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Stochastic Dynamics of Quantum Physical Systems

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Abstract

In this article we define and build the one method of description of stochastic evolution of a physical quantum system. For each quantum state $\omega \in E_U$ we construct the probability measure μ_ω in the space (P_U, S), where P_U is the space of the pure states of the quantum system, S the Borel σ-algebra in P_U. Farther, for any Hermit's positive element with norm $\|u\| = 1$, in the C^*-algebra of observables U, we define the probability measure μ_u on the set of states E_U. If strongly continuous group $\{\alpha_t\}$ of * automorphisms on U describes the evolution of structure of observables, according to this, we have a picture of evolution of distribution of states of quantum system relatively to each observable u.

Subject Areas

Modern Physics

Keywords

Quantum Physical System, State, Observable, Probability Measure, Statistical Structure

1. Introduction

As known [1] a quantum physical system can be represented by a couple $(U, \Im)$, where U is some C^*-algebra which are hermit's elements that are called observables and some set $\Im$ of positive functional with norm one, called the quantum states of this physical system [2].

We say what the functional ω_1 majorizes functional ω_2 if $\omega_1 - \omega_2$ is a positive functional [2].

The state ω quantum physical system is called the pure state if it majorizes only functional type $\lambda\omega, 0 \le \lambda \le 1$ [2]. Denote the set of all pure states on C^*-algebra U

by P_U.

In the set of all linear continuous functional on C^*-algebra U we have topological structure, called as * weakly topological structure [2] and defined by pre-basis:

$$V(\omega, u_1, u_2, \cdots, u_n) = \left\{\omega' \in U^* \middle| \left|\omega(u_i) - \omega'(u_i)\right| < \varepsilon, i = 1, 2, \cdots, n\right\},$$

where $\omega, \omega' \in U^*, u_i \in U$; according to this in the set P_U we have the topological structure induced from this topological structure.

It is very known if U commutative C^*-algebra, then every positive linear functional defines complex valued measure on the P_U, which is separable and locally compact space of pure states under the * weekly topological structure. This measure is defined by corresponding $\mu \leftrightarrow F$, $F(f) = \int_{P_U} f(x)\mathrm{d}\mu$, for all $f \in P_U$ continuous function. In non-commutative case we cannot define the measure on P_U in this way. In the work [3] for each linear functional, with norm one, we define the probability measure P_U for commutative and non-commutative cases in other ways. It gives us the opportunity to present a quantum physical system as a statistical structure [4]. Representation of quantum physical system in this form, in our opinion, is more comfortable for the solution of problem of quantum system, for example for testing hypotheses [4]. In this paper, using this representation, we have tried to consider the dynamic of quantum physical system as a random process.

2. Quantum Physical System as a Statistical Structure

Denote by $\Re$ the set of Hermit's elements of U C^*-algebra.

Easy to show that every linear functional on the U C^*-algebra uniquely will be defined by its values on Banach subspace of Hermit's elements,as it's known [2] that every element u of C^*-algebra U uniquely represented as $u_1 + iu_2$, where u_1 and u_2 are Hermit's.

Every a Hermit's element $u \in \Re$ in the C^*-algebra U any can be represented by integral

$$u = \int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^u,$$

where $\{p_\lambda^u\}_{\lambda \in R}$ $(p_\lambda^u)^2 = p_\lambda^u$ projectors and represents the partition of unityof Hermit's element $u \in \Re$ [5].

Correspond to projector $p_\alpha^2 = p_\alpha \in U$ the family $\{p_\lambda^{p_\alpha}\}_{\lambda \in R}$ of elements of the C^*-algebra U which has the condition: $p_\lambda^{p_\alpha} = 0$, if $\lambda < 1$, $p_\lambda^{p_\alpha} = p_\alpha$, if $\lambda = 1$ and $p_\lambda^{p_\alpha} = E$, if $\lambda > 1$, where E is the unit element in the algebra U. It is clear that.

$$p_\alpha = \int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^{p_\alpha}.$$

If $u \in U$, then $u = u_1 + iu_2$, where u_1 and u_2 are Hermit's elements. The representation such u will be

$$u = u_1 + iu_2 = \int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^{u_1} + i \int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^{u_1} ,$$

Obviously, if ω some linear continuous functional on C^*-algebra U, then from the last equality we will have

$$\omega(u) = \omega(u_1 + iu_2) = \omega\left(\int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^{u_1}\right) + \omega\left(\int_{-\infty}^{\infty} \lambda \mathrm{d}p_\lambda^{u_2}\right) = \int_{-\infty}^{\infty} \lambda \mathrm{d}\omega\left(p_\lambda^{u_1}\right) + i\int_{-\infty}^{\infty} \lambda \mathrm{d}\omega\left(p_\lambda^{u_2}\right),$$

where u_1 and u_2 are Hermit's elements.

Let, $\{p_\alpha\}$ be the set of all one dimensional projectors on C^*-algebra U and ω pure state, then the last equality follows that this state has the non zero meaning only on some projector $p_\alpha \in \{p_\alpha\}$ and the meaning 0 on the other one dimensional projectors. Otherwise, we can always construct such functional which will not have the type $\lambda\omega, 0 \le \lambda \le 1$ and well majorized by the pure state ω. So how, if functional ω is a pure state, then $\omega(u^2) \ge 0$ for all Hermit's elements u.

Let, $\omega(p_{\alpha'}) \ne 0$ and $\omega_{\alpha'}$ such functional, which has the non zero meaning ε only on some projector $p_{\alpha'} \in \{p_\alpha\}$ and the meaning 0 on the other one dimensional projectors. It is clear, if we take ε sufficiently small, thenwe can achieve, that for every $u \in \Re$ will have place inequality

$$(\omega - \omega_{\alpha'})(u^2) = \int_{-\infty}^{\infty} \lambda^2 \mathrm{d}(\omega - \omega_{\alpha'})(p_\lambda^u) \ge 0 .$$

It means, that the pure state ω majorize the functional $\omega_{\alpha'}$ which does not have the type $\lambda\omega, 0 \le \lambda \le 1$, but this is impossible. It follows that the pure states are sach functional which satisfy the condition $\omega_\alpha(p_\beta) = \delta^{\alpha\beta}$.

An integral representation of Hermit's elements follows that for the pure states ω_α has a place of equality $\omega_\alpha(u) = \lambda_\alpha^u$, where λ_α^u is some element of spectrum of Hermit element $u \in \Re$. It gives opportunity to identify every pre state with the set of number $\{\lambda_\alpha^u\}_{u\in\Re}$, where $p_\alpha(u) = \lambda_\alpha^u$.

Consider the Tikhonov's product $\Sigma = \bigotimes_{u\in\Re} \sigma_u$, where $\sigma_u \subset R$ spectrum of element $u \in \Re$.

It is clear, $P_U \subset \Sigma$, because P_U the set of such elements in product $\Sigma = \bigotimes_{u\in\Re} \sigma_u$ which represents linear continue maps with respect to the topological structure in $\Re$ which is defined by the norm:

$$P_U = \left\{\omega_\alpha : \Re \to \bigcup_{u\in\Re} \sigma_u \mid \omega_\alpha(u) = \lambda_\alpha^u, \omega_\alpha(k_1u_1 + k_2u_2) = k_1\omega_\alpha(u_1) + k_2\omega_\alpha(u_2)\right\}.$$

Consequently in the set P_U we have $\Sigma = \bigotimes_{u\in\Re} \sigma_u$, induced from Tikhonov's product $\Sigma = \bigotimes_{u\in\Re} \sigma_u$ topological structure. This topological structure coincide with theinduced topological structure from * weakly topological structure on set of functionals on C^* algebra U.

We can also identify the set P_U with the set of one dimensional projectors $\{p_\alpha\}$. We call P_U as physical space of quantum system.

In the work [3] we have proved:

Theorem 1. Every state $\omega \in \Im$ in space P_U with * weakly topological structure, defined on the Borel σ-algebra of a probability measure μ_ω.

This measure constructed as: for the subset $\left\{p_{\alpha_\beta}\right\}$ of P_U, measure $\mu_\omega\left(\left\{p_{\alpha_\beta}\right\}\right)$ is norm of positive functional ν, $\|\nu\| \le 1$, if this exists, which values on the elements of this subset are coincide to corresponding to values of the state ω. *i.e.* $\mu_\omega\left(\left\{p_{\alpha_\beta}\right\}\right) = \|\nu\|$.

Every measure μ_ω describes distribution elementary particle in physical space of quantum system P_U in the state ω.

If C^*-algebra U has a unit, then in the space U^* with * weakly topological structure the set of all state $\Im$ is convex compact set and represent convex linear combination of pure states $\zeta_1, \zeta_2, \cdots, \zeta_n$ from the set P_U:

$$\omega = k_1\zeta_1 + k_2\zeta_2 + \cdots + k_n\zeta_n,\ k_i \ge 0, \sum_{i=1}^{n} k_i = 1$$

or limit of sequence $\omega_1, \omega_2, \cdots, \omega_l, \cdots$, where

$$\omega_l = k_1^l\zeta_1^l + k_2^l\zeta_2^l + \cdots + k_{n_l}^l\zeta_{n_l}^l,\ k_i^l \ge 0, \sum_{i=1}^{n_l} k_i^l = 1 \quad [2].$$

This means, that elements of set P_U are the extreme points of set [2].

Because each state $\omega \in \Im_u$ defines a probability measure μ_ω on couple (P_U, S), where S is borel C^*-algebra therefore it is easy to show, that every μ_ω represent convex linear combination

$$\mu_\omega = \sum_{i=1}^{n} k_i \mu_{\varsigma_i},\ k_i \ge 0, \sum_{i=1}^{n} k_i = 1$$

of Dirak measures μ_{ς_i} where $\xi_i \in P_U, i = 1, 2, \cdots, n$ or limit of sequense $\left\{\mu_{\omega_n}\right\}_{n \in N}$, where $\mu_{\omega_n} = k_1^n \mu_{\varsigma_1^n} + k_2^n \mu_{\varsigma_2^n} + \cdots + k_{n_l}^n \mu_{\varsigma_l^n},\ k_l^n \ge 0, \sum_{i=1}^{l} k_i^n = 1$ [2].

For every state $\omega \in \Im$ we have $\int_{-\infty}^{\infty} \mathrm{d}\omega\left(E_\lambda^u\right) = 1$ therefore it is easy that the value of quantum state on observable $u \in \Re$ is the middle value of this observable. The value $\omega(u) \in R$ is called the middle value of observable $u \in \Re$ of quantum physical system in the state $\omega \in \Im$.

All told above follows that a quantum physical system is an object, so-called statistical structure [4]:

$$\left(U, P_U, \Im, S, \{\mu_\omega\}, \omega \in \Im\right),$$

where U some C^*-algebra, Hermit element of which are called observables of this system, P_U is the space of quantum system, S Borel σ-algebra in P_U, μ_ω the probability measure defined by state $\omega \in \Im$ and which describes distribution elementary particle in physical space of quantum system P_U in the state ω.

3. A Stochastic Dynamics of Quantum System

Theorem 2. Every Hermit's element u, $\|u\| = 1$ in σ-algebra U defines probability measure on the set of states E_U.

Proof: It is well-known that the map $\pi : U \to U^{**}$ defined by formula $\pi(u)(\omega) = F_u(\omega) = \omega(u)$ is isometric embedding U as Banach space in the double conjugate space U^{**} [3]. If ω is a state then $\omega(u^*u) \ge 0$ [2]; it follows that if $u \in U$ is positive element, then $u = v^*v, v \in U$ and $\omega(u) \ge 0$.

Thus if $u \in U$ is positive element then $F_u(\omega) \ge 0$ for each state on U. Because π is isometric, and therefore $\|F_u\| = \|u\|$.

$$(F_u + F_v)(\omega) = F_u(\omega) + F_v(\omega) = \omega(u) + \omega(v) = \omega(u+v) = F_{u+v}(\omega).$$

If u is hermit's element $u = \int_{-\infty}^{\infty} \lambda \mathrm{d} p_\lambda^u$, because, for such elements $F_u = \int_{-\infty}^{\infty} \lambda \mathrm{d} F_{p_\lambda^u}$ and

$$F_u(f) = \int_{-\infty}^{\infty} \lambda \mathrm{d} F_{p_\lambda^u}(\omega) = \int_{-\infty}^{\infty} \lambda \mathrm{d}\omega(p_\lambda^u).$$

Let u, $\|u\| = 1$ be hermit's positive element in U, then spectrum $\sigma_u \subset [0,1]$. Let E_U be the set of all states on U, if $O \subset E_U$ is a set of states; we assume the measure $\mu_u(O)$ of this set is λ, if O consists for all such element ω for which $\omega(u) \le \lambda$, $\lambda \in \sigma_u$, $\sup_\omega\{\omega(u)\} = \lambda$. Since $\sigma_u \subset [0,1]$, 0 and 1 are elements of σ_u [5], and therefore $\mu_u(E_U) = 1$. It is clear that, if $\lambda_1, \lambda_2 \in \sigma_u, \lambda_1 < \lambda_2$ then

$$\{f \mid \omega(u) \le \lambda_1\} \subset \{\omega \mid \omega(u) \le \lambda_2\}.$$

If we assume $\mu_u(\{\omega \mid \omega(u) \le \lambda_2\} \setminus \{\omega \mid \omega(u) \le \lambda_1\}) = \lambda_2 - \lambda_1$, then we get a measure on E_U.

The sets for which we define measure, make σ-algebra in E_U. This is not a Borel's σ-algebra in space E_U whit the * weekly topology. Denote it by S_u^*. Thus, we define on (E_U, S_u^*) probability measure. The theorem is proved.

Consider the family of measures $\{E_U, S_u^*, \mu_u, u \in \Re_1^+\}$ defined above, where $\Re_1^+$ is the set of positive hermit's elements whit norm 1, S_u^* is corresponding to hermit's element $u \in \Re_1^+$, σ-algebra in E_U.

Let $(U, P_U, \Im, S, \{\mu_\omega\}, \omega \in \Im)$ statistical structure represent a quantum physical system, $\Im \subset E_U$. For each $u \in \Re_1^+$ we can define the measure $\mu_\Im^u$ on the set of states $\Im$ of given quantum physical system such:

$$\mu_\Im^u(O) = \lambda \text{ if } O \subset \Im,\ O = \Im \cap \mu_u^{-1}(\lambda).$$

Literally, we have defined measure $\mu_\Im^u$ on the set of measures $\{\mu_\omega\}_{\omega \in \Im}$, of which each element μ_ω describes distribution elementary particles in physical space P_U of quantum system in the state ω.

If $\{a_t\}_{t \in R}$ is strongly one parametric group of maps of C^*-algebra U whit unity and $a_t(1) = 1$ for all $t \in R$, then following conditions are equivalent [2]:

1) All a_t^* automorphisms of U;
2) $\|a_t\| \le 1$ for all $t \in R$;
3) $a_t(U_+) \subset U_+$, where U_+ is the set of positive elements in U;

4) $a_t^*(E_U) \subset E_U$, for all $t \in R$.

Each defined measure $\mu_{\Im}^{u}$ describes distribution of states in $\Im$ relatively to middle value of observable u over states in $\Im$, or distribution of elementary particles in physical space P_U of quantum system in the states $\omega \in \Im$ relatively to middle value of observable u over states in $\Im$.

It follows: If strongly continuous one parametric group of automorphisms $\{a_t\}_{t \in R}$ describes dynamic of structure of observables, according to this, we have a picture of evolution of distribution of states quantum system $(U, P_U, \Im, S, \{\mu_\omega\}, \omega \in \Im)$ relatively to each observable u.

Such, the representation of quantum physical system as a statistical structure allows formalizing the dynamics of the quantum system as a random process.

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