

Peristaltic Transport of Magnetohydrodynamic Carreau Nanofluid with Heat and Mass Transfer inside Asymmetric Channel

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Abstract

In this work, the peristaltic motion of a nano non-Newtonian fluid which obeys Carreau model through a porous medium inside an asymmetric channel is investigated. The hall current effects with Joule heating and viscous dissipation are considered. The problem is modulated mathematically by a set of nonlinear partial differential equations which describe the conservation of mass, momentum, energy and concentration of nanoparticles. The non-dimensional form of these equations is simplified under the assumption of long wavelength and low Reynolds number, and then resulting equations of coupled nonlinear differential equations are tackled numerically with appropriate boundary conditions. Graphical results are presented for dimensionless velocity, temperature, concentration and pressure gradient in order to illustrate the variations of various parameters of this problem on these obtained solutions.

Keywords

Peristaltic Motion, MHD, Carreau Nanofluid, Asymmetric Channel, Porous Medium, Heat and Mass Transfer

1. Introduction

Nowadays, the study of nanofluids flow has the interest of researches because of its applications in medicine, biochemistry and industrial engineering. Nanofluids are moderately new category of fluids which consist of a base-fluid with nano-sized particles (1 - 100 nm) suspended within them. Choi [1] may be the first author to use the term “nanofluid”, where it was reported that one of the prom-

ising nanofluids applications in heat could transfer enhancement. In [2], Choi *et al.* showed that the addition of a small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increase the thermal conductivity of the fluid up to approximately two times. An analysis of nanofluids examined by Buongiorno [3] was induced that this massive increase in the thermal conductivity occurs due to the presence of the Brownian diffusion and the thermophoretic diffusion of the nanoparticles. Recent articles on the nanofluid are cited in ReFS [4] [5] [6] [7] [8].

Peristaltic motion in a channel or tube is considered as a type of flow that has great value in several physiological processes and industries. Peristalsis is a mechanism for mixing and transporting fluids through expansion and contraction of the wave propagation along the channel walls. This mechanism is seen in many biological system such as urine transport from kidney to bladder through the ureter, transport of lymph in the lymphatic vessels, swallowing food through the esophagus, the movement of chime in the gastrointestinal track, ovum movement in the fallopian tube, transport of spermatozoa vasomotion of small blood vessels such as venules and capillaries and blood flow in arteries, transport of corrosive fluids, sanitary fluid transport and blood pumps in heart lung machine etc. This analysis was first investigated by Latham [9]. He presented fluid motion in peristaltic pump and also discussed the characteristic of pressure rise versus flow rate. After the work of Latham [9], Jaffrin and Shapiro [10] investigated the peristaltic pumping. They made the study under the assumption of long wavelength and low Reynolds number approximation. This work was further extended for Newtonian and non-Newtonian fluids with different flow geometries and boundary conditions as elaborated in the references [11]-[17].

Recently, Eldabe *et al.* [18] have discussed peristaltic transport of a magneto non-Newtonian fluid through a porous medium in a horizontal finite channel. Safia Akram [19] has investigated the effects of nanofluid on peristaltic flow of a Carreau fluid model in an inclined magnetic field. Arshad *et al.* [20] have analyzed the peristaltic transport of a Carreau fluid in a compliant rectangular duct. Khalid Nowar [21] has studied peristaltic flow of a nanofluid under the effect of Hall current and porous medium. Noreen *et al.* [22] have examined numerical simulation of peristaltic flow of a Carreau naofluid in an asymmetric channel. Tasawar *et al.* [23] have discussed the radiative peristaltic flow of Jeffery nanofluid with slip conditions and joule heating.

Few attempts have been devoted to peristaltic flows in presence of heat and mass transfer; such investigations are of great importance, which is due to their extensive applications in medical and bio-engineering sciences, as it may be relevant in many processes in human body like oxygenation in lungs, hemodialysis and nutrients diffuse out of blood. Sohail and nadeem [24] have examined the effects of heat and mass transfer on peristaltic flow of Carreau fluid in a vertical annulus. Eldabe *et al.* [25] have studied the magneto-hydrodynamic flow and heat transfer for a peristaltic motion of Carreau fluid through a porous medium. Eldabe *et al.* [26] have studied the peristaltic motion of non-Newtonian fluid

with heat and mass transfer through a porous medium in channel under uniform magnetic field. Sohail *et al.* [27] have discussed the effects of heat and mass transfer on peristaltic flow of a nanofluid between eccentric cylinders. Eldabe *et al.* [28] have discussed the peristaltic pumping of a conduction Sisko fluid through porous medium with heat and mass transfer. Ramesh and Devaker [29] have studied the effects of heat and mass transfer on the peristaltic transport of MHD couple stress fluid through porous medium in a vertical asymmetric channel.

The main aim of this work is to study the peristaltic motion of a Carreau nanofluid with heat and mass transfer through a porous medium in an asymmetric channel under the effects of Hall current, viscous dissipation and Joule heating. The analysis is performed under the well-established long wavelength and low Reynolds number approximations. A detailed mathematical formulation is presented and numerical solution graphically for velocity, temperature, nanoparticle phenomena and pressure gradient have been presented.

2. Mathematical Formulation of the Problem

Consider the peristaltic flow of an incompressible viscous electrically conducting nanofluid which obeys Carreau model inside a two dimensional vertical asymmetric channel of width $d_1 + d_2$ through a porous medium. Asymmetry in the channel is produced by propagation of waves along the channel walls traveling with different amplitudes, phases but with constant speed. In the Cartesian coordinates system (X, Y) , the right-hand side wall $Y = H_1$ and the left-hand side wall $Y = H_2$ are given by

$$Y = H_1(X, t) = d_1 + a_1 \cos\left(\frac{2\pi}{\lambda}(X - ct)\right) \quad (1)$$

$$Y = H_2(X, t) = -d_2 - b_1 \cos\left(\frac{2\pi}{\lambda}(X - ct) + \phi\right) \quad (2)$$

where, a_1 and b_1 are the amplitudes of right and left walls respectively, λ is the wavelength, t is the time, and ϕ is the phase which varies in the range $0 \leq \phi \leq \pi$. When $\phi = 0$ then symmetric channel with waves out of the phase can be described (*i.e.* both walls move outward or inward simultaneously), and for $\phi = \pi$, the waves are in phase. Further d_1, d_2, a_1, b_1 and ϕ satisfy the condition:

$$a_1^2 + b_1^2 + 2a_1b_1 \cos\phi \leq (d_1 + d_2)^2, \quad (3)$$

so that the walls will not intersect with each other.

A strong uniform magnetic field with magnetic flux density $\mathbf{B} = (0, 0, B_0)$ is applied and the Hall effects are taken into account. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number ($Re_m \ll 1$), also it is assumed that there is no applied polarization voltage so that the total electric field $\mathbf{E} = \mathbf{0}$. The expression for the current density \mathbf{J} including the Hall effect and neglecting ion-slip and thermoelectric effects [The generalized Ohm's law] is given by [30]:

$$\mathbf{J} = \sigma \left[\mathbf{E} + \mathbf{V} \times \mathbf{B} - \frac{1}{en_e} (\mathbf{J} \times \mathbf{B}) \right] \quad (4)$$

where σ is the electrical conductivity of the fluid, \mathbf{V} is the velocity vector, e is the electric charge of electrons, n_e is the number density of electrons. Equation (1) can be solved in \mathbf{J} to yield the Lorentz force vector in the form:

$$\mathbf{J} \times \mathbf{B} = \frac{-\sigma B_0^2}{1+m^2} \left[(U-mV)\hat{i} + (mU+V)\hat{j} \right] \quad (5)$$

where U and V are the X and Y components of the velocity vector, $m = \frac{\sigma B_0}{en_e}$ is

the hall parameter. The heat transfer and nanoparticle processes are maintained by considering temperature T_0, T_1 and nanoparticle phenomena C_0, C_1 to the right and left sides wall, respectively.

The constitutive equation for a Carreau fluid is given by [24]:

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + (\Gamma \dot{\gamma})^2 \right]^{\frac{n-1}{2}}, \quad (6)$$

$$\tau_{ij} = \eta_0 \left[1 + \frac{n-1}{2} (\Gamma \dot{\gamma})^2 \right] \dot{\gamma}_{ij}, \quad (7)$$

in which τ_{ij} is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, n is the power law index, and $\dot{\gamma}$ is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \dot{\gamma}_{ij} \dot{\gamma}_{ji}} = \sqrt{\frac{1}{2} \mathbf{\Pi}} \quad (8)$$

where $\mathbf{\Pi} = \text{trac} \left(\text{grad} \mathbf{V} + (\text{grad} \mathbf{V})^T \right)^2$ is the second invariant strain tensor.

The governing equations for the present problem are described as:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (9)$$

Equations of motion

$$\begin{aligned} \rho_f \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) \\ = -\frac{\partial P}{\partial X} + \frac{\partial \tau_{xx}}{\partial X} + \frac{\partial \tau_{xy}}{\partial Y} - \frac{\eta_0}{k_1} U + \frac{\sigma B_0^2}{1+m^2} (mV-U) + \rho_f g \alpha (T-T_0) + \rho_f g \alpha (C-C_0) \end{aligned} \quad (10)$$

$$\rho_f \left(\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{\partial \tau_{yx}}{\partial X} + \frac{\partial \tau_{yy}}{\partial Y} - \frac{\eta_0}{k_1} V + \frac{\sigma B_0^2}{1+m^2} (mU+V) \quad (11)$$

Energy equation

$$\begin{aligned} (\rho c)_f \left(\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right) \\ = K \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) + \left[\tau_{xx} \frac{\partial U}{\partial X} + \tau_{xy} \frac{\partial U}{\partial Y} + \tau_{yx} \frac{\partial V}{\partial X} + \tau_{yy} \frac{\partial V}{\partial Y} \right] + \frac{\sigma B_0^2}{1+m^2} (U^2 + V^2) + (\rho c)_p \left\{ \begin{aligned} & D_B \left[\frac{\partial C}{\partial X} \frac{\partial T}{\partial X} + \frac{\partial C}{\partial Y} \frac{\partial T}{\partial Y} \right] \\ & + \frac{D_T}{T_0} \left[\left(\frac{\partial T}{\partial X} \right)^2 + \left(\frac{\partial T}{\partial Y} \right)^2 \right] \end{aligned} \right\} \end{aligned} \quad (12)$$

Concentration equation

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = D_B \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + \frac{D_T}{T_0} \left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) \quad (13)$$

where ρ_f is the density of the fluid, P is the pressure, k_1 is the permeability of the porous medium, g is the acceleration due to gravity, α is the volume expansion coefficient, T is the temperature of the fluid, C is the nanoparticle concentration, $(\rho c)_f$ is the heat capacity of the fluid, K is the thermal conductivity, $(\rho c)_p$ is the effective heat capacity of the nanoparticle material, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion.

Introducing a wave frame (x, y) moving with the velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad p(x) = P(X, t) \quad (14)$$

in which $(x, y), (u, v)$ and P are the coordinates, velocity components and pressure in the wave frame.

Defining the following non-dimensional quantities

$$\begin{aligned} \bar{x} &= \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{d_1}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{c}, \quad \bar{t} = \frac{t}{\lambda}, \quad \delta = \frac{d_1}{\lambda}, \quad d = \frac{d_2}{d_1}, \quad a = \frac{a_1}{d_1}, \\ b &= \frac{b_1}{d_1}, \quad h_1 = \frac{H_1}{d_1}, \quad h_2 = \frac{H_2}{d_2}, \quad \bar{p} = \frac{d_1^2 p}{\eta_0 c \lambda}, \quad Re = \frac{\rho_f c d_1}{\eta_0}, \quad \bar{\psi} = \frac{\psi}{c d_1}, \\ \bar{\tau}_{xx} &= \frac{\lambda}{\eta_0 c} \tau_{xx}, \quad \bar{\tau}_{xy} = \frac{d_1}{\eta_0 c} \tau_{xy}, \quad \bar{\tau}_{yy} = \frac{d_1}{\eta_0 c} \tau_{yy}, \quad We = \frac{\Gamma c}{d_1}, \quad \bar{\gamma} = \frac{d_1 \dot{\gamma}}{c}, \\ s &= \frac{k_1}{d_1^2}, \quad M = \sqrt{\frac{\sigma}{\eta_0}} B_0 d_1, \quad \bar{\theta} = \frac{T - T_0}{T_1 - T_0}, \quad \bar{\Omega} = \frac{C - C_0}{C_1 - C_0}, \quad Pr = \frac{\mu}{\rho_f \alpha}, \\ Gr &= \frac{\rho_f g \alpha d_1^2 (T_1 - T_0)}{c \eta_0}, \quad Br = \frac{\rho_f g \alpha d_1^2 (C_1 - C_0)}{c \eta_0}, \quad E_c = \frac{c_1^2}{c_f (T_1 - T_0)}, \\ S_c &= \frac{\mu}{\rho_f D_B}, \quad Nb = \frac{\tau D_B (C_1 - C_0)}{\alpha}, \quad N_t = \frac{\tau D_B (T_1 - T_0)}{\alpha}, \\ \alpha &= \frac{K}{(\rho c)_f}, \quad \tau = \frac{(\rho c)_p}{(\rho c)_f}. \end{aligned} \quad (15)$$

where δ is the dimensionless wave number, Re is the Reynolds number, We is the Weissenberg number, s is the porosity parameter, M is the magnetic parameter, Gr is the local temperature Grashof number, Br is the local nanoparticle Grashof number, Pr is the Prandtl number, E_c is the Eckert number, S_c is the Schmidt number, Nb is the Brownian motion parameter and N_t is the thermophoresis parameter.

Using Equation (14) and the above set of non-dimensional quantities (15) into Equations (10)-(13), the resulting equations in terms of stream function

$$\psi \left(u = \frac{\partial \psi}{\partial x}, \quad v = -\delta \frac{\partial \psi}{\partial y} \right) \text{ can be written after dropping bars in the following}$$

Non-dimensional form as:

$$\begin{aligned} & \text{Re} \delta \left[\left(\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \frac{\partial \psi}{\partial y} \right] \\ &= -\frac{\partial p}{\partial x} + \delta^2 \frac{\partial}{\partial x} (\tau_{xx}) + \frac{\partial}{\partial y} (\tau_{xy}) - \delta \frac{mM^2}{1+m^2} \left(\frac{\partial \psi}{\partial x} \right) - \frac{M^2}{1+m^2} \left(\frac{\partial \psi}{\partial y} + 1 \right) - \frac{1}{s} \left(\frac{\partial \psi}{\partial y} + 1 \right) + Gr\theta + Br\Omega \end{aligned} \quad (16)$$

$$\begin{aligned} & \text{Re} \delta^3 \left[\left(-\frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial}{\partial y} \right) \frac{\partial \psi}{\partial x} \right] \\ &= -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial}{\partial x} (\tau_{yx}) + \delta \frac{\partial}{\partial y} (\tau_{yy}) - \delta \frac{mM^2}{1+m^2} \left(\frac{\partial \psi}{\partial y} - 1 \right) - \delta^2 \frac{M^2}{1+m^2} \left(\frac{\partial \psi}{\partial y} + 1 \right) - \delta \frac{1}{s} \left(\frac{\partial \psi}{\partial x} \right) \end{aligned} \quad (17)$$

$$\begin{aligned} \text{Re} \delta \left[\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right] &= \frac{1}{P_r} \left(\frac{\partial^2 \theta}{\partial y^2} + \delta^2 \frac{\partial^2 \theta}{\partial x^2} \right) + E_c \left[\begin{aligned} & \left(\delta^2 \tau_{xx} \frac{\partial^2 \psi}{\partial x \partial y} + \tau_{xy} \frac{\partial^2 \psi}{\partial y^2} \right) \\ & \left(-\delta^2 \tau_{yx} \frac{\partial^2 \psi}{\partial x^2} - \delta \tau_{yy} \frac{\partial^2 \psi}{\partial y \partial x} \right) \end{aligned} \right] \end{aligned} \quad (18)$$

$$+ E_c \frac{M^2}{1+m^2} \left[\frac{\delta^2 \left(\frac{\partial \psi}{\partial x} \right)^2}{\left(\frac{\partial \psi}{\partial y} + 1 \right)^2} \right] + E_c \frac{M^2}{1+m^2} \left[\frac{\delta^2 \left(\frac{\partial \psi}{\partial x} \right)^2}{\left(\frac{\partial \psi}{\partial y} + 1 \right)^2} \right] + \frac{N_t}{\text{Pr}} \left[\delta^2 \left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2 \right]$$

$$\text{Re} \delta S_c \left[\frac{\partial \psi}{\partial y} \frac{\partial \Omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \Omega}{\partial y} \right] = \left(\frac{\partial^2 \Omega}{\partial y^2} + \delta^2 \frac{\partial^2 \Omega}{\partial x^2} \right) + \frac{N_t}{N_b} \left(\frac{\partial^2 \theta}{\partial y^2} + \delta^2 \frac{\partial^2 \theta}{\partial x^2} \right) \quad (19)$$

where

$$\tau_{xx} = 2 \left[1 + \frac{n-1}{2} We^2 \dot{\gamma}^2 \right] \frac{\partial^2 \psi}{\partial x \partial y}, \quad (20)$$

$$\tau_{xy} = \left[1 + \frac{n-1}{2} We^2 \dot{\gamma}^2 \right] \left(\frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right) = \tau_{yx}, \quad (21)$$

$$\tau_{yy} = 2\delta \left[1 + \frac{n-1}{2} We^2 \dot{\gamma}^2 \right] \frac{\partial^2 \psi}{\partial x \partial y}, \quad (22)$$

and,

$$\dot{\gamma} = \left[2\delta^2 \left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right)^2 + 2\delta^2 \left(\frac{\partial^2 \psi}{\partial y \partial x} \right)^2 \right]^{\frac{1}{2}} \quad (23)$$

Under the assumption of long wavelength $\delta \ll 1$ and low Reynolds number, Equations (16)-(19) become

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left[1 + \frac{n-1}{2} We^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \right] \frac{\partial^2 \psi}{\partial y^2} - \left(\frac{M^2}{1+m^2} + \frac{1}{s} \right) \left[\frac{\partial \psi}{\partial y} + 1 \right] + Gr\theta + Br\Omega \quad (24)$$

$$0 = -\frac{\partial p}{\partial y} \quad (25)$$

$$0 = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + E_c \left[\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + \frac{n-1}{2} We^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right)^4 \right] + E_c \frac{M^2}{1+m^2} \left(\frac{\partial \psi}{\partial y} + 1 \right)^2 + \frac{N_b}{\text{Pr}} \left(\frac{\partial \Omega}{\partial y} \frac{\partial \theta}{\partial y} \right) + \frac{N_t}{\text{Pr}} \left(\frac{\partial \theta}{\partial y} \right)^2 \quad (26)$$

$$0 = \frac{\partial^2 \Omega}{\partial y^2} + \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial y^2} \quad (27)$$

Eliminating the pressure from Equations (24) and (25) yields

$$\frac{\partial^2}{\partial y^2} \left[\left(1 + \frac{n-1}{2} We^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \right) \frac{\partial^2 \psi}{\partial y^2} - \left(\frac{M^2}{1+m^2} + \frac{1}{s} \right) \psi \right] + Gr \frac{\partial \theta}{\partial y} + Br \frac{\partial \Omega}{\partial y} = 0 \quad (28)$$

The non-dimensional boundaries will take the form

$$h_1 = 1 + a \cos(2\pi x), \quad h_2 = -d - b \cos(2\pi x + \phi) \quad (29)$$

where a, b, ϕ, d satisfies the relation

$$a^2 + b^2 + 2ab \cos \phi \leq (1+d)^2 \quad (30)$$

The corresponding boundary conditions are

$$\begin{aligned} \psi &= \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 0, \quad \Omega = 0, \quad \text{at } y = h_1 \\ \psi &= -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} = -1, \quad \theta = 1, \quad \Omega = 1, \quad \text{at } y = h_2 \end{aligned} \quad (31)$$

where F is the dimensionless average flux in the wave frame defined by

$$F = \int_{h_2(x)}^{h_1(x)} \frac{\partial \psi}{\partial y} dy \quad (32)$$

The time mean Q in the wave frame is defined by

$$Q = F + 1 + d \quad (33)$$

3. Results and Discussion

In this work, a program was designed by Mathematica Software (version 10) simulate the application of parametric ND Solve package to find the numerical behaviors of our dimensionless system. Graphical results to the velocity u , the temperature θ , the nano particle concentration Ω and the pressure gradient $\frac{dp}{dx}$ are obtained under the impact of emerging parameters at moving boundaries of the fluid.

3.1. Velocity Profile

Figures 1-5 represent the impact of the Weissenberg number we , the power law index n , the magnetic parameter M , the thermophoresis parameter n_t and flow rate Q on the velocity profile. **Figure 1** and **Figure 2** depict that the behaviour of the velocity near the channel walls and at the center is not similar in view of we and n . The velocity field increases with the increase of we and n near the channel walls, however it decreases at the center of the channel. **Figure 3** and **Figure 4** show that the effect of M and n_t on the velocity profile is the same and opposite to that of we and n , as by increasing M and n_t the velocity decreases near the channel walls and increases at the center of the channel, but a reduction in u occurs and variation in u becomes narrow in some sections of the

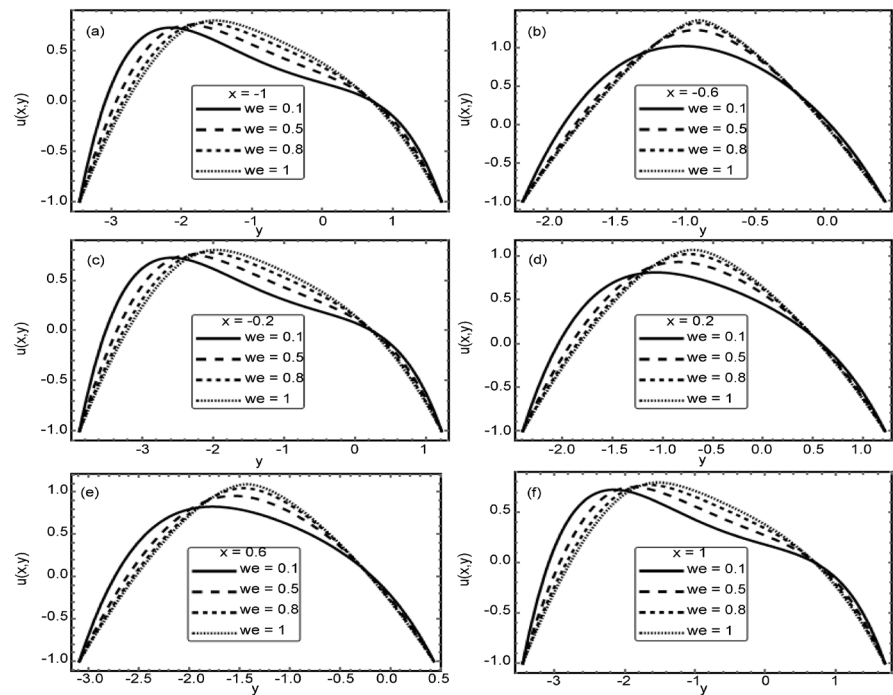


Figure 1. The velocity profile u is plotted against y for several values of We at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

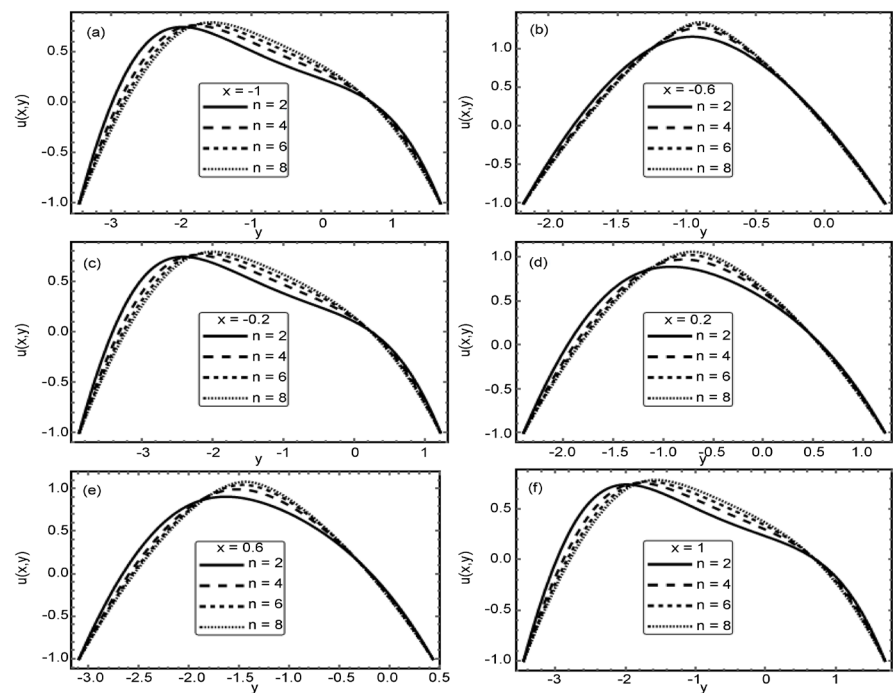


Figure 2. The velocity profile u is plotted against y for several values of n at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

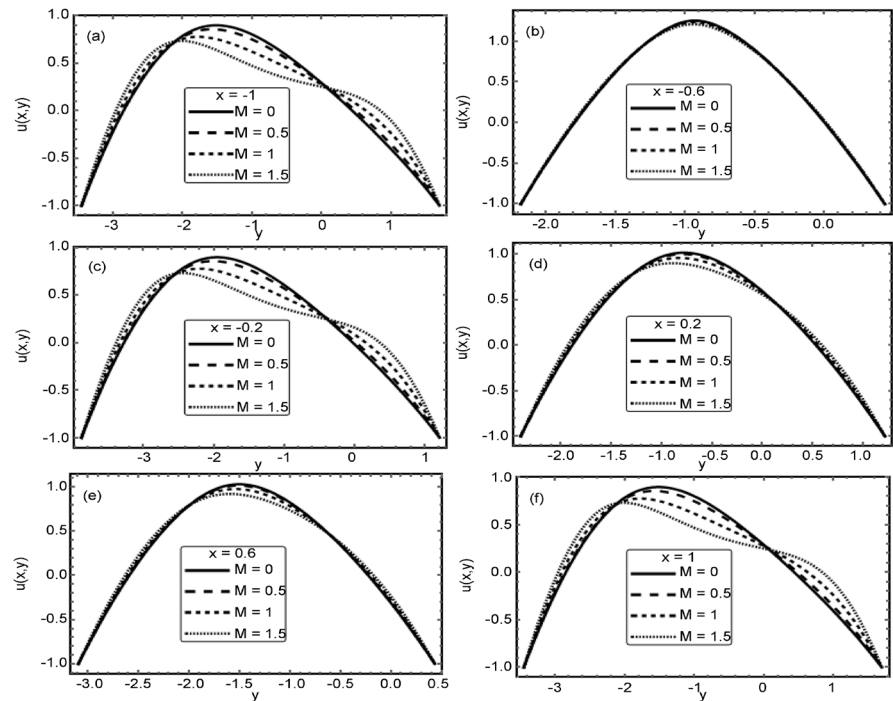


Figure 3. The velocity profile u is plotted against y for several values of M at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

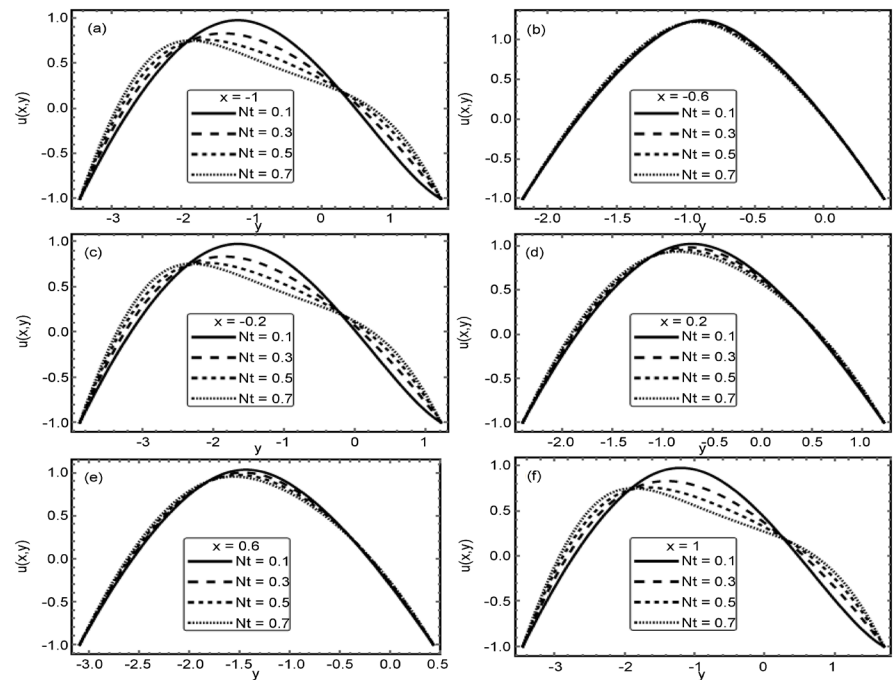


Figure 4. The velocity profile u is plotted against y for several values of Nt at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

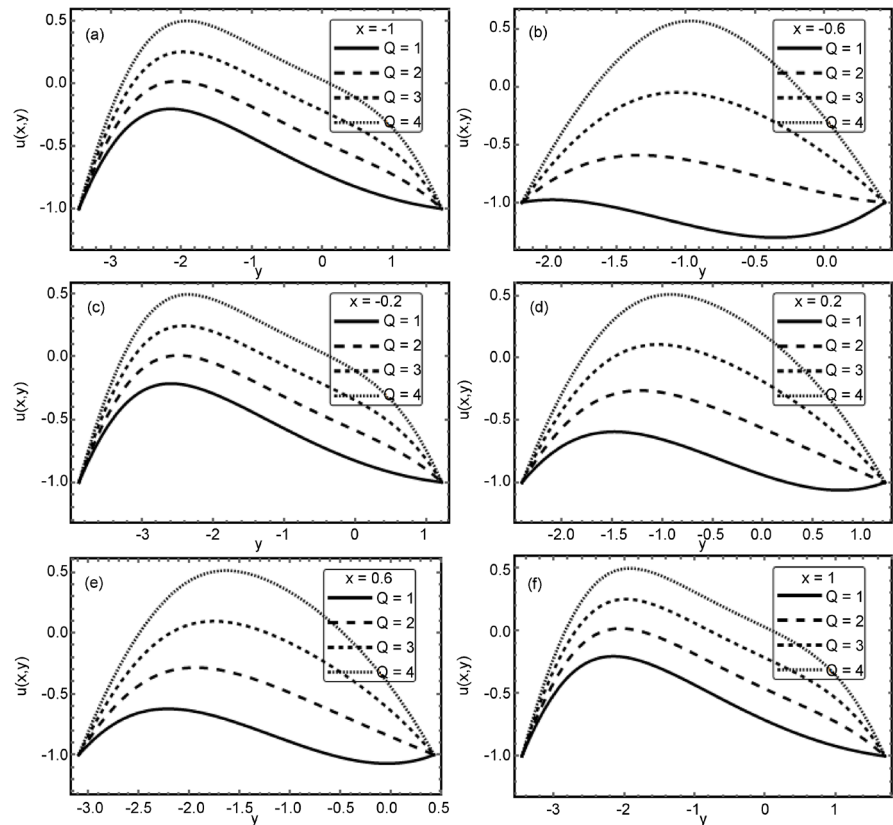


Figure 5. The velocity profile u is plotted against y for several values of Q for $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

fluid motion see (Figure 3(b), Figure 3(d) and Figure 3(e)), and Figure 4(b), Figure 4(d) and Figure 4(e). It is noticed through Figure 5 that the velocity profile increases with increasing Q .

3.2. Temperature Profile

The variation of temperature profile for different values of the magnetic parameter M , the Brownian motion parameter Nb , the Prandtl number Pr the local nanoparticle Grashof number br and the thermophoresis parameter n_t are plotted in Figures 6-10. It is clear that the temperature profile increases when there is an increase in M , Nb and Pr ; however it decreases with the increase in br and n_t .

3.3. Concentration Profile

Figures 11-15 describe the variation of the concentration profile for several values of the magnetic parameter M , the prandtl number Pr , the thermophoresis parameter n_b , the Brownian motion parameter Nb and the local nanoparticle Grashof number br . From Figures 11-13 it is clear that by increasing M , Pr , and n_b , the concentration profile decreases, while from Figure 14 and Figure 15 we observe that the concentration profile increases with the increase in Nb and n_t .

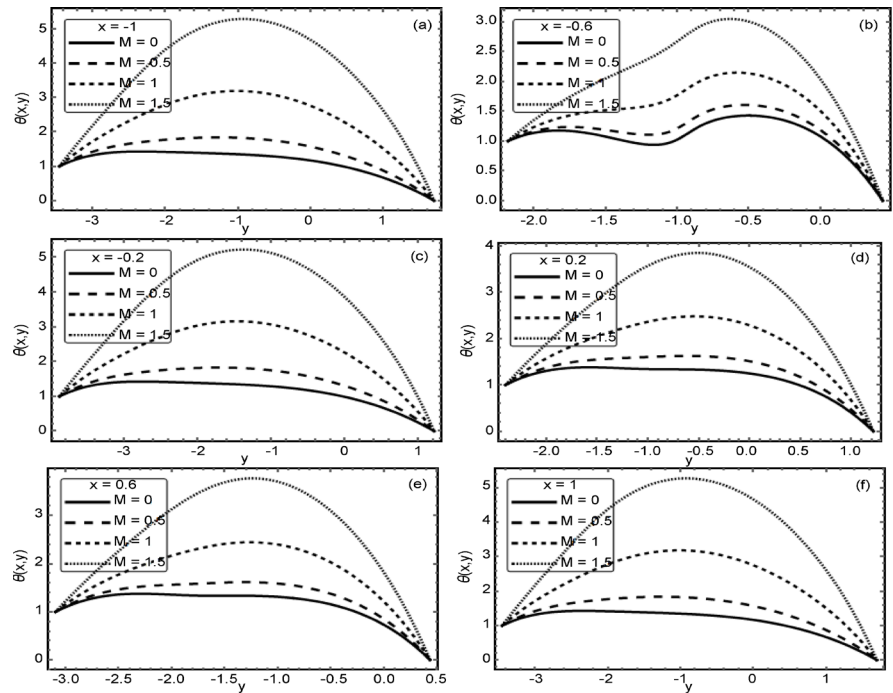


Figure 6. The temperature profile θ is plotted against y for several values of M at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

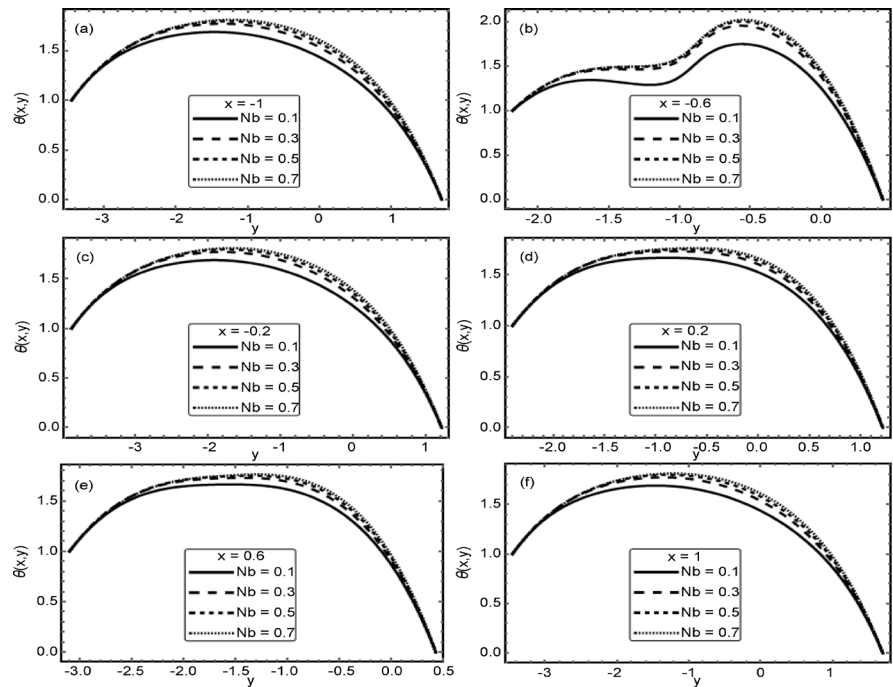


Figure 7. The temperature profile θ is plotted against y for several values of Nb at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 0.5$, $N_t = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

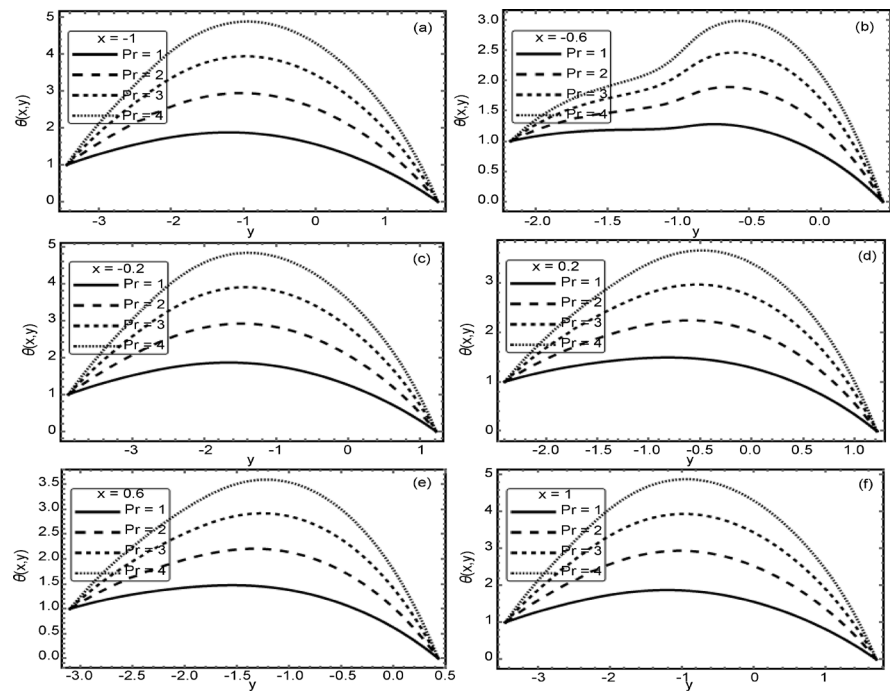


Figure 8. The temperature profile θ is plotted against y for several values of Pr at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 0.5$, $N_l = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

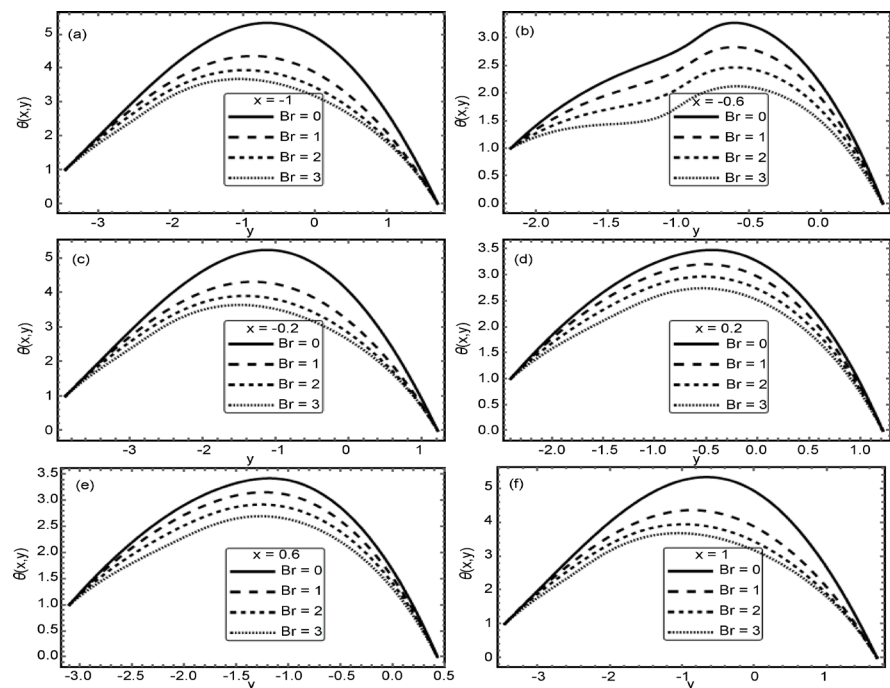


Figure 9. The temperature profile θ is plotted against y for several values of Br at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_l = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

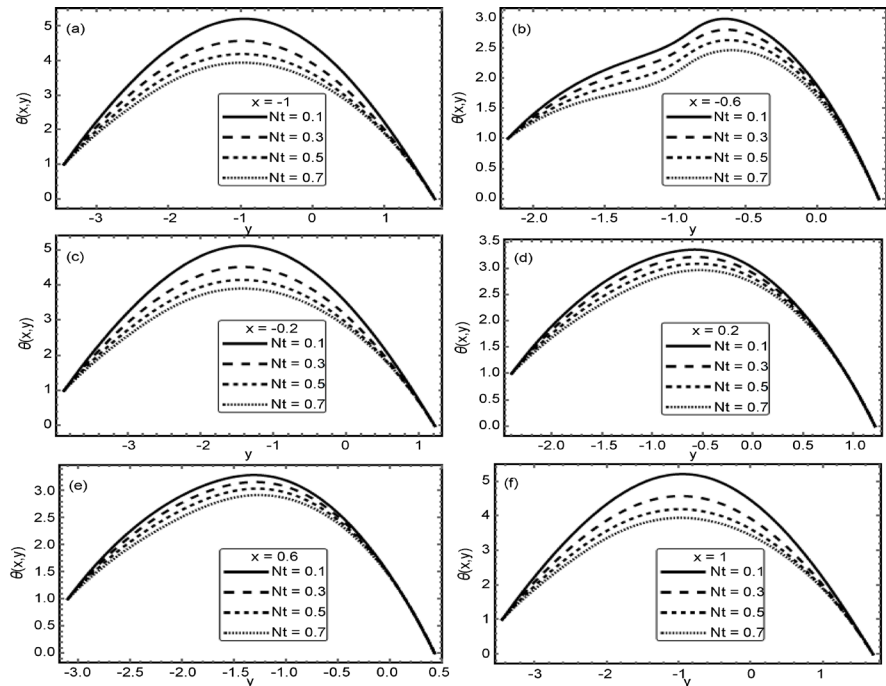


Figure 10. The temperature profile θ is plotted against y for several values of Nt at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

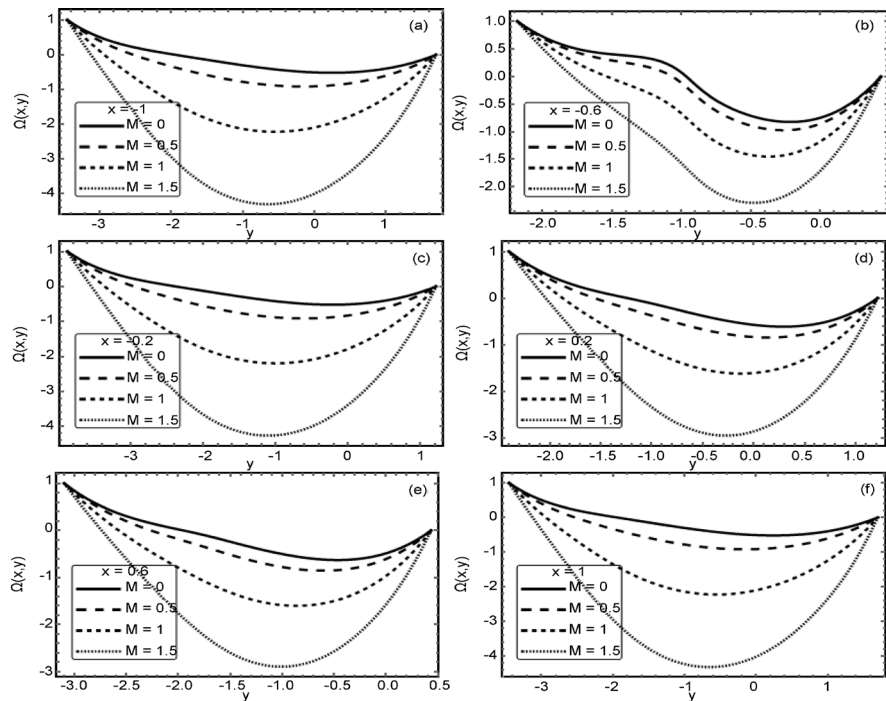


Figure 11. The concentration profile Ω is plotted against y for several values of M at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $Nt = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

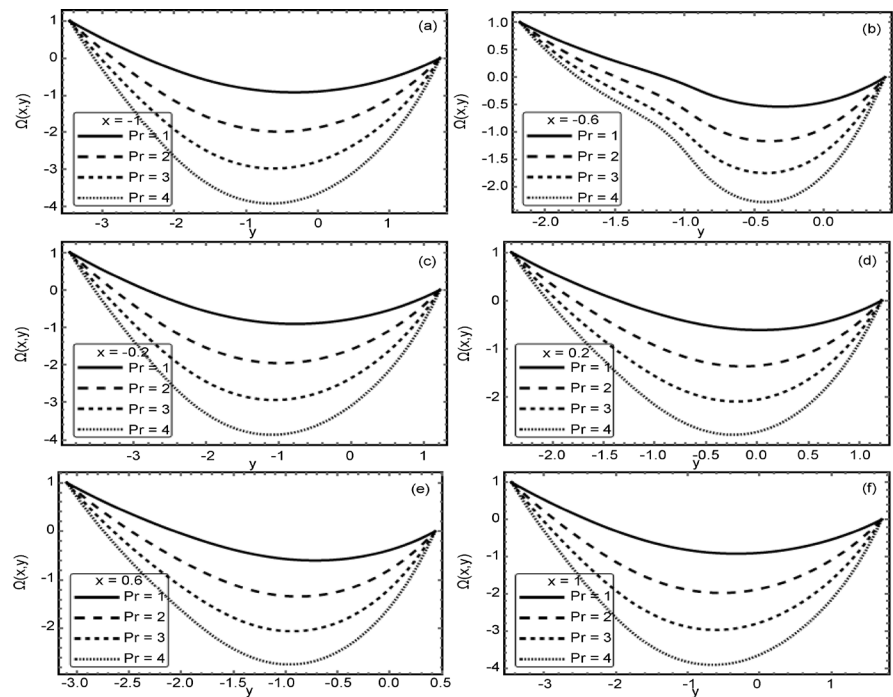


Figure 12. The concentration profile Ω is plotted against y for several values of Pr at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $M = 1.2$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

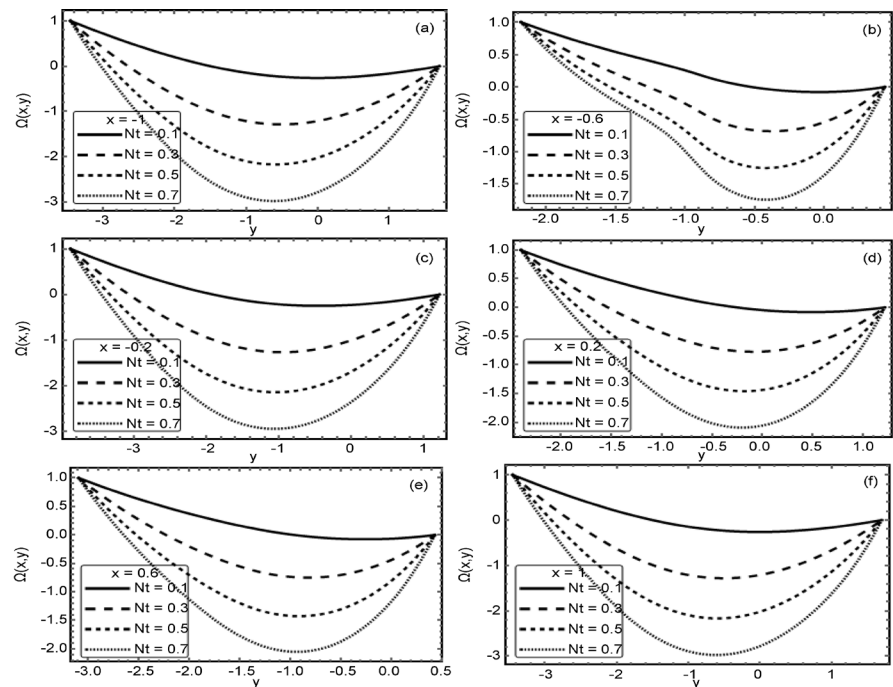


Figure 13. The concentration profile Ω is plotted against y for several values of Nt at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

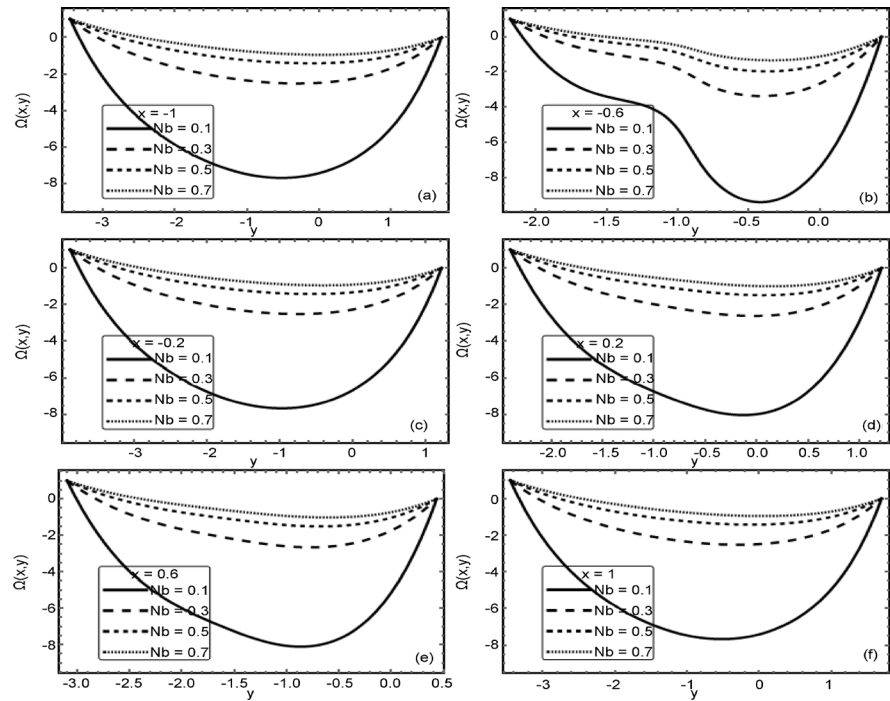


Figure 14. The concentration profile Ω is plotted against y for several values of Nb at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 0.5$, $N_t = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

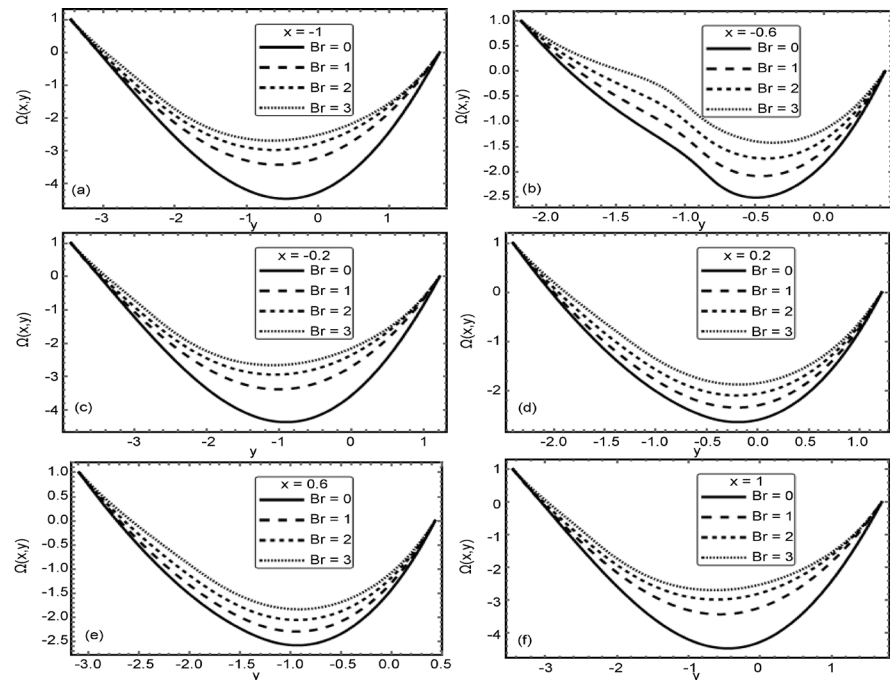


Figure 15. The concentration profile Ω is plotted against y for several values of Br at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $M = 1.2$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $s = 2$, $F = 1$, when (a) $x = -1$, (b) $x = -0.6$, (c) $x = -0.2$, (d) $x = 0.2$, (e) $x = 0.6$, (f) $x = 1$.

3.4. Pressure Gradient Profile

Figures 16-21 examine the influence of the magnetic parameter M , the power law index n , the Weissenberg number we , the local nanoparticle Grashof

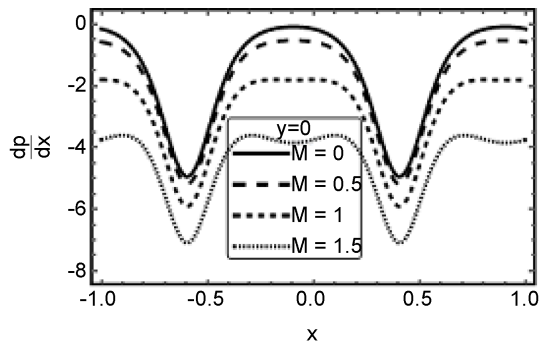


Figure 16. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of M at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, $y = 0$.

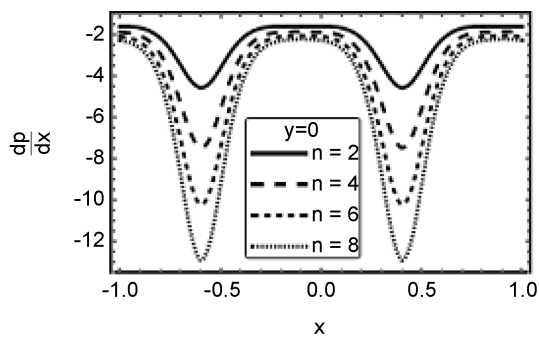


Figure 17. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of n at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, $y = 0$.

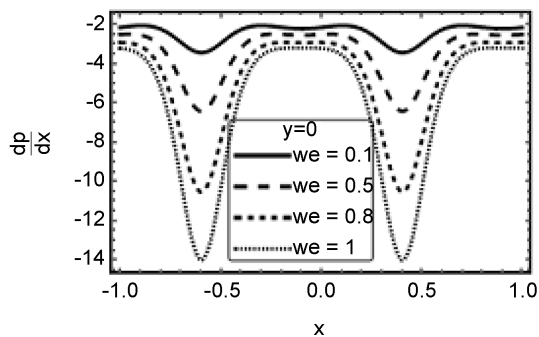


Figure 18. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of We at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, $y = 0$.

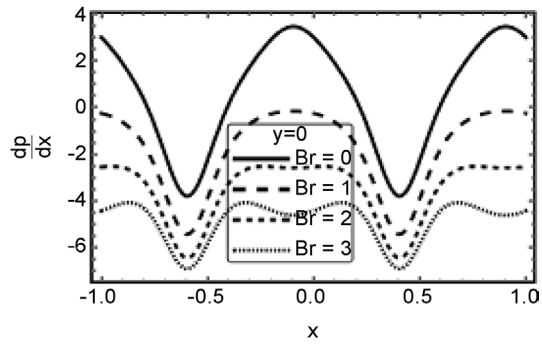


Figure 19. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of Br at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Gr = 1$, $s = 2$, $F = 1$, $y = 0$.

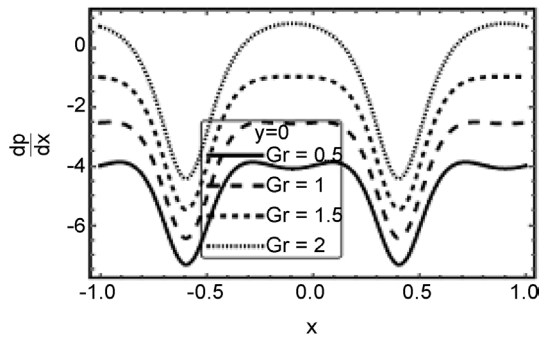


Figure 20. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of Gr at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $Nb = 0.7$, $n = 3$, $Br = 2$, $s = 2$, $F = 1$, $y = 0$.

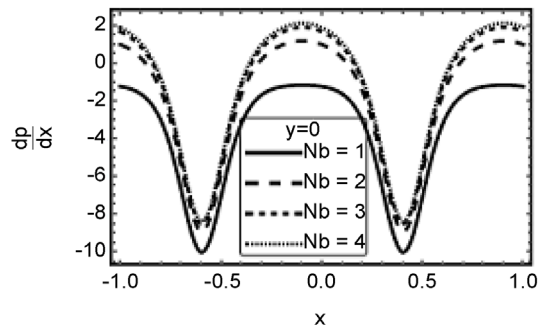


Figure 21. The pressure gradient $\frac{dp}{dx}$ is plotted against x for several values of Nb at $a = 0.7$, $b = 0.9$, $d = 3$, $\phi = \frac{\pi}{3}$, $Pr = 3$, $Ec = 0.2$, $We = 0.5$, $m = 1$, $M = 1.2$, $N_t = 0.7$, $n = 3$, $Gr = 1$, $Br = 2$, $s = 2$, $F = 1$, $y = 0$.

number br , the local temperature Grashof number g_r and the Brownian motion parameter Nb on the pressure gradient. It has been observed that because of successive contraction and relaxation of peristaltic walls the pressure gradient

shows oscillatory behavior. It is clear from **Figures 16-19**, that the magnitude of the pressure gradient decreases in view of an increase in M , n , we and br . The situation is reversed in **Figure 20** and **Figure 21**, the magnitude of the pressure gradient increases with an increase in gr and Nb . It is also noticed that the pressure gradient has its minimum values at the narrow parts of the channel and achieve its maximum values at the wider parts.

4. Conclusions

The peristaltic flow of a Carreau nanofluid through a porous medium with heat and mass transfer in the presence of Hall current, Joule heating and viscous dissipation is studied under assumption of long wavelength and low Reynolds number. The main results are summarized as follow:

- The velocity of Carreau nanofluid increases at the neighborhood of the channel walls and decreases near the center of the channel by increasing the Weissenber number We and the power law index n .
- It is observed that the magnetic parameter M and the thermophoresis parameter n_t have opposite effects on the velocity to that of we and n .
- The temperature profile increases with an increase in the Brownian motion parameter Nb and decreases with an increase in the thermophoresis parameter n_t .
- The concentration profile decreases with an increase in the Brownian motion parameter Nb and increases with an increase in the thermophoresis parameter n_t .
- The pressure gradient decreases with the increase of the magnetic parameter M , the power law index n , the Weissenber number we and the local nanoparticle Grashof number br , while it increases with the increase of the local temperature Grashof number gr and the thermophoresis parameter n_t .

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