory equations contain one undefined universal constant q such that the case of $q = 0$ corresponds to the standard special relativity with isotropic one-way speed of light in all inertial frames. The measurable effects following from the theory equations can provide estimates for q and define deviations from the standard relativity that way.

Applying the theory to the problem of calculating the CMB temperature distribution eliminates the inconsistency of the usual approach when formulas of the standard special relativity, which does not allow a preferred frame, are used to define effects caused by motion with respect to the preferred frame. The CMB temperature angular dependence predicted by the present theory coincides with that obtained on the basis of the standard relativity equations while the mean temperature is corrected by the terms second order in the observer velocity.

The paper is organized, as follows. In Section 2, following the Introduction, the issue of anisotropy of the light propagation in special relativity is discussed in more details. In Section 3, the conceptual framework of the analysis is presented. In Section 4, the method is outlined and the coordinate transformations between inertial frames incorporating anisotropy of the light propagation, with the anisotropy parameter varying from frame to frame, are derived. In Section 5, the transformations are specified using the argument that the anisotropy of the light propagation is due to the observer motion with respect to the preferred frame. Consequences of the transformations are considered in Section 6. In Section 7, the results are applied to the problem of calculating the CMB effective temperature distribution as seen by a moving observer. The approach and results are discussed in Section 8.

2. Anisotropy of the Light Propagation in Special Relativity

Anisotropy of the one-way speed of light is traditionally placed into the context of conventionality of distant simultaneity and clock synchronization [3] [4] [5] [6]. Simultaneity at distant space points of an inertial system is defined by a clock synchronization that makes use of light signals. Let a pulse of light is emitted from the master clock and reflected off the remote clock. If t_0 and t_R are respectively the times of emission and reception of the light pulse at the master clock and t is the time of reflection of the pulse at the remote clock then the conventionality of simultaneity is a statement that one is free to choose the time t to be anywhere between t_0 and t_R. This freedom may be parameterized by a parameter k_ϵ, as follows

$$t = t_0 + \frac{1+k_\epsilon}{2}\left(t_R - t_0\right); \quad \left|k_\epsilon\right| < 1 \tag{1}$$

Any choice of $k_\epsilon \neq 0$ corresponds to assigning different one-way speeds of light signals in each direction which must satisfy the condition that the average is equal to c. Speed of light in each direction is therefore

$$V_\pm = \frac{c}{1 \pm k_\epsilon} \tag{2}$$

The "standard" (Einstein) synchronization entailing equal speeds in opposite directions corresponds to $k_\epsilon = 0$. If the described procedure is used for setting up throughout the frame of a set of clocks using signals from some master clock placed at the spatial origin, a difference in the standard and nonstandard clock synchronization may be reduced to a change of coordinates [3] [4] [5] [6]

$$t = t^{(s)} + \frac{k_\epsilon x}{c}, \quad x = x^{(s)} \tag{3}$$

where $t^{(s)} = \left(t_0 + t_R\right)/2$ is the time setting according to Einstein (standard) synchronization procedure.

The analysis can be extended to the three dimensional case. If a beam of light propagates (along straight lines) from a starting point and through the reflection over suitable mirrors covers a closed part the experimental fact is that the speed of light as measured over closed part is always c (*Round-Trip Light Principle*). In accordance with that experimental fact, if the speed of light is allowed to be anisotropic it must depend on the direction of propagation as [4] [5]

$$V = \frac{c}{1 + \boldsymbol{k}_\epsilon \boldsymbol{n}} = \frac{c}{1 + k_\epsilon \cos\theta_k} \tag{4}$$

where $\boldsymbol{k}_\epsilon$ is a constant vector and θ_k is the angle between the direction of propagation $\boldsymbol{n}$ and $\boldsymbol{k}_\epsilon$. Similar to the one-dimensional case, the law (4) may be considered as a result of the transformation from "standard" coordinatization of the four-dimensional space-time manifold, with $k_\epsilon = 0$, to the "nonstandard" one with $k_\epsilon \neq 0$:

$$t = t^{(s)} + \frac{\boldsymbol{k}_\epsilon \boldsymbol{r}}{c}, \quad \boldsymbol{r} = \boldsymbol{r}^{(s)} \tag{5}$$

The conventionality of simultaneity in the special theory of relativity, and the related issue of anisotropy of the one-way speed of light, have been much debated issues. A common view is that, due to freedom in the choice of the anisotropy parameter k_ϵ, the one-way speed of light is irreducibly conventional. The purpose of the following discussion is to show that, if there is an anisotropy in a physical system, the arguments for conventionality of the one-way speed of light are not valid and, what is more, a specific value of the one-way speed of light, *together with corresponding synchronization*, is selected in some objective way.

The arguments for conventionality of the one-way speed are based first on the possibility of introducing the transformations treated as replacing the Lorentz transformations of special relativity in the case of the anisotropic one-way speed of light (2) with $k_\epsilon \neq 0$. Such transformations have been repeatedly derived in the literature using kinematic arguments, the works [7] [8] [9] should be mentioned first. In what follows, they will be called the "ϵ-Lorentz transformations", the name is due to [8] [9]. Although the ϵ-Lorentz transformations can be obtained from the standard Lorentz transformations by a change of coordinates (3) and so they are in fact the Lorentz transformations of the standard special relativity represented using the "nonstandard" coordinatization of the four-dimensional space-time manifold, they are usually considered as describing the special relativity kinematics in an anisotropic system (for example, the most highly cited paper by Edwards [7] is entitled "Special relativity in anisotropic space"). Below, the arguments are presented showing that 1) the ϵ-Lorentz transformations, commonly considered as incorporating anisotropy, are in fact not applicable to an anisotropic system and 2) in the case of isotropic system, the particular case of the transformations corresponding to the isotropic one-way speed of light and Einstein synchronization (standard Lorentz transformations) is privileged.

The first statement is related to the issue of *invariance of the interval*. Invariance of the interval is commonly considered as an integral part of the physics of special relativity which is used as a starting point for derivation of the space-time transformations between inertial frames. Nevertheless, invariance of the interval is not a straightforward consequence of the basic principles of the theory. The two principles constituting the conceptual basis of the special relativity, the *principle of relativity* which states the equivalence of all inertial frames as regards the formulation of the laws of physics and *universality of the speed of light* in inertial frames, taken together lead to the condition of *invariance of the equation of light propagation* with respect to the coordinate transformations between inertial frames. Thus, in general, not the invariance of the interval but invariance of the equation of light propagation should be a starting point for derivation of the transformations. Therefore the use of the interval invariance is usually preceded by a proof of its validity (see, e.g., [10] [11]) based on invariance of the equation of light propagation. However, those proofs are not valid if an anisotropy is present and the same arguments lead to the conclusion that, in the presence of anisotropy, the interval is not invariant but modified by a conformal factor [12]. The "ϵ-Lorentz transformations", like the standard Lorentz transformations, leave the interval invariant and therefore they are applicable only to the case of no anisotropy.

The second statement, that, in the case of isotropy, the particular case of the isotropic one-way speed of light and Einstein synchronization is privileged, relies on the *correspondence principle*. The correspondence principle was taken by Niels Bohr as the guiding principle to discoveries in the old quantum theory. Since then it was considered as a guideline for the selection of new theories in physical science. In the context of special relativity, the correspondence principle is traditionally mentioned as a statement that Einstein's theory of special relativity reduces to classical mechanics in the limit of small velocities in comparison to the speed of light. Being applied to the special relativity kinematics, the correspondence principle implies that *the transformations between inertial frames should turn into the Galilean transformations in the limit of small velocities*. The "ϵ-Lorentz transformations" do not satisfy the correspondence principle unless $k_\epsilon = 0$ [12] which means that the isotropic one-way speed of light and Einstein synchrony are selected if no anisotropy is present in a physical system. Similarly, in the case of an *anisotropic* system, there should also exist a privileged value of the one-way speed selected by the size of the anisotropy.

The above comments are related to the case when synchronization is implemented using light signals. Nevertheless, if another method of synchronization, as, for example, the "external synchronization" [2], is used it cannot change the value of the one-way speed of light. It can change the form of transformations for the time and space variables but, again, changing the synchronization method is equivalent to a change of coordinates (see more details in Section 8).

It is worth to mention, in connection with the issues of the correspondence principle and synchronization problem, a discussion in the literature (see, e.g., [13] [14] [15] [16]) initiated by the paper of Ohanian [13] "The role of dynamics in the synchronization problem". Ohanian argued that dynamical considerations, applied to inertial systems, necessarily entail the standard synchronization rule. He shows that the nonstandard synchronization procedure, when discussing Newtons (classical) mechanics, would result in a change in the mathematical form of the equation of motion such that the Newtons second law involves what he calls "pseudo-forces". He concludes that in an inertial reference frame any synchronization, other than the Einsteinian one, is forbidden.

Ohanian's approach has been criticized by several authors (for example, by Macdonald [14] and Martinez [15], see also a reply of Ohanian [16] to comments by Macdonald and Martinez) but their analyses are too concentrated on such issues as a synchronization convention and the origin of the Einsteinian synchronization while more apparent inconsistencies of Ohanian's analysis are not sufficiently emphasized. Below we briefly discuss some of them. First, the Newtons second law of classical mechanics is used as a relation for choosing a synchronization rule or, in other terms, for choosing the value of anisotropy parameter for the one-way speed of light. The issues of light speed and its anisotropy are alien to Newtonian mechanics with absolute time and so such an approach is an inconsistent mixture of relativistic and classical concepts. (It would be more consistent to use in that context the correspondence principle as applied to dynamical equations of relativistic physics but, in general, using dynamical equations in the problem of clock synchronization is doubtful, see comments below.) Next, there is no reason for choosing Newtons second law, even if it were in a relativistic form, as a basic relation and considering it as more fundamental than any kinematics in the context of such purely kinematic issues as clock synchronization and light speed. Note also that (as emphasized in [14] and [15]) it is in contradiction with a consensus on considering the law of inertia as independent and prior to the force law in the definition of inertial frames. Further, a change in the mathematical form of dynamical equations resulting from different synchrony conventions do not correspond to any differences whatsoever in the actual material behavior of physical systems and so using the requirement that a dynamical equation took a specific form (even if it is the simplest one) as a basis for distinguishing a specific synchronization is not justified.

3. Conceptual Framework

The special relativity kinematics applicable to an anisotropic system should be developed based on the first principles of special relativity but without refereeing to the relations of the standard relativity theory. The principles constituting the conceptual basis of special relativity, the relativity principle, according to which physical laws should have the same forms in all inertial frames, and the universality of the speed of light in inertial frames, lead to the requirement of invariance of the equation of light propagation with respect to the coordinate transformations between inertial frames. In the present context, it should be invariance of the equation of propagation of light which incorporates the anisotropy of the one-way speed of light, with the law of variation of the speed with direction consistent with the experimentally verified round-trip light principle, as follows

$$V = \frac{c}{1 + \boldsymbol{kn}} = \frac{c}{1 + k\cos\theta_k} \tag{6}$$

where $\boldsymbol{k}$ is a (constant) vector characteristic of the anisotropy. The change of notation, as compared with (4), from k_ϵ to k is intended to indicate that $\boldsymbol{k}$ is a parameter value corresponding to the size of the really existing anisotropy while k_ϵ defines the anisotropy in the one-way speeds of light due to the nonstandard synchrony equivalent to the coordinate change (5). The anisotropic equation of light propagation incorporating the law (6) has the form [12]

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - 2kc\,\mathrm{d}t\mathrm{d}x - \left(1-k^2\right)\mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = 0 \quad (7)$$

where (x, y, z) are coordinates and t is time. It is assumed that the x-axis is chosen to be along the anisotropy vector $\boldsymbol{k}$. Note that although the form (7) is usually attributed to the one-dimensional formulation it can be shown that, in the three-dimensional case, the equation has the same form if the anisotropy vector $\boldsymbol{k}$ is directed along the x-axis (see [12]).

Further, in the development of the anisotropic relativistic kinematics, a number of other physical requirements, associativity, reciprocity and so on are to be satisfied which all are covered by the condition that the transformations between the frames form a group. Thus, the group property should be taken as another first principle. The formulation based on the invariance and group property suggests using the *Lie group theory* apparatus for defining groups of space-time transformations between inertial frames.

At this point, it should be clarified that there can exist two different cases: 1) The size of anisotropy does not depend on the observer motion and so is the same in all inertial frames (groups of transformations for this case are studied in [12]); 2) The anisotropy is due to the observer motion with respect to a preferred frame and so the size of anisotropy varies from frame to frame (it is a subject of the present study). In the latter case, the anisotropy parameter becomes a variable which takes part in the transformations so that groups of transformations in *five* variables $\{x, y, z, t, k\}$ are studied. The preferred frame, commonly defined by that the propagation of light in that frame is isotropic, is naturally present in that framework as the frame in which $k = 0$. However, it does not violate the relativity principle since the transformations from/to that frame are not distinguished from other members of the group. Nevertheless, the fact, that the anisotropy of the one-way speed of light in an arbitrary inertial frame is due to motion of that frame relative to the preferred frame, is a part of the paradigm which is used in the analysis.

The procedure of obtaining the transformations consists of the following steps: 1) The infinitesimal invariance condition is applied to the equation of light propagation which yields determining equations for the infinitesimal group generators; 2) The determining equations are solved to define the group generators and the correspondence principle is applied to specify the solutions; 3) Having the group generators defined the finite transformations are determined as solutions of the Lie equations; 4) The group parameter is related to physical parameters using some obvious conditions; 5) Finally, the conceptual argument, that the size of anisotropy of the one-way speed of light in an arbitrary inertial frame depends on its velocity relative to the preferred frame, is used to specify the results and place them into the context of special relativity with a preferred frame. Note that implementing the steps 1)-4) for the case of no isotropy yields the standard Lorentz transformations [12].

The transformations between inertial frames derived in such a way contain a scale factor and thus do not leave the interval between two events invariant but modify it by a conformal factor (square of the scale factor). Applying the conformal invariance in physical theories originates from the papers by Bateman [17] and Cunningham [18] who discovered the form-invariance of Maxwells equations for electromagnetism with respect to conformal space-time transformations. Since then conformal symmetries have been successfully exploited for many physical systems (see, e.g., reviews [19] [20]). Transformations which conformally modify Minkowski metric have been introduced in the context of the special relativity kinematics in the presence of space anisotropy in [21] (see also references therein) and [22] (see also [23]). As a matter of fact, those works are not directly related to the subject of the present study as they consider the case of a constant anisotropy degree, not dependent on the frame motion. Nevertheless, it is worthwhile to note that in the works [21] [22] the assumption that the form of the metric changes by a conformal factor is *imposed* while, in the framework of the present analysis, conformal invariance of the metric *arises* as an intrinsic feature of special relativity based on invariance of the anisotropic equation of light propagation and the group property (see [12] for a more detailed discussion of the works [21] [22]).

4. Transformations between Inertial Frames with a Varying Anisotropy Parameter

In this section, groups of transformations between inertial frames that leave the equation for light propagation, incorporating the anisotropic law (6), form-invariant are defined. The parameter of anisotropy k is allowed to vary from frame to frame which, in particular, implies that there exists a preferred frame in which the speed of light is isotropic. The transformations are required to form a one-parameter group with the group parameter $a = a(v)$ (such that $v \ll 1$ corresponds to $a \ll 1$). Note that the group property is used not as in the traditional analysis which commonly proceeds along the lines initiated by [24] and [25] which are based on the linearity assumption and relativity arguments. The difference can be seen from the derivation of the standard Lorentz transformations using the above procedure [12].

Consider two arbitrary inertial reference frames S and S' in the standard configuration with the y- and z-axes of the two frames being parallel while the relative motion is along the common x-axis. The space and time coordinates in S and S' are denoted respectively as $\{X, Y, Z, T\}$ and $\{x, y, z, t\}$. The velocity of the S' frame along the positive x direction in S, is denoted by v. It is assumed that the frame S' moves relative to S along the direction determined by the vector $\boldsymbol{k}$ from (6). This assumption is justified by that one of the frames in a set of frames with different values of k is a preferred frame, in which k = 0, so that the transformations must include, as a particular case, the transformation to that preferred frame. Since the anisotropy is attributed to the fact of motion with respect to the preferred frame it is expected that the axis of anisotropy is along the direction of motion (however, the direction of the anisotropy vector can be both coinciding and opposite to that of velocity).

The equations for light propagation in the frames S and S' are

$$c^2\mathrm{d}T^2 - 2Kc\mathrm{d}T\mathrm{d}X - \left(1-K^2\right)\mathrm{d}X^2 - \mathrm{d}Y^2 - \mathrm{d}Z^2 = 0, \tag{8}$$

$$c^2\mathrm{d}t^2 - 2kc\mathrm{d}t\mathrm{d}x - \left(1-k^2\right)\mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 = 0 \tag{9}$$

where the anisotropy parameters K and k in the frames S and S' are different. The relativity principle implies that the transformations of variables from $\{X,Y,Z,T,K\}$ to $\{x,y,z,t,k\}$ leave the form of the equation of light propagation invariant so that (8) is converted into (9) under the transformations. The transformations form a one-parameter group

$$\begin{aligned} &x = f\left(X,Y,Z,T,K;a\right),\ y = g\left(X,Y,Z,T,K;a\right),\\ &z = h\left(X,Y,Z,T,K;a\right),\ t = q\left(X,Y,Z,T,K;a\right);\ k = p\left(K;a\right) \end{aligned} \tag{10}$$

where a is the group parameter. Remark that k is a transformed variable taking part in the group transformations. Based on the symmetry arguments it is assumed that the transformations of the variables x and t do not involve the variables y and z and vice versa:

$$\begin{aligned} &x = f\left(X,T,K;a\right),\ t = q\left(X,T,K;a\right),\\ &y = g\left(Y,Z,K;a\right),\ z = h\left(Y,Z,K;a\right);\ k = p\left(K;a\right) \end{aligned} \tag{11}$$

The correspondence principle requires that, in the limit of small velocities $v \ll c$ (small values of the group parameter $a \ll 1$), the formula for transformation of the coordinate x turns into that of the Galilean transformation:

$$x = X - vT \tag{12}$$

Remark that the small v limit is not influenced by the presence of anisotropy of the light propagation. It is evident that there should be no traces of light anisotropy in that limit, the issues of the light speed and its anisotropy are alien to the framework of Galilean kinematics.

The group property and the invariance of the equation of light propagation suggest applying the infinitesimal Lie technique (see, e.g., [26] [27]). The infinitesimal transformations corresponding to (11) are introduced, as follows

$$\begin{aligned} &x \approx X + \xi\left(X,T,K\right)a,\ t \approx T + \tau\left(X,T,K\right)a,\\ &y \approx Y + \eta\left(Y,Z,K\right)a,\ z \approx Z + \zeta\left(Y,Z,K\right)a,\ k \approx K + a\chi\left(K\right) \end{aligned} \tag{13}$$

and Equations (8) and (9) are used to derive determining equations for the group generators $\tau\left(X,T,K\right)$, $\xi\left(X,T,K\right)$, $\eta\left(Y,Z,K\right)$, $\zeta\left(Y,Z,K\right)$ and $\chi\left(K\right)$. The infinitesimal group generators can be partially specified by applying the correspondence principle. Equation (12) is used to calculate the group generator $\xi\left(X,T\right)$, as follows

$$\xi = \left(\frac{\partial x}{\partial a}\right)_{a=0} = \left(\frac{\partial\left(X - v\left(a\right)T\right)}{\partial a}\right)_{a=0} = -bT;\ b = v'\left(0\right) \tag{14}$$

It can be set b = 1 without loss of generality since this constant can be eliminated by redefining the group parameter. Thus, the generator ξ is defined by

$$\xi = -T \tag{15}$$

Then substituting the infinitesimal transformations (13), with ξ defined by (15), into Equation (9) with subsequent linearizing with respect to *a* and using Equation (8) to eliminate $\mathrm{d}T^2$ yields

$$\begin{aligned}&\left(-Kc^2\tau_X + \left(1-K^2\right)\left(K + c\tau_T\right) + \chi\left(K\right)cK\right)\mathrm{d}X^2 \\ &+ c\left(c^2\tau_X + cK\tau_T + 1 + K^2 - \chi\left(K\right)c\right)\mathrm{d}X\mathrm{d}T + \left(K + c\tau_T - c\eta_Y\right)\mathrm{d}Y^2 \\ &+ \left(K + c\tau_T - c\zeta_Z\right)\mathrm{d}Z^2 - c\left(\eta_Z + \zeta_Y\right)\mathrm{d}Y\mathrm{d}Z = 0\end{aligned} \tag{16}$$

where subscripts denote differentiation with respect to the corresponding variable. In view of arbitrariness of the differentials $\mathrm{d}X$, $\mathrm{d}Y$, $\mathrm{d}Z$ and, $\mathrm{d}T$, the equality (16) can be valid only if the coefficients of all the monomials in (16) vanish which results in an overdetermined system of determining equations for the group generators.

The generators τ, η and ζ found from the determining equations yielded by (16) are

$$\begin{aligned}&\tau = -\frac{1-K^2-\chi\left(K\right)c}{c^2}X - \frac{2K}{c}T + c_2, \\ &\eta = -\frac{K}{c}Y + \omega Z + c_3,\ \zeta = -\frac{K}{c}Z - \omega Y + c_4\end{aligned} \tag{17}$$

where c_2, c_3 and c_4 are arbitrary constants. The common kinematic restrictions that one event is the spacetime origin of both frames and that the x and X axes slide along another can be imposed to make the constants c_2, c_3 and c_4 vanishing (space and time shifts are eliminated). In addition, it is required that the $\left(x,z\right)$ and $\left(X,Z\right)$ planes coincide at all times which results in $\omega = 0$ and so excludes rotations in the plane $\left(y,z\right)$.

The finite transformations are determined by solving the Lie equations which, after rescaling the group parameter as $\hat{a} = a/c$ together with $\hat{\chi} = \chi c$ and omitting hats afterwards, take the forms

$$\frac{\mathrm{d}k\left(a\right)}{\mathrm{d}a} = \chi\left(k\left(a\right)\right);\quad k\left(0\right) = K, \tag{18}$$

$$\frac{\mathrm{d}x\left(a\right)}{\mathrm{d}a} = -ct\left(a\right),\ \frac{\mathrm{d}\left(ct\left(a\right)\right)}{\mathrm{d}a} = -\left(1-k\left(a\right)^2 - \chi\left(k\left(a\right)\right)\right)x\left(a\right) - 2k\left(a\right)ct\left(a\right), \tag{19}$$

$$\frac{\mathrm{d}y\left(a\right)}{\mathrm{d}a} = -k\left(a\right)y\left(a\right),\quad \frac{\mathrm{d}z\left(a\right)}{\mathrm{d}a} = -k\left(a\right)z\left(a\right); \tag{20}$$

$$x\left(0\right) = X,\ t\left(0\right) = T,\ y\left(0\right) = Y,\ z\left(0\right) = Z. \tag{21}$$

Because of the arbitrariness of $\chi\left(k\left(a\right)\right)$, the solution of the system of Equations (18), (19) and (20) contains an arbitrary function $k\left(a\right)$. Using (18) to replace $\chi\left(k\left(a\right)\right)$ in the second equation of (19) we obtain solutions of Equations (19) subject to the initial conditions (21) in the form

$$x = R\left(X\left(\cosh a + K\sinh a\right) - cT\sinh a\right), \tag{22}$$

$$ct = R\left(cT\left(\cosh a - k\left(a\right)\sinh a\right) - X\left(\left(1 - Kk\left(a\right)\right)\sinh a + \left(K - k\left(a\right)\right)\cosh a\right)\right) \tag{23}$$

where R is defined by

$$R = \mathrm{e}^{-\int_0^a k(\alpha)\mathrm{d}\alpha} \tag{24}$$

To complete the derivation of the transformations the group parameter a is to be related to the velocity v using the condition

$$x = 0 \ \text{ for } X = vT \tag{25}$$

which yields

$$a = \frac{1}{2}\ln\frac{1+\beta - K\beta}{1-\beta - K\beta}; \ \beta = \frac{v}{c} \tag{26}$$

Substituting (26) into (22) and (23) yields

$$x = \frac{R}{\sqrt{\left(1 - K\beta\right)^2 - \beta^2}}\left(X - cT\beta\right),$$

$$ct = \frac{R}{\sqrt{\left(1 - K\beta\right)^2 - \beta^2}}\left(cT\left(1 - K\beta - k\beta\right) - X\left(\left(1 - K^2\right)\beta + K - k\right)\right) \tag{27}$$

where k is the value of $k\left(a\right)$ calculated for a given by (26).

Solving Equations (20) and using (26) in the result yields

$$y = RY, \ z = RZ \tag{28}$$

Calculating the interval

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - 2kc\mathrm{d}t\mathrm{d}x - \left(1 - k^2\right)\mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \tag{29}$$

with (27) and (28) yields

$$\mathrm{d}s^2 = R^2\mathrm{d}S^2, \ \mathrm{d}S^2 = c^2\mathrm{d}T^2 - 2Kc\mathrm{d}T\mathrm{d}X - \left(1 - K^2\right)\mathrm{d}X^2 - \mathrm{d}Y^2 - \mathrm{d}Z^2 \tag{30}$$

Thus, in the case when the anisotropy exists, the interval invariance is replaced by *conformal invariance* with the conformal factor dependent on the relative velocity of the frames and the anisotropy degree.

Considering inverse transformations from the frame S' to S one has to take into account that, in the presence of the light speed anisotropy, the reciprocity principle is modified [3] [8]. The reasoning behind this is that all speeds are to be affected by the anisotropy of the light speed since the speeds are timed by their coincidences at master and remote clocks, and the latter are altered. Therefore the relative velocity v_- of S to S' is not equal to the relative velocity v of S' to S. The modified reciprocity relation is commonly obtained using kinematic arguments [28], but, in the framework of our analysis, it straightforwardly follows from the group property of the transformations $a_- = -a$, where a is given by (26) and a_- is also defined by Equation (26) but with β replaced by β_- and K replaced by k, as follows

$$a_{-} = \frac{1}{2}\ln\frac{1-\beta_{-}+k\beta_{-}}{1+\beta_{-}+k\beta_{-}}, \quad \beta_{-} = \frac{v_{-}}{c} \tag{31}$$

Thus, the modified reciprocity relation is obtained in the form

$$\beta_{-} = \frac{\beta}{1-(k+K)\beta} \tag{32}$$

For deriving consequences of the transformations it is convenient to write the inverse transformations in terms of β (not β_{-}), as follows

$$X = \frac{R^{-1}}{\sqrt{(1-K\beta)^2-\beta^2}}\left(x(1-K\beta-k\beta)+ct\beta\right),$$

$$cT = \frac{R^{-1}}{\sqrt{(1-K\beta)^2-\beta^2}}\left(ct+x\left(\left(1-K^2\right)\beta+K-k\right)\right) \tag{33}$$

$$Y = R^{-1}y, \; Z = R^{-1}z \tag{34}$$

The formulas for the velocity transformation are readily obtained from (27) and (28), as follows

$$u_x = \frac{c(U_X-c\beta)}{Q}, u_y = \frac{cU_Y\sqrt{(1-K\beta)^2-\beta^2}}{Q}, u_z = \frac{cU_Z\sqrt{(1-K\beta)^2-\beta^2}}{Q},$$

$$Q = c(1-K\beta)+U_X\left(K^2\beta-K-\beta\right)+k\left(U_X-c\beta\right) \tag{35}$$

where (U_X, U_Y, U_Z) and (u_x, u_y, u_z) are the velocity components in the frames S and S' respectively.

The transformations (24)-(28) contain an indefinite function $k(a)$. The scale factor R also depends on that function. The transformations are specified in the next section.

5. Specifying the Transformations

In the derivation of the transformations in the previous section, the arguments, that there exists a preferred frame in which the light speed is isotropic and that the anisotropy of the one-way speed of light in a specific frame is due to its motion relative to the preferred frame, have not been used. In the framework of the derivation, nothing distinguishes the frame in which $k = 0$ from others and the transformations from/to that frame are members of a group of transformations that are equivalent to others. Thus, the theory developed above is a counterpart of the standard special relativity kinematics which incorporates an anisotropy of the light propagation, with the anisotropy parameter varying from frame to frame. Below the transformations between inertial frames derived in Section 2 are specified based on that anisotropy of the one-way speed of light in an inertial frame is caused by its motion with respect to the preferred frame.

First, this leads to the conclusion that the anisotropy parameter k in an arbitrary frame s moving with respect to the preferred frame with velocity $\bar{\beta} = \bar{v}/c$ should be given by some (universal) function of that velocity, as follows

$$k = F\left(\bar{\beta}\right) \tag{36}$$

Indeed, Equations (18) and (26) imply that $k = k\left(a\left(\beta, K\right), K\right)$ which being specified for the transformation from the preferred frame to the frame s by setting $K = 0, \beta = \bar{\beta}$ yields (36). (It could be expected, in general, that a size of the anisotropy depends on the velocity relative to the preferred frame but, in the present analysis, it is not a presumption but a part of the framework.)

Next, consider three inertial reference frames $\bar{S}$, S and S'. As in the preceding analysis, the standard configuration, with the y- and z-axes of the three frames being parallel and the relative motion being along the common x-axis (and along the direction of the anisotropy vector), is assumed. The space and time coordinates and the anisotropy parameters in the frames $\bar{S}$, S and S' are denoted respectively as $\{\bar{x}, \bar{y}, \bar{z}, \bar{t}, \bar{k}\}$, $\{X, Y, Z, T, K\}$ and $\{x, y, z, t, k\}$. The frame S' moves relative to S with velocity v and velocities of the frames S and S' relative to the frame $\bar{S}$ are respectively $\bar{v}_1$ and $\bar{v}_2$. A relation between $\bar{v}_2$, v and $\bar{v}_1$ can be obtained from the equation expressing a group property of the transformations, as follows

$$a_2 = a_1 + a \tag{37}$$

where a_2, a_1 and a are the values of the group parameter corresponding to the transformations from $\bar{S}$ to S', from $\bar{S}$ to S and from S to S' respectively. Those values are expressed through the velocities and the anisotropy parameter values by a properly specified Equation (26) which, upon substituting into Equation (37), yields

$$\frac{1}{2}\ln\frac{1+\bar{\beta}_2-\bar{k}\bar{\beta}_2}{1-\bar{\beta}_2-\bar{k}\bar{\beta}_2} = \frac{1}{2}\ln\frac{1+\bar{\beta}_1-\bar{k}\bar{\beta}_1}{1-\bar{\beta}_1-\bar{k}\bar{\beta}_1} + \frac{1}{2}\ln\frac{1+\beta-K\beta}{1-\beta-K\beta} \tag{38}$$

where

$$\bar{\beta}_2 = \frac{\bar{v}_2}{c},\ \bar{\beta}_1 = \frac{\bar{v}_1}{c},\ \beta = \frac{v}{c} \tag{39}$$

Exponentiation of Equation (38) yields

$$\bar{\beta}_2 = \frac{\bar{\beta}_1 + \beta\left(1-\left(\bar{k}+K\right)\bar{\beta}_1\right)}{1+\beta\left(\bar{k}-K+\left(1-\bar{k}^2\right)\bar{\beta}_1\right)} \tag{40}$$

Let us now choose the frame $\bar{S}$ to be a preferred frame. Then, $\bar{k} = 0$ and, according to (36), for the frames S and S' we have

$$K = F\left(\bar{\beta}_1\right),\ k = F\left(\bar{\beta}_2\right) \tag{41}$$

With $\bar{\beta} = f\left(k\right)$ being a function inverse to $k = F\left(\bar{\beta}\right)$, using in (40) the equalities inverse to those of (41) together with $\bar{k} = 0$ yields

$$f\left(k\right) = \frac{f\left(K\right)+\beta\left(1-Kf\left(K\right)\right)}{1+\beta\left(-K+f\left(K\right)\right)} \tag{42}$$

If the function $f\left(k\right)$ were known, the relation (42), that implicitly defines the anisotropy parameter k in the frame S' as a function of the anisotropy parameter K in the frame S and the relative velocity v of the frames, would provide a formula for the transformation of the anisotropy parameter k. This would allow to specify the transformations (27) and (28) by substituting that formula for k into the equation of transformation for t and calculating the scale factor R using that formula with β expressed as a function of a group parameter a from (26).

Although the function $F(\bar{\beta})$ is not known, a further specification can be made based on the argument that an expansion of the function $F(\bar{\beta})$ in a series with respect to $\bar{\beta}$ should not contain a quadratic term since it is expected that a direction of the anisotropy vector changes to the opposite if a direction of a motion with respect to a preferred frame is reversed: $F(\bar{\beta}) = -F(-\bar{\beta})$. Thus, with accuracy up to the third order in $\bar{\beta}$, the dependence of the anisotropy parameter on the velocity with respect to a preferred frame can be approximated by

$$k = F(\bar{\beta}) \approx q\bar{\beta}, \quad \bar{\beta} = f(k) \approx k/q \tag{43}$$

Introducing the last equation of (43) into (42) yields

$$k = \frac{q\left(K + \beta\left(q - K^2\right)\right)}{q + \beta K(1-q)} \tag{44}$$

which is the expression to be substituted for k into (27). To calculate the scale factor in (27) and (28), β is expressed as a function of a group parameter a from (26), as follows

$$\beta = \frac{\sinh a}{K\sinh a + \cosh a} \tag{45}$$

which, being substituted into (44), yields

$$k(a) = \frac{q(K\cosh a + q\sinh a)}{K\sinh a + q\cosh a} \tag{46}$$

Then using (46) in (24), with (26) substituted for a in the result, yields

$$R = \left(\frac{q^2\left(1+\beta(1-K)\right)\left(1-\beta(1+K)\right)}{\left(q+\beta K(1-q)\right)^2}\right)^{\frac{q}{2}} \tag{47}$$

Thus, after the specification, the transformations between inertial frames incorporating anisotropy of light propagation are defined by Equations (27) and (28) with k given by (44) and the scale factor given by (47). It is readily checked that the specified transformations satisfy the correspondence principle. All the equations contain only one undefined parameter, a universal constant q.

It should be clarified that, although the specification relies on the approximate relation (43), the transformations, with k and R defined by (44) and (47), are *not* approximate and they do possess the group property. The transformations (27) and (28) form a group, even with $k(a)$ (or $k(K,\beta)$) undefined, provided that the transformation of k obeys the group property. Since the relation (42), defining that transformation, is a particular case of the relation (40) obtained from Equation (37) expressing the group property, the transformation of k satisfies the group property with any form of the function $F(\beta_s)$, and, in particular, with that defined by (43). Nevertheless, a straightforward check can be made that the specified transformation (46) obeys the group properties. Using the notation

$$\kappa(a,k)=\frac{q(k\cosh a+q\sinh a)}{k\sinh a+q\cosh a} \tag{48}$$

and introducing, in addition to S and S', the frame S_0 with the anisotropy parameter k_0, one can check that

$$\kappa(a,\kappa(a_0,k_0))=\kappa(a+a_0,k_0) \tag{49}$$

Similarly it is readily verified that $\kappa(-a,\kappa(a,k))=k$ and $\kappa(0,k)=k$. Alternatively, one can calculate the group generator $\chi(k)$ as

$$\chi(k)=\left.\frac{\partial\kappa(a,k)}{\partial a}\right|_{a=0}=q-\frac{k^2}{q} \tag{50}$$

and solve the initial value problem

$$\frac{\mathrm{d}k(a)}{\mathrm{d}a}=q-\frac{k(a)^2}{q},\quad k(0)=K \tag{51}$$

to be assured that it, as expected, yields (46). Thus, as a matter of fact, what is specified using the approximate relation (43) is the form of the group generator $\chi(k)$ in the group of transformations defined on the basis of the first principles.

The relation (42) allows defining a form of the group generator $\kappa(k)$ for arbitrary $F(\bar{\beta})$, not restricted by the approximate relation (43). Representing (42) in the form

$$f(k(K;a))=\frac{f(K)+\beta(a)(1-Kf(K))}{1+\beta(a)(-K+f(K))} \tag{52}$$

substituting (45) for $\beta(a)$ and differentiating the result with respect to *a*, with $\partial k(K;a)/\partial a$ separated, yields

$$\frac{\partial k(K;a)}{\partial a}=\frac{1-f(K)^2}{(\cosh a+f(K)\sinh a)^2 f'(k(a))} \tag{53}$$

Then the relation (52), with β substituted from (45), is used again to express $f(K)$ through $f(k)$ and *a*. Substituting that expression into (53) yields

$$\frac{\mathrm{d}k(a)}{\mathrm{d}a}=\frac{1-f^2(k(a))}{f'(k(a))} \tag{54}$$

Equation (54) is the Lie equation defining (with the initial condition $k(0)=K$)

the group transformation $k(K;a)$ which implies that the expression on the right-hand side is the group generator

$$\kappa(k) = \frac{1 - f^2(k)}{f'(k)} \tag{55}$$

6. Time Dilation, Aberration Law and Doppler Effect

Time dilation. Consider a clock C' placed at rest in S' at a point on the x-axis with the coordinate $x = x_1$. When the clock records the times $t = t_1$ and $t = t_2$ the clock in S which the clock C' is passing by at those moments will record times T_1 and T_2 given by the transformations (33) where it should be evidently set $x_2 = x_1$. Subtracting the two relations we obtain the time dilation relation

$$\Delta T = \frac{R^{-1}}{\sqrt{(1 - K\beta)^2 - \beta^2}} \Delta t \tag{56}$$

If clock were at rest in the frame S the time dilation relation would be

$$\Delta t = \frac{R(1 - K\beta - k\beta)}{\sqrt{(1 - K\beta)^2 - \beta^2}} \Delta T = \frac{R}{\sqrt{(1 - k\beta_-)^2 - \beta_-^2}} \Delta T \tag{57}$$

with β_- defined by (32).

Aberration law. The light aberration law can be derived using the formulas (35) for the velocity transformation. The relation between directions of a light ray in the two inertial frames S and S' is obtained by setting $U_X = c\cos\Theta / (1 + K\cos\Theta)$ and $u_x = c\cos\theta / (1 + k\cos\theta)$ in the first equation of (35). Then solving for $\cos\theta$ yields

$$\cos\theta = \frac{\cos\Theta - \beta(1 + K\cos\Theta)}{1 - \beta(\cos\Theta + K)} \tag{58}$$

where θ and Θ are the angles between the direction of motion and that of the light propagation in the frames of a moving observer and of an immovable source respectively. (Equation (58) could be obtained in several other ways, for example, straight from the transformations (27) and (28) by rewriting them in spherical coordinates and then specifying to radial light rays.) Introducing $\tilde{\theta} = \theta - \pi$ and $\tilde{\Theta} = \Theta - \pi$ as the angles between the direction of motion and the line of sight one gets the aberration law

$$\cos\tilde{\theta} = \frac{\cos\tilde{\Theta} + \beta(1 - K\cos\tilde{\Theta})}{1 + \beta(\cos\tilde{\Theta} - K)} \tag{59}$$

Doppler effect. Consider a source of electromagnetic radiation (light) in a reference frame S very far from the observer in the frame S' moving with velocity v with respect to S along the X-axis with Θ being the angle between the direction of the observer motion and that of the light propagation as measured in a frame of the source. Let two pulses of the radiation are emitted from the source with the time interval $(\delta T)_e$ (period). Then the interval $(\delta T)_r$

between the times of arrival of the two pulses to the observer, as measured by a clock in the frame of the source S, is

$$(\delta T)_r = (\delta T)_e + \frac{\delta L}{V} \tag{60}$$

where δL is a difference of the distances traveled by the two pulses, measured in the frame of the source S, and V is the speed of light in the frame S given by

$$\delta L = v(\delta T)_r \cos\Theta, \quad V = \frac{c}{1 + K\cos\Theta} \tag{61}$$

Substituting (61) into (60) yields

$$(\delta T)_e = (\delta T)_r \left(1 - \beta\cos\Theta(1 + K\cos\Theta)\right) \tag{62}$$

The interval $(\delta t)_r$ between the moments of receiving the two pulses by the observer in the frame S', as measured by a clock at rest in S', is related to $(\delta T)_r$ by the time dilation relation (56), as follows

$$(\delta T)_r = \frac{R^{-1}}{\sqrt{(1-K\beta)^2 - \beta^2}} (\delta t)_r \tag{63}$$

Thus, the periods of the electromagnetic wave measured in the frames of the source and the receiver are related by

$$(\delta T)_e = \frac{R^{-1}\left(1 - \beta\cos\Theta(1 + K\cos\Theta)\right)}{\sqrt{(1-K\beta)^2 - \beta^2}} (\delta t)_r \tag{64}$$

so that the relation for the frequencies is

$$\nu_r = \nu_e \frac{R^{-1}\left(1 - \beta\cos\Theta(1 + K\cos\Theta)\right)}{\sqrt{(1-K\beta)^2 - \beta^2}} \tag{65}$$

where ν_e is the emitted wave frequency and ν_r is the wave frequency measured by the observer moving with respect to the source. (This formula could be derived in several other ways, for example, using the condition of invariance of the wave phase.)

To complete the derivation of the formula for the Doppler shift, the relation (65) is to be transformed such that the angle θ between the wave vector and the direction of motion measured in the frame of the observer S' figured instead of Θ which is the corresponding angle measured in the frame of the source. Using the aberration formula (58), solved for $\cos\Theta$, as follows

$$\cos\Theta = \frac{\cos\theta + \beta(1 - K\cos\theta)}{1 + \beta(\cos\theta - K)} \tag{66}$$

in the relation (65) yields

$$\nu_r = \nu_e \frac{R^{-1}\left(1 + \beta\cos\theta(1 - K\cos\theta)\right)\sqrt{(1-K\beta)^2 - \beta^2}}{(1 - K\beta + \beta\cos\theta)^2} \tag{67}$$

Finally, introducing the angle $\tilde{\theta} = \theta - \pi$ between the line of sight and the direction of the observer motion one obtains the relation for a shift of frequencies due to the Doppler effect in the form

$$\nu_r = \nu_e \frac{R^{-1}\left(1-\beta\cos\tilde{\theta}\left(1+K\cos\tilde{\theta}\right)\right)\sqrt{\left(1-K\beta\right)^2-\beta^2}}{\left(1-K\beta-\beta\cos\tilde{\theta}\right)^2} \tag{68}$$

7. The CMB Effective Temperature

Let us apply the equations of the anisotropic special relativity developed above to describe effects caused by an observer motion (our galaxy's peculiar motion) with respect to the CMB frame. It is more consistent than using equations of the standard special relativity in that context—the standard relativity framework is in contradiction with existence of a preferred frame while the anisotropic special relativity naturally combines a preferred frame concept with the special relativity principles. Let choose the frame S to be a preferred frame and the frame S' to be a frame of an observer moving with respect to the preferred frame. Then the coordinate transformations from the preferred frame S to the frame S' of the moving observer are obtained by setting $K = 0$ in equations (27), (28), (47) and (44) which yields

$$x = \left(X - cT\beta\right)\left(1-\beta^2\right)^{\frac{q-1}{2}},\ ct = \left(cT\left(1-q\beta^2\right) - X\beta\left(1-q\right)\right)\left(1-\beta^2\right)^{\frac{q-1}{2}}$$

$$y = Y\left(1-\beta^2\right)^{\frac{q}{2}}, \quad z = Z\left(1-\beta^2\right)^{\frac{q}{2}} \tag{69}$$

where q is a universal constant. Equation of aberration of light (59) with $K = 0$ converts into the common aberration law of the standard theory

$$\cos\tilde{\theta} = \frac{\cos\tilde{\Theta} + \beta}{1+\beta\cos\tilde{\Theta}} \tag{70}$$

while Equation (65), describing the Doppler frequency shift for the light emitted at the last scattering surface (LSS) and received by a moving observer, differs from its counterpart of the standard relativity by the factor R^{-1}, as follows

$$\nu_r = \nu_e \frac{R^{-1}\left(1-\beta\cos\Theta\right)}{\sqrt{1-\beta^2}} \tag{71}$$

The inverse R^{-1} of (47) for $K = 0$ takes the form

$$R^{-1} = \left(1-\beta^2\right)^{-\frac{q}{2}} \tag{72}$$

Substituting (72) into (71) yields

$$\nu_r = \nu_e \left(1-\beta^2\right)^{-\frac{1}{2}-\frac{q}{2}}\left(1-\beta\cos\Theta\right) \tag{73}$$

Thus, in terms of the angle Θ between the direction of the observer motion and that of the light propagation as measured in a frame of the source, the Doppler frequency shift is a pure dipole pattern as it is in the standard relativity. However, the amplitude of the shift includes an additional factor which depends on the value of the universal constant q.

Equation (68) incorporating the effect of light aberration and thus relating the frequency ν_e of the light emitted at the LSS to the frequency ν_r measured by a moving observer, with the use of (72) becomes

$$\nu_r = \nu_e \frac{\left(1-\beta^2\right)^{\frac{1}{2}-\frac{q}{2}}}{1-\beta\cos\tilde{\theta}} \tag{74}$$

where $\tilde{\theta}$ is the angle between the line of sight and the direction of the observer motion as measured in the frame of the observer. In the context of the CMB anisotropy, one should switch from the frequencies to effective thermodynamic temperatures of the CMB blackbody radiation using the relation [29]

$$\frac{T\left(\tilde{\theta}\right)}{\nu_r} = \frac{T_0}{\nu_e} \tag{75}$$

where T_0 is the effective temperature measured by the observer which sees strictly isotropic blackbody radiation, and $T\left(\tilde{\theta}\right)$ is the effective temperature of the blackbody radiation for the moving observer looking in the fixed direction $\tilde{\theta}$. Substituting (74) into (75) yields

$$T\left(\tilde{\theta}\right) = \frac{M}{1-\beta\cos\tilde{\theta}}; \quad M = T_0\left(1-\beta^2\right)^{\frac{1}{2}-\frac{q}{2}} \tag{76}$$

Thus, the angular distribution of the CMB effective temperature seen by an observer moving with respect to the CMB frame is not altered by the light speed anisotropy. However, the anisotropy influences the mean temperature which differs from the value yielded by applying the standard relativity by the factor $\left(1-\beta^2\right)^{-\frac{q}{2}}$ (it may be also considered as a correction to the temperature T_0). Dependence of the amplitude factor M (normalized by T_0) on β for different values of the parameter q is shown in **Figure 1**. It is seen that, for negative values of q, the amplitude factor decreases with β, like as it does in the standard SR (q = 0), but the dependence becomes steeper. For positive values of q, the factor M may both decrease and increase with β and it does not depend on β for a specific value q = 1. Note, however, that q is expected to be negative both from intuitive considerations and on the basis of some arguments considering of which is beyond the scope of the current study.

Developing Equation (76) up to the second order in β yields

$$T\left(\tilde{\theta}\right) = T_0\left(1 + q\frac{\beta^2}{2} + \beta\cos\tilde{\theta} + \frac{\beta^2}{2}\cos 2\tilde{\theta}\right) \tag{77}$$

which implies that, up to the order β^2, the amplitudes of the dipole and quadrupole patterns remain the same, only the constant term is modified.

It is worth reminding that, even though the specified law (43) is linear in β, it does include the second order term which is identically zero. Thus, describing the anisotropy effects, which are of the order of β^2, by Equations (76) and (77) is legitimate.

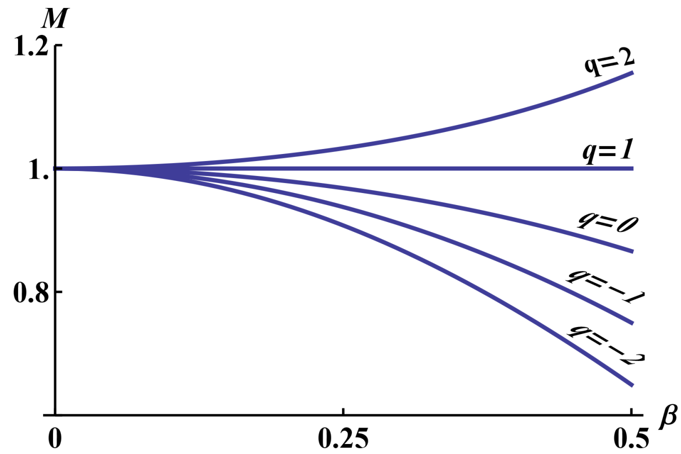

Figure 1. Dependence of the amplitude factor M (normalized by T_0) on the observer velocity β for different values of the parameter q.

8. Discussion

Analysis of the present paper, incorporating the existence of a preferred frame of reference into the special relativity framework, does not abolish the basic principles of special relativity but simply uses the freedom in applying those principles. A degree of anisotropy of the one-way velocity, which is commonly considered as irreducibly conventional, acquires meaning of a characteristic of the really existing anisotropy caused by motion of an inertial frame relative to the preferred frame. In that context, the fact, that there exists the inescapable entanglement between remote clock synchronization and one-way speed of light (if the synchronization is made using light signals), does not imply conventionality of the one-way velocity but means that, in the synchronization procedure, the one-way speed determined by the size of the anisotropy is used. The analysis yields equations differing from those of the standard relativity. The deviations depend on the value of an universal constant q where $q = 0$ corresponds to the standard relativity theory with the isotropic one-way speed of light in all the frames. The measurable effects following from the theory equations can be used to provide estimates for q and validate the theory.

Applying the theory to the problem of calculating the CMB temperature distribution is conceptually attractive since it removes the inconsistency of the usual approach when formulas of the standard special relativity, in which a preferred frame is not allowed, are applied to define effects caused by motion with respect to the preferred frame. It is worthwhile to note that even though it were found that the constant q is very small, which would mean that applying the present theory yields results practically identical to those of the standard relativity, this would not reduce the importance of the present framework which reconciles the principles of special relativity with the existence of the privileged CMB frame. As a matter of fact, it would justify the application of the standard relativity in that situation.

It is worthwhile, at the end of the discussion, to return to the much debated issues of conventionality of simultaneity and relativity of simultaneity in special relativity and discuss the approach and results of the present paper in the light of the debates. First of all, an important difference between motivations (and, correspondingly, conceptual frameworks) of the analyses devoted to those issues and the approach of the present study should be clarified and emphasized again.

The concept of anisotropy of light propagation is always discussed in the literature in relation with the concept of remote clock synchronization. Considering different synchronization procedures, as the rule, is aimed at obtaining the transformations possessing some specific properties. For example, in the work by Tangherlini [30], a special method of synchronizing two clocks in an inertial frame is proposed in order to achieve a universal synchronization, such that spatially separated clocks remain synchronous between themselves thus establishing the common time of the moving system. In [30], it is achieved by using clocks synchronized with absolute signals, that is, signals travelling with infinite or arbitrarily large velocity. Using these signals, one arrives at the view of an absolute rest frame (or ether frame), in which the velocity of light is the same in all directions, but for observers in motion relative to this frame velocity of light is not the same in all directions. Another method of synchronization of spatially separated clocks, which leads to the same transformations that Tangherlini obtained in [30], is the so-called "external synchronization" (see, e.g., [2] [31] [32]). The external synchronization is based on the assumption that there is a preferred ("rest") inertial frame in which the one-way speed of light in vacuum is c in all directions. The clocks from the rest system, S, are synchronized using Einsteins procedure with light signals. Then, in any moving inertial frame S', the common time can be established using these already synchronized clocks of the rest inertial frame. It can be done simply by adjusting clocks of moving inertial frame to zero during those moments of time when they meet in space a clock at rest that shows zero as well. Applying any of two synchronization methods described above, together with the postulate of constancy of the two-way speed of light, yields the transformations

$$x' = \gamma(x - vt),\ t' = \frac{t}{\gamma};\ \gamma = \frac{1}{\sqrt{1-\beta^2}},\ \beta = \frac{v}{c} \tag{78}$$

where (x,t) and (x',t') are space and time coordinates of a certain event in the rest frame S and in a moving frame S' respectively and v is a velocity of the frame S' relative to S. Thus, using the synchronization method, that is different from synchronization by light signals, yields the transformations (78) which exhibit absolute simultaneity. They also exhibit non-invariant one-way speed of light so that, in that approach, the anisotropy of the velocity of light in a moving inertial frame is a feature that emerges due to synchronization procedure designed to keep simultaneity unchanged between all inertial frames of reference.

The principal difference of the present analysis from those in the literature on the synchronization problem is that, in the present analysis, the one-way speed of light in an inertial frame is a primary issue and its anisotropy is governed entirely by a physical lawc (36) (or its approximate version (43)). If a preferred frame is identified then the law (36) defines unequivocally the anisotropy size. Note that there is no ambiguity in determining the velocity $\bar{\beta}$ since it is measured in a preferred frame where the one-way speed of light is c in all directions. At the same time, the relativity principle is not violated since transformations of the parameter of anisotropy k from one inertial frame to another possess a group property and, in this respect, transformations from/to the preferred frame with $k = 0$ are not distinguished from other members of the group of transformations. Specifying the function $F(\bar{\beta})$ (or the inverse function $f(k)$) is equivalent to specifying the group generator for the variable k according to (55). In such a framework, synchronization is a concomitant issue if the remote clocks are set using light signals. In particular, since the transformations (27) are derived based on invariance of the equation of anisotropic light propagation, they correspond to the synchronization procedure using light signals with the one-way velocities defined by the relation (6) but, provided that the velocity of the frame relative to the preferred frame $\bar{\beta}$ is known, in the relation (6), *k is a definite value determined by the law* (36). That value cannot be altered by changing the synchronization method.

The same is valid if another method of synchronization, as, for example, the above discussed "external synchronization", is used. Changing the synchronization method results in a change of the form of transformations for the time and space variables which is equivalent to a change of coordinates. The Lorentz transformations

$$x'_L = \gamma(x - vt),\ t'_L = \gamma\left(t - \frac{vx}{c^2}\right) \tag{79}$$

can be obtained from the Tangherlini transformations (78) by the change of coordinates [32]

$$t'_L = t' - \frac{vx'}{c^2} \tag{80}$$

where t' and x' are defined by (78). Substituting (78) in (80), one gets the Lorentz transformations. The same can be done for the transformations (27) obtained in the present paper. In the case of the transformations from a preferred frame with the anisotropy parameter $K = 0$ to an arbitrary frame with the anisotropy parameter k, the transformations (27) take the form

$$x = R\gamma(X - cT\beta),\ ct = R\gamma\left(cT(1 - k\beta) - X(\beta - k)\right),\ \gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{81}$$

where R is the scale factor defined by (24) (for the sake of clearness, we do not use the law (43) in these calculations). The transformations that exhibit absolute simultaneity, a counterpart of the Tangherlini transformations, are

$$x^{(T)} = R\gamma(X - cT\beta),\ ct^{(T)} = R\frac{cT}{\gamma} \tag{82}$$

and the change of variables converting (82) into (81) is

$$ct = ct^{(T)} - (\beta - k) x^{(T)} \tag{83}$$

It is readily verified that substituting (82) in (83) yields (81). Thus, any event that can be described by the transformations (81) can be described as well by the transformations with absolute simultaneity (82). Descriptions using clocks set as in (81) and clocks set as in (82) are equivalent in a sense that they are describing one and the same reality, which is independent of the coordinates chosen.

It is also worth remarking that alterations, as compared with the standard relativity, in the formulas describing physical effects caused by motion with respect to a preferred frame depend only on the scale factor R as, for example, a correction to the distribution (76) of the CMB effective temperature seen by an observer moving with respect to the CMB frame. In (76), R is given by the expression

$$R = \left(1 - \beta^2\right)^{\frac{q}{2}} \tag{84}$$

defining R as a function of the velocity of a moving frame measured in a preferred frame. The expression (84) has been obtained from (47) evaluated for $K = 0$ and so it corresponds to the approximate law (43) but it is possible to represent R defined by the general expression (24) as a function of $\bar{\beta}$ for arbitrary $F(\bar{\beta})$. It is evident that the form $R(\bar{\beta})$ of the scale factor does not depend on the synchronization (or on the space-time coordinates) chosen.

To conclude the discussion, the present analysis, which combines the basic principles of special relativity with the existence of a preferred frame, stands apart from the ample literature devoted to the conventionality of simultaneity, relativity of simultaneity and synchronization issues. In the present analysis, anisotropy of the one way speed of light in an inertial frame is governed by a physical law which is not influenced by changing the synchronization procedure. Synchronization emerges as a complementary issue needed for defining transformations of the space-time coordinates but physical effects are not changed by the way the clocks have been set.

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