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Edge Product Cordial Labeling of Some Cycle Related Graphs

Udayan M. Prajapati[1], Nittal B. Patel[2]

[1]St. Xavier's College, Ahmedabad, India
[2]Shankersinh Vaghela Bapu Institute of Technology, Gandhinagar, India
Email: udayan64@yahoo.com, nittalbpatel000@gmail.com

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Abstract

For a graph $G=(V(G),E(G))$ having no isolated vertex, a function $f:E(G)\to\{0,1\}$ is called an edge product cordial labeling of graph G, if the induced vertex labeling function defined by the product of labels of incident edges to each vertex is such that the number of edges with label 0 and the number of edges with label 1 differ by at most 1 and the number of vertices with label 0 and the number of vertices with label 1 also differ by at most 1. In this paper, we discuss edge product cordial labeling for some cycle related graphs.

Keywords

Graph Labeling, Edge Product Cordial Labeling

1. Introduction

We begin with simple, finite, undirected graph $G=(V(G),E(G))$ having no isolated vertex where $V(G)$ and $E(G)$ denote the vertex set and the edge set respectively, $|v(G)|$ and $|E(G)|$ denote the number of vertices and edges respectively. For all other terminology, we follow Gross [1]. In the present investigations, C_n denotes cycle graph with n vertices. We will give brief summary of definitions which are useful for the present work.

Definition 1. *A graph labeling is an assignment of integers to the vertices or edges or both subject to the certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex (or an edge) labeling.*

For an extensive survey on graph labeling and bibliography references, we refer to Gallian [2].

Definition 2. *For a graph G, the edge labeling function is defined as* $f:E(G)\to\{0,1\}$ *and induced vertex labeling function* $f^*:V(G)\to\{0,1\}$ *is given as if* $e_1,e_2,\cdots,e_n$ *are all the edges incident to the vertex v then* $f^*(v)=f(e_1)f(e_2)\cdots f(e_n)$.

Let $v_f(i)$ *be the number of vertices of G having label i under* f^* *and* $e_f(i)$ *be the number of edges of G having label i under f for* $i=1,2$.

f is called an edge product cordial labeling of graph G if $|v_f(0)-v_f(1)|\le 1$ *and* $|e_f(0)-e_f(1)|\le 1$. *A graph G is called edge product cordial if it admits an edge product cordial labeling.*

Vaidya and Barasara [3] introduced the concept of the edge product cordial labeling as an edge analogue of the product cordial labeling.

Definition 3. *The wheel* W_n $(n>4)$ *is the graph obtained by adding a new vertex joining to each of the vertices of* C_n. *The new vertex is called the apex vertex and the vertices corresponding to* C_n *are called rim vertices of* W_n. *The edges joining rim vertices are called rim edges.*

Definition 4. *The helm* H_n *is the graph obtained from a wheel* W_n *by attaching a pendant edge to each of the rim vertices.*

Definition 5. *The closed helm* CH_n *is the graph obtained from a helm* H_n *by joining each pendant vertex to form a cycle. The cycle obtained in this manner is called an outer cycle.*

Definition 6. *The web* Wb_n *is the graph obtained by joining the pendant vertices of a helm* H_n *to form a cycle and then adding a pendant edge to each of the vertices of the outer cycle.*

Definition 7. *The Closed Web graph* CWb_n *is the graph obtained from a web graph* Wb_n *by joining each of the outer pendent vertices consecutively to form a cycle.*

Definition 8. [4] *The Sunflower graph* SF_n *is the graph obtained by taking a wheel with the apex vertex* v_0 *and the consecutive rim vertices* $v_1,v_2,\cdots,v_n$ *and additional vertices* $w_1,w_2,\cdots,w_n$ *where* w_i *is joined by edges to* v_i *and* $v_{i+1}(\mathrm{mod}\ n)$.

Definition 9. [4] *The Lotus inside a circle* LC_n *is a graph obtained from a cycle* $C_n:u_1u_2\cdots u_nu_1$ *and a star graph* $K_{1,n}$ *with center vertex* v_0 *and end vertices* $v_1,v_2,\cdots,v_n$ *by joining each* v_i *to* u_i *and* $u_{i+1}(\mathrm{mod}\ n)$.

Definition 10. [5] *Duplication of a vertex v of a graph G produces a new graph* G' *by adding a new vertex* v' *such that* $N(v')=N(v)$. *In other words,* v' *is said to be a duplication of v if all the vertices which are adjacent to v in G are also adjacent to* v' *in* G'.

Definition 11. [5] *Duplication of a vertex* v_k *by a new edge* $e=v_k'v_k''$ *in a graph G produces a new graph* G' *such that* $N(v_k')=\{v_k,v_k''\}$ *and* $N(v_k'')=\{v_k,v_k'\}$.

Definition 12. *The Flower graph* Fl_n *is the graph obtained from a helm* H_n *by joining each pendant vertex to the apex of the helm* H_n.

2. Main Result

Theorem 1. *Closed web graph* CWb_n *is not an edge product cordial graph.*

Proof. Let v_0 be the apex vertex and $v_1,v_2,v_3,\cdots,v_n$ be the consecutive rim vertices of W_n. Let $e_i = v_i v_{i+1}$, for each $i = 1, 2, \cdots, n$ (here subscripts are modulo n). Let $f_i = v_0 v_i$. Let $v_1^a, v_2^a, \cdots, v_n^a$ be the new vertices corresponding to $v_1, v_2, v_3, \cdots, v_n$ respectively. Form the edge f_i^a by joining v_i and v_i^a. Form the cycle by creating edges $e_1^a\left(=v_1^a v_2^a\right), e_2^a\left(=v_2^a v_3^a\right), \cdots, e_n^a\left(=v_n^a v_1^a\right)$. The resulting graph is called a helm graph. Let $v_1^b, v_2^b, \cdots, v_n^b$ be the new vertices corresponding to $v_1^a, v_2^a, \cdots, v_n^a$ respectively. Form the edge f_i^c by joining v_i^a and v_i^b. Form the cycle by creating the edges $e_1^v\left(=v_1^b v_2^b\right), e_2^b\left(=v_2^b v_3^b\right), \cdots, e_n^b\left(=v_n^b v_1^b\right)$. The resulting graph is a closed web graph CWb_n. Thus $\left|E\left(CWb_n\right)\right| = 6n$ and $\left|V\left(CWb_n\right)\right| = 3n+1$. We are trying to define the mapping $f : E\left(CWb_n\right) \to \{0,1\}$ we consider following two cases:

Case 1: If n is odd then in order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $3n$ edges out of $6n$ edges. So in this context, the edges with label 0 will give rise at least $\left\lceil \frac{3n}{2} \right\rceil + 1$ vertices with label 0 and at most $\left\lfloor \frac{3n}{2} \right\rfloor$ vertices with label 1 out of $3n+1$ vertices. Therefore $\left|v_f(0) - v_f(1)\right| \geq 2$. So CWb_n is not an edge product cordial graph for odd n.

Case 2: If n is even then in order to satisfy the edge condition for edge product cordial graph it is essential to assign label 0 to $3n$ edges out of $6n$ edges. So in this context, the edges with label 0 will give rise at least $\frac{3n}{2} + 2$ vertices with label 0 and at most $\frac{3n}{2} - 1$ vertices with label 1 out of $3n+1$ vertices.

From both the cases $\left|v_f(0) - v_f(1)\right| \geq 2$, so CWb_n is not an edge product cordial graph.

Theorem 2. *Lotus inside circle* LC_n *is not an edge product cordial graph.*

Proof. Let v_0 be the apex vertex and $v_1, v_2, v_3, \cdots, v_n$ be the consecutive vertices of star graph $K_{1,n}$. Let $e_i = v_0 v_i$ for each $i = 1, 2, \cdots, n$. Let $v_1^a, v_2^a, \cdots, v_n^a$ be the new vertices corresponding to $v_1, v_2, v_3, \cdots, v_n$ respectively. Form the cycle by creating the edges $e_1^a\left(=v_1^a v_2^a\right), e_2^a\left(=v_2^a v_3^a\right), \cdots, e_n^a\left(=v_n^a v_1^a\right)$. Let $e_i^l = v_i v_i^a$ and $e_i^r = v_i v_{i+1}^a$. The resulting graph is Lotus inside circle Lc_n. Thus $\left|E\left(LC_n\right)\right| = 4n$ and $\left|V\left(LC_n\right)\right| = 2n+1$. We are trying to define the mapping $f : E\left(LC_n\right) \to \{0,1\}$. In order to satisfy an edge condition for edge product cordial graph it is essential to assign label 0 to $2n$ edges out of $4n$ edges. So in this context, the edges with label 0 will give rise to at least $n+2$ vertices with label 0 and at most $n-1$ vertices with label 1 out of $2n+1$ vertices. Therefore $\left|v_f(0) - v_f(1)\right| \geq 3$.

So, LC_n is not an edge product cordial graph.

Theorem 3. *Sunflower graph* SF_n *is an edge product cordial graph for* $n \geq 3$.

Proof. Let SF_n be the sunflower graph, where v_0 is the apex vertex, $v_1, v_2, \cdots, v_n$ be the consecutive rim vertices of W_n and $w_1, w_2, \cdots, w_n$ be the additional vertices where w_i is joined to v_i and $v_{i+1} (\bmod n)$. Let $e_1, e_2, e_3, \cdots, e_n$ be the consecutive rim edges of the W_n. $e_1', e_2', \cdots, e_n'$ are the corresponding edges joining apex vertex v_0 to the vertices $v_1, v_2, \cdots, v_n$ of the cycle. Let for each $1 \leq i \leq n$, e_i^l be the edges joining w_i to v_i and e_i^r be the edges joining w_i to $v_{i+1} (\bmod n)$. Thus $\left|E\left(SF_n\right)\right| = 4n$ and

$\left|V\left(SF_n\right)\right| = 2n+1$. Define the mapping $f: E\left(SF_n\right) \to \{0,1\}$ as follows:

$$f(e) = \begin{cases} 0 & \text{if } e \in \{e_i, e_i'\}, \quad \forall i = 1,2,3,\cdots,n; \\ 1 & \text{if } e \in \{e_i^l, e_i^r\}, \quad \forall i = 1,2,3,\cdots,n. \end{cases}$$

In view of the above defined labeling pattern we have, $v_f(0) = \{v_0, v_1, v_2, \cdots, v_n\}$ and $v_f(1) = \{w_1, w_2, \cdots, w_n\}$. So, $v_f(0) = v_f(1) + 1 = n+1$ and $e_f(0) = e_f(1) = 2n$. Thus $\left|v_f(0) - v_f(1)\right| \le 1$ and $\left|e_f(0) - e_f(1)\right| \le 1$. Thus *f* admits an edge product cordial on SF_n. Hence, SF_n is an edge product cordial graph.

Illustration 1. *Graph* SF_6 *and its edge product cordial labeling is shown in* Figure 1*.*

Theorem 4. *The graph obtained from duplication of each of the vertices* w_i *for* $i = 1,2,\cdots,n$ *by a new vertex in the sunflower graph* SF_n *is an edge product cordial graph if and only if n is even.*

Proof. Let SF_n be sunflower graph, where v_0 is the apex vertex, $v_1, v_2, \cdots, v_n$ be the consecutive rim vertices of W_n and $w_1, w_2, \cdots, w_n$ be the additional vertices where w_i is joined to v_i and $v_{i+1} (\bmod n)$. Let $e_1, e_2, e_3, \cdots, e_n$ be the consecutive rim edges of the W_n. $e_1', e_2', \cdots, e_n'$ are the corresponding edges joining the apex vertex v_0 to the vertices $v_1, v_2, \cdots, v_n$ of the cycle. Let for each $1 \le i \le n$, e_i^l be the edges joining w_i to v_i and e_i^r be the edges joining w_i to $v_i + 1 (\bmod n)$. Let G be the graph obtained from SF_n by duplication of the vertices $w_1, w_2, \cdots, w_n$ by the new vertices $u_1, u_2, \cdots, u_n$ respectively. Let for each $1 \le i \le n$, f_i^l be the edges joining u_i to v_i and f_i^r be the edges joining u_i to $v_{i+1} (\bmod n)$. Thus $\left|E(G)\right| = 6n$ and $\left|V(G)\right| = 3n+1$. We consider the following two cases:

Case 1: If n is odd, define the mapping $g: E(G) \to \{0,1\}$ in order to satisfy edge condition for edge product cordial graph it is essential to assign label 0 to $3n$ edges out of $6n$ edges. So in this context, the edges with label 0 will give rise at least $\left\lceil \frac{3n}{2} \right\rceil + 1$

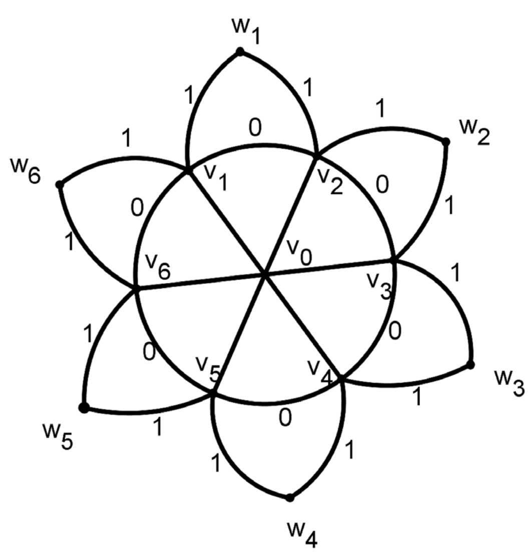

Figure 1. SF_6 and its edge product cordial labeling

vertices with label 0 and at most $\left\lfloor \frac{3n}{2} \right\rfloor$ vertices with label 1 out of $3n+1$ vertices. Therefore $\left|v_g(0)-v_g(1)\right| \geq 2$. So the duplication of vertex w_i by vertex u_i in sunflower graph is not edge product cordial for odd n.

Case 2: If n is even, define the mapping $g: E(G) \to \{0,1\}$ as follows:

$$g(e)=\begin{cases} 0 & \text{if } e \in \{e_i, e_i'\}, \forall i=1,2,3,\cdots,n; \\ 1 & \text{if } e \in \{e_i^l, e_i^r\}, \forall i=1,2,3,\cdots,\frac{n}{2}; \\ 0 & \text{if } e \in \{e_i^l, e_i^r\}, \forall i=\frac{n}{2}+1,\frac{n}{2}+2,\cdots,n; \\ 1 & \text{if } e \in \{f_i^l, f_i^r\}, \forall i=1,2,3,\cdots,n. \end{cases}$$

In view of the above defined labeling pattern we have,

$v_g(0)=\left\{v_0, v_1, v_2, \cdots, v_n, w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \cdots, w_n\right\}$ and $v_g(1)=\left\{u_1, u_2, \cdots, u_n, w_1, w_2, \cdots, w_{\frac{n}{2}}\right\}$.

So $v_g(0)=v_g(1)+1=\frac{3n}{2}+1$ and $e_g(0)=e_g(1)=3n$. Thus $\left|v_g(0)-v_g(1)\right| \leq 1$ and $\left|e_g(0)-e_g(1)\right| \leq 1$. Thus g admits an edge product cordial labeling on G. So, G is an edge product cordial for even n.

Illustration 2. *Graph G obtained from* SF_6 *by duplication of each of the vertices* $w_1, w_2, w_3, w_4, w_5, w_6$ *by new vertices* $u_1, u_2, u_3, u_4, u_5, u_6$ *and its edge product cordial labeling is shown in* Figure 2.

Theorem 5. *The graph obtained from duplication of each of the vertices* w_i *for* $i=1,2,\cdots,n$ *by a new edges* f_i *in the sunflower graph* SF_n *is an edge product cordial graph.*

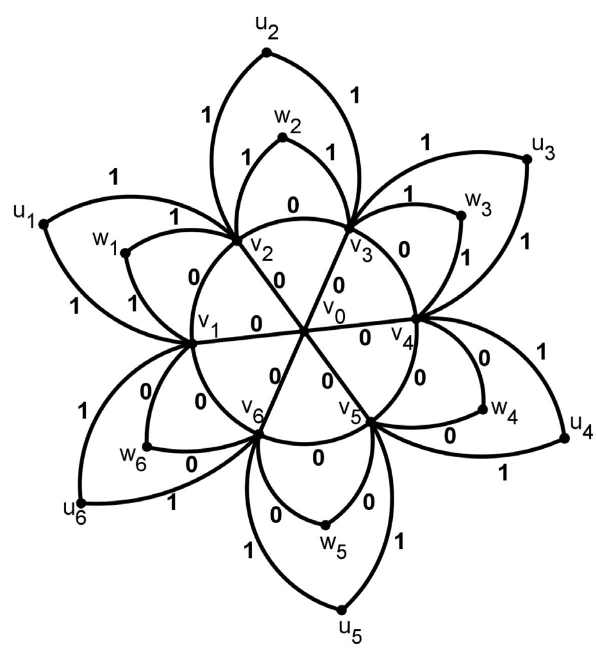

Figure 2. G and its edge product cordial labeling.

Proof. Let SF_n be sunflower graph, where v_0 is the apex vertex, $v_1, v_2, \cdots, v_n$ be the consecutive rim vertices of W_n and $w_1, w_2, \cdots, w_n$ be the additional vertices where w_i is joined to v_i and $v_{i+1}(\bmod n)$. Let $e_1, e_2, e_3, \cdots, e_n$ be the consecutive rim edges of W_n. $e_1', e_2', \cdots, e_n'$ are the corresponding edges joining apex vertex v_0 to the vertices $v_1, v_2, \cdots, v_n$ of the cycle. Let for each $1 \le i \le n$, e_i^l be the edges joining w_i to v_i and e_i^r be the edges joining w_i to $v_i + 1(\bmod n)$. Let G be the graph obtained from SF_n by duplication of the vertices $w_1, w_2, \cdots, w_n$ by corresponding new edges $f_1, f_2, \cdots, f_n$ with new vertices u_i^l and u_i^r such that u_i^l and u_i^r join to the vertex w_i. Let for each $1 \le i \le n$, f_i be the edges joining the vertices u_i^l and u_i^r and for each $1 \le i \le n$, f_i^l be the edges joining u_i^l to w_i and f_i^r be the edges joining u_i^r to w_i. Thus $|E(G)| = 7n$ and $|V(G)| = 4n+1$. We consider the following two cases:

Case 1: If n is odd, define the mapping $g: E(G) \to \{0,1\}$ as follows:

$$g(e)=\begin{cases} 1 & \text{if } e = e_i, \forall i = 1,2,3,\cdots,\frac{n-1}{2}; \\ 1 & \text{if } e = e_i', \forall i = 2,3,\cdots,\frac{n-1}{2}; \\ 1 & \text{if } e \in \{e_i^l, e_i^r, f_i, u_i^l, u_i^r\}, \forall i = 1,2,3,\cdots,\frac{n+1}{2}; \\ 0 & \text{if } e \in \{e_i, e_i'\}, \forall i = \frac{n+1}{2}, \frac{n+3}{2}, \frac{n+5}{2}, \cdots, n; \\ 0 & \text{if } e = e_1'; \\ 0 & \text{if } e \in \{e_i^l, e_i^r, f_i, u_i^l, u_i^r\}, \forall i = \frac{n+3}{2}, \frac{n+5}{2}, \frac{n+7}{2}, \cdots, n. \end{cases}$$

In view of the above defined labeling pattern we have,

$$v_g(0) = \left\{ v_0, v_1, v_{\frac{n-1}{2}+1}, v_{\frac{n-1}{2}+2}, \cdots, v_n, u^l_{\frac{n+1}{2}+1}, u^l_{\frac{n+1}{2}+2}, \cdots, u_n^l, u^r_{\frac{n+1}{2}+1}, u^r_{\frac{n+1}{2}+2}, \cdots, u_n^r, w_{\frac{n+1}{2}+1}, w_{\frac{n+1}{2}+2}, \cdots, w_n \right\}$$

and $v_g(1) = \left\{ v_2, v_3, \cdots, v_{\frac{n-1}{2}}, u_1^l, u_2^l, \cdots, u^l_{\frac{n+1}{2}}, u_1^r, u_2^r, \cdots, u^r_{\frac{n+1}{2}}, w_1, w_2, \cdots, w_{\frac{n+1}{2}} \right\}$. So,

$v_g(0) = v_g(1) + 1 = 2n+1$ and $e_g(1) = \left\lceil \frac{7n}{2} \right\rceil$, $e_g(0) = \left\lfloor \frac{7n}{2} \right\rfloor$.

Case 2: If n is even, define $g: E(G) \to \{0,1\}$ as follows:

$$g(e)=\begin{cases} 1 & \text{if } e = e_i, \forall i = 1,2,3,\cdots,\frac{n-2}{2}; \\ 1 & \text{if } e = e_i', \forall i = 2,3,4,\cdots,\frac{n-2}{2}; \\ 1 & \text{if } e \in \{e_i^l, e_i^r\}, \forall i = 1,2,3,\cdots,\frac{n}{2}; \\ 1 & \text{if } e \in \{f_i, u_i^l, u_i^r\}, \forall i = 1,2,3,\cdots,\frac{n+2}{2}; \\ 0 & \text{if } e \in \{e_i, e_i'\}, \forall i = \frac{n}{2}, \frac{n+2}{2}, \frac{n+4}{2}, \cdots, n; \\ 0 & \text{if } e = e_1'; \\ 0 & \text{if } e \in \{e_i^l, e_i^r\}, \forall i = \frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \cdots, n; \\ 0 & \text{if } \quad e \in \{f_i, u_i^l, u_i^r\}, \forall i = \frac{n+4}{2}, \frac{n+6}{2}, \frac{n+8}{2}, \cdots, n. \end{cases}$$

In view of the above defined labeling pattern we have,

$$v_g(0)=\left\{v_0,v_1,v_{\frac{n-2}{2}+1},v_{\frac{n-2}{2}+2},\cdots,v_n,u^l_{\frac{n+2}{2}+1},u^l_{\frac{n+2}{2}+2},\cdots,u^l_n,u^r_{\frac{n+2}{2}+1},u^r_{\frac{n+2}{2}+2},\cdots,u^r_n,w_{\frac{n}{2}+1},w_{\frac{n}{2}+2},\cdots,w_n\right\}$$

and $v_g(1)=\left\{v_2,v_3,\cdots,v_{\frac{n-2}{2}},u^l_1,u^l_2,\cdots,u^l_{\frac{n+2}{2}},u^r_1,u^r_2,\cdots,u^r_{\frac{n+2}{2}},w_1,w_2,\cdots,w_{\frac{n}{2}}\right\}$. So,

$v_g(0)=v_g(1)+1=2n+1$ and $e_g(0)=e_g(1)=\dfrac{7n}{2}$.

From both the cases $|v_g(0)-v_g(1)|\le 1$ and $|e_g(0)-e_g(1)|\le 1$. Thus, g admits an edge product cordial labeling on G. So the graph G is an edge product cordial graph.

Illustration 3. *Graph G obtained from* SF_5 *by duplication of each of the vertices* w_1,w_2,w_3,w_4,w_5 *by corresponding new edges* f_1,f_2,f_3,f_4,f_5 *and its edge product cordial labeling is shown in* Figure 3.

Theorem 6. *The graph obtained by duplication of each of the vertices in the sunflower graph* SF_n *is not an edge product cordial graph.*

Proof. Let SF_n be the sunflower graph, where v_0 is the apex vertex, $v_1,v_2,\cdots,v_n$ be the consecutive vertices of the cycle C_n and $w_1,w_2,\cdots,w_n$ be the additional vertices where w_i is joined to v_i and $v_{i+1}(\mathrm{mod}\,n)$. Let $e_1,e_2,e_3,\cdots,e_n$ be the consecutive edges of the cycle C_n. $e'_1,e'_2,\cdots,e'_n$ are the corresponding edges joining the apex vertex v_0 to the vertices $v_1,v_2,\cdots,v_n$ of the cycle. Let for each $1\le i\le n$, e^l_i be the edge joining w_i to v_i and e^r_i be the edge joining w_i to $v_{i+1}(\mathrm{mod}\,n)$. Let G be the graph obtained from SF_n by duplication of each of the vertices $v_0,v_1,v_2,\cdots,v_n,w_1,w_2,\cdots,w_n$ by the vertices $v'_0,v'_1,v'_2,\cdots,v'_n,w'_1,w'_2,\cdots,w'_n$ respectively.

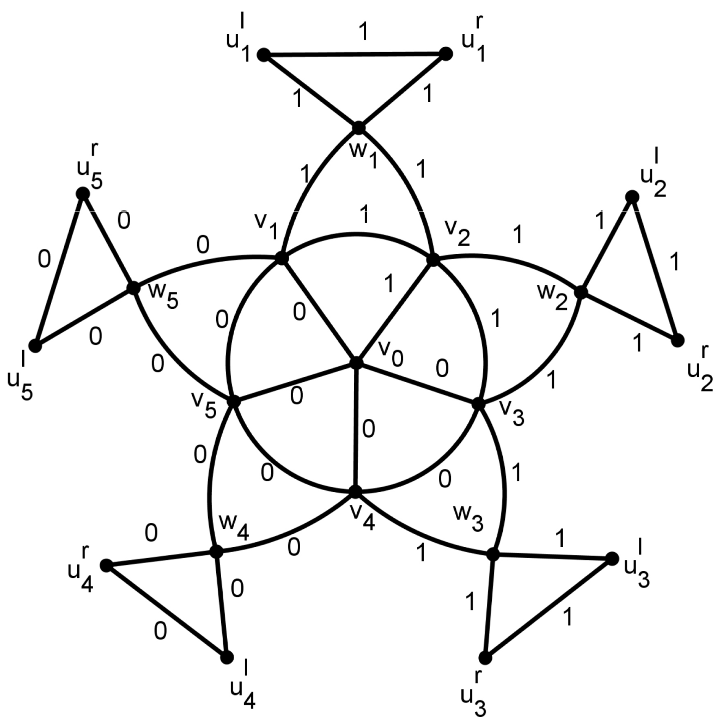

Figure 3. G and its edge product cordial labeling.

Let $e_1^a, e_2^a, \cdots, e_{2n}^a$ be the edges joining the vertex w_i' to the adjacent vertex of w_i for $i = 1, 2, \cdots, n$. $e_1^b, e_2^b, \cdots, e_{5n}^b$ are the edges joining the vertex v_i' to the adjacent vertex of v_i for $i = 1, 2, \cdots, n$ and $e_1^c, e_2^c, \cdots, e_n^c$ are the edges joining the vertex v_0' to the adjacent vertex of c in the sunflower graph. Thus $|E(G)| = 12n$ and $|V(G)| = 4n + 2$. We are trying to define $f : E(G) \to \{0,1\}$.

In order to satisfy the edge condition for edge product cordial graph, it is essential to assign label 0 to $6n$ edges out of $12n$ edges. So in this context, the edge with label 0 will give rise at least $\left\lceil \frac{4n+1}{2} \right\rceil + 1$ vertices with label 0 and at most $\left\lfloor \frac{4n+1}{2} \right\rfloor$ vertices with label 1 out of $4n + 2$ vertices. Therefore, $|v_f(0) - v_f(1)| \ge 2$. So the graph G is not an edge product cordial graph.

Theorem 7. *The graph obtained by subdividing the edges $w_i v_i$ and $w_i v_{i+1}$ (subscripts are mod n) for all i = 1,2,...,n by a vertex in the sunflower graph SF_n is an edge product cordial.*

Proof. Let SF_n be the sunflower graph, where v_0 is the apex vertex, $v_1, v_2, \cdots, v_n$ be the consecutive rim vertices of W_n and $w_1, w_2, \cdots, w_n$ be the additional vertices where w_i is joined to v_i and $v_{i+1} (\bmod n)$. Let $e_1, e_2, e_3, \cdots, e_n$ be the consecutive rim edges of w_n. $e_1', e_2', \cdots, e_n'$ are the corresponding edges joining apex vertex v_0 to the vertices $v_1, v_2, \cdots, v_n$ of the cycle. Let G be the graph obtained from SF_n by subdividing the edges $w_i v_i$ and $w_i v_{i+1} (\bmod n)$ for all $i = 1, 2, \cdots, n$ by a vertex u_i^l and u_i^r respectively. Let for each $1 \le i \le n$, e_i^l be the edges joining u_i^l to v_i and e_i^r be the edges joining u_i^r to $v_{i+1} (\bmod n)$. Let for each $1 \le i \le n$, f_i^l be the edges joining w_i to u_i^l and f_i^r be the edges joining w_i to u_i^r. Thus, $|E(G)| = 6n$ and $|V(G)| = 4n + 1$. We consider the following two cases:

Case 1: If n is odd, define $g : E(G) \to \{0,1\}$ as follows:

$$g(e) = \begin{cases} 0 & \text{if } e \in \{e_i, e_i'\}, \forall i = 1, 2, 3, \cdots, n; \\ 0 & \text{if } e = e_i^l, \forall i = \frac{n+3}{2}, \frac{n+5}{2}, \frac{n+7}{2}, \cdots, n; \\ 0 & \text{if } e = e_i^r, \forall i = \frac{n+1}{2}, \frac{n+3}{2}, \frac{n+5}{2}, \cdots, n; \\ 1 & \text{if } e = e_i^l, \forall i = 1, 2, 3, \cdots, \frac{n+1}{2}; \\ 1 & \text{if } e = e_i^r, \forall i = 1, 2, 3, \cdots, \frac{n-1}{2}; \\ 1 & \text{if } e \in \{f_i^l, f_i^r\}, \forall i = 1, 2, 3, \cdots, n. \end{cases}$$

In view of the above defined labeling pattern we have,

$v_g(0) = \left\{ v_0, v_1, v_2, \cdots, v_n, u^l_{\frac{n+1}{2}+1}, u^l_{\frac{n+1}{2}+2}, \cdots, u^l_n, u^r_{\frac{n-1}{2}+1}, u^r_{\frac{n-1}{2}+2}, \cdots, u^r_n \right\}$ and

$v_g(1) = \left\{ u_1^l, u_2^l, \cdots, u^l_{\frac{n+1}{2}}, u_1^r, u_2^r, \cdots, u^r_{\frac{n-1}{2}} w_1, w_2, \cdots, w_n \right\}$. So $v_g(0) = v_g(1) + 1 = 2n + 1$ and $e_g(0) = e_g(1) = 3n$.

Case 2: If n is even, define $g: E(G) \to \{0,1\}$ as follows:

$$g(e) = \begin{cases} 0 & \text{if } e \in \{e_i, e_i'\}, \forall i = 1,2,3,\cdots,n; \\ 0 & \text{if } e \in \{e_i^l, e_i^r\}, \forall i = \frac{n+2}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \cdots, n; \\ 1 & \text{if } e \in \{e_i^r, e_i^l\}, \forall i = 1,2,3,\cdots,\frac{n}{2}; \\ 1 & \text{if } e \in \{f_i^l, f_i^r\}, \forall i = 1,2,3,\cdots,n. \end{cases}$$

In view of the above defined labeling pattern we have,

$v_g(0) = \left\{ v_0, v_1, v_2, \cdots, v_n, u^l_{\frac{n}{2}+1}, u^l_{\frac{n}{2}+2}, \cdots, u^l_n, u^r_{\frac{n}{2}+1}, u^r_{\frac{n}{2}+2}, \cdots, u^r_n \right\}$ and $v_g(1) = \left\{ u_1^l, u_2^l, \cdots, u^l_{\frac{n}{2}}, u_1^r, u_2^r, \cdots, u^r_{\frac{n}{2}} w_1, w_2, \cdots, w_n \right\}$. So $v_g(0) = v_g(1) + 1 = 2n+1$ and $e_g(0) = e_g(1) = 3n$.

From both the cases $|v_g(0) - v_g(1)| \le 1$ and $|e_g(0) - e_g(1)| \le 1$. Thus g admits an edge product cordial labeling on G. So the graph G is an edge product cordial graph.

Illustration 4. *Graph G obtained from* SF_8 *by subdividing the edges* $w_i v_i$ *and* $w_i v_{i+1} (\bmod n)$ *for all* $i = 1,2,\cdots,8$ *by a vertex* u_i^l *and* u_i^r *respectively for* $i = 1,2,\cdots,8$ *and its edge product cordial labeling is shown in* Figure 4.

Theorem 8. *The graph obtained by flower graph* Fl_n *by adding n pendant vertices to the apex vertex* v_0 *is an edge product cordial graph.*

Proof. The Flower graph Fl_n is the graph obtained from a helm H_n by joining

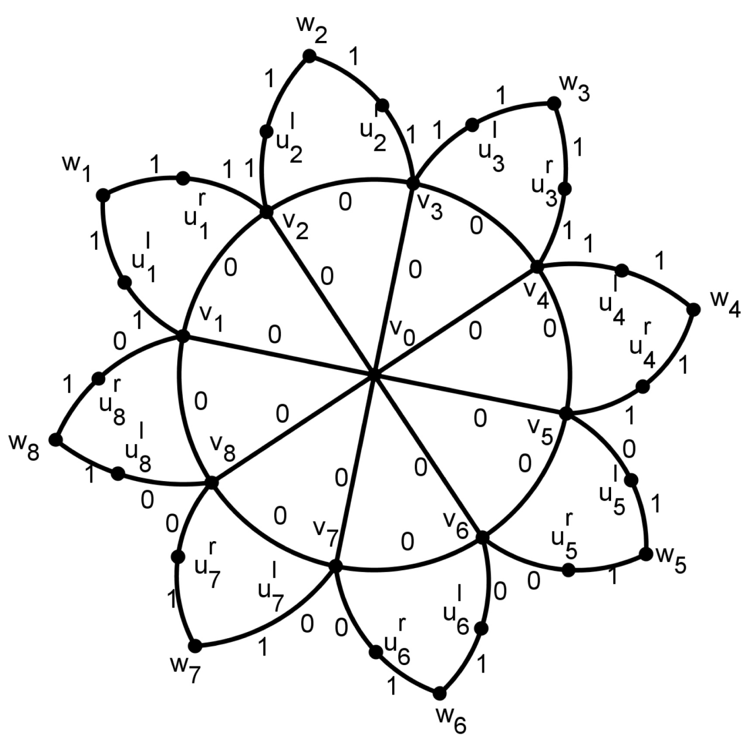

Figure 4. G and its edge product cordial labeling.

each pendant vertex to the apex vertex of the helm H_n. Let G be the graph obtained from the flower graph Fl_n by adding n pendant vertices $u_1,u_2,\cdots,u_n$ to the apex vertex v_0. Let $v_1,v_2,\cdots,v_n$ be the rim vertices and $w_1,w_2,\cdots,w_n$ be the pendant vertices to $v_1,v_2,\cdots,v_n$ respectively.

Let $e_1,e_2,e_3,\cdots,e_n$ be the consecutive rim edges of W_n. $e'_1,e'_2,\cdots,e'_n$ are the corresponding edges joining the apex vertex v_0 to the vertices $v_1,v_2,\cdots,v_n$ of the cycle. Let $e_i^a = w_i v_i$ for each $i=1,2,\cdots,n$. Let $e_i^b = w_i v_0$ for each $i=1,2,\cdots,n$. Let $e_i^c = u_i v_0$ for each $i=1,2,\cdots,n$. Thus $|E(G)|=5n$ and $|V(G)|=3n+1$. We consider two cases:

Case 1: If n is even, define the mapping $f:E(G)\to\{0,1\}$ as follows:

$$f(e)=\begin{cases}0 & \text{if } e\in\{e_i,e'_i\},\forall i=1,2,3,\cdots,n;\\ 1 & \text{if } e\in\{e_i^a,e_i^b\},\forall i=1,2,3,\cdots,n;\\ 1 & \text{if } e=e_i^c,\forall i=1,2,3,\cdots,\frac{n}{2};\\ 0 & \text{if } e=e_i^c,\forall i=\frac{n+2}{2},\frac{n+4}{2},\frac{n+6}{2},\cdots,n.\end{cases}$$

In view of the above defined labeling pattern we have,

$v_f(0)=\left\{v_0,v_1,v_2,\cdots,v_n,u_{\frac{n}{2}}+1,u_{\frac{n}{2}}+2,\cdots,u_n\right\}$ and

$v_f(1)=\left\{w_1,w_2,\cdots,w_n,u_1,u_2,\cdots,u_{\frac{n}{2}}\right\}$. So $v_f(0)=v_f(1)+1=\frac{3n}{2}+1$ and

$e_f(0)=e_f(1)=\frac{5n}{2}$.

Case 2: If n is odd, define the mapping $f:E(G)\to\{0,1\}$ as follows:

$$f(e)=\begin{cases}0 & \text{if } e\in\{e_i,e'_i\},\forall i=1,2,3,\cdots,n;\\ 1 & \text{if } e\in\{e_i^a,e_i^b\},\forall i=1,2,3,\cdots,n;\\ 1 & \text{if } e=e_i^c,\forall i=1,2,3,\cdots,\frac{n+1}{2};\\ 0 & \text{if } e=e_i^c,\forall i=\frac{n+3}{2},\frac{n+5}{2},\frac{n+7}{2},\cdots,n.\end{cases}$$

In view of the above defined labeling pattern we have,

$v_f(0)=\left\{v_0,v_1,v_2,\cdots,v_n,u_{\frac{n+1}{2}+1},u_{\frac{n+1}{2}+2},\cdots,u_n\right\}$ and

$v_f(1)=\left\{w_1,w_2,\cdots,w_n,u_1,u_2,\cdots,u_{\frac{n+1}{2}}\right\}$. So $v_f(0)=v_f(1)=\frac{3n+1}{2}$ and $e_f(1)=\left\lceil\frac{5n}{2}\right\rceil$,

$e_f(0)=\left\lfloor\frac{5n}{2}\right\rfloor$.

From both the cases $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$. Thus, f admits an edge product cordial labeling on G. So G is an edge product cordial graph.

Illustration 5. *Graph G obtained from* Fl_5 *by adding* 5 *pendent vertices* u_1,u_2,u_3,u_4,u_5 *to the apex vertex* v_0 *and its edge product cordial labeling is shown in* Figure 5.

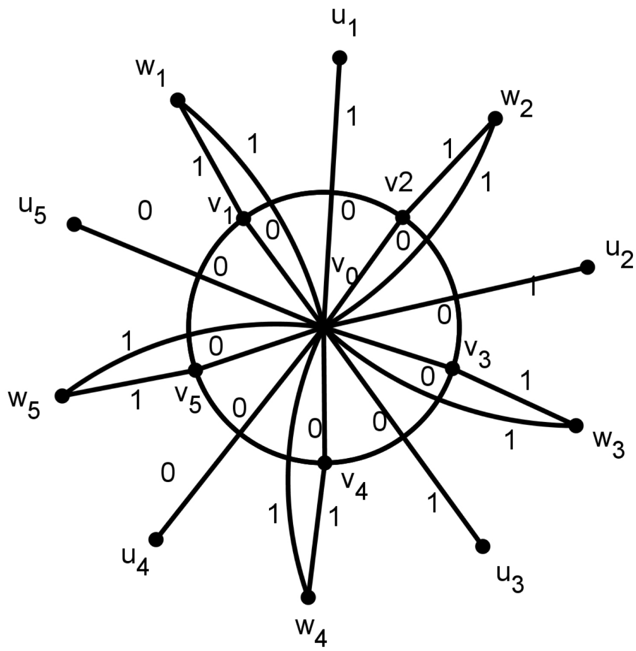

Figure 5. G and its edge product cordial labeling.

3. Concluding Remarks

We investigated eight results on the edge product cordial labeling of various graph generated by a cycle. Similar problem can be discussed for other graph families.

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