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Unrelated Parallel-Machine Scheduling Problems with General Truncated Job-Dependent Learning Effect

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Abstract

In this paper, we consider scheduling problems with general truncated job-dependent learning effect on unrelated parallel-machine. The objective functions are to minimize total machine load, total completion (waiting) time, total absolute differences in completion (waiting) times respectively. If the number of machines is fixed, these problems can be solved in $O(n^{m+2})$ time respectively, where m is the number of machines and n is the number of jobs.

Keywords

Scheduling, Unrelated Parallel Machines, Truncated Job-Dependent Learning

1. Introduction

In modern planning and scheduling problems, there are many real situations where the processing time of jobs may be subject to change due to learning effect. An extensive survey of different scheduling models and problems with learning effects could be found in Biskup [1]. More recently, Janiak *et al.* [2] studied a single processor problem with a S-shaped learning model. They proved that the makespan minimization problem is strongly NP-hard. Lee [3] considered scheduling jobs with general position-based learning curves. For some single machine and a two-machine flowshop scheduling problems, they presented the optimal solution respectively. Lee [4] considered single-machine scheduling jobs with general learning effect and past-sequence-dependent setup time. For some single machine scheduling problems, they presented the optimal solution respectively. Lee and Wu [5], and Wu and Lee [6] considered scheduling jobs with learning effects. They proved that some single machine and flowshop scheduling problems can be solved in polynomial time respectively. Lee *et al.* [7] considered a single-machine scheduling problem with release times and learning effect. Lee *et al.* [8] considered a makespan minimization uniform parallel-machine scheduling problem with position-based learning curves. Lee and Chung [9], Sun *et al.* [10] [11], and Wang *et al.* [12] considered flow shop scheduling with learning effects. Wu *et al.* [13], Wu *et al.* [14], Wu *et al.* [15] and Wang *et al.* [16] considered scheduling problems with the

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truncated learning effect.

Recently, Wang *et al.* [17] considered several scheduling problems on a single machine with truncated job-dependent learning effect, *i.e.*, the actual processing time of job J_j is $p_{jr}^A = p_j \max\{r^{a_j}, b\}$ if it is scheduled in the rth position of a sequence, where $a_j \le 0$ is the job-dependent learning index of job J_j, and b is a truncation parameter with $0<b<1$. In this paper, we study scheduling problems with general truncated job-dependent learning effect on unrelated parallel-machine. The objective is to minimize total machine load, total completion (waiting) time, total absolute differences in completion (waiting) times respectively.

2. Problems Description

There are n independent jobs $N=\{J_1, J_2, \cdots J_n\}$ to be processed on m unrelated paralle-machine $M=\{M_1, M_2, \cdots M_m\}$. Let $(n_1, n_2, \cdots n_m)$ denote a job-allocation vector, where n_i denotes the number of jobs assigned to machine M_i, and $\sum_{i=1}^m n_i = n$. In this paper, we assume that the actual processing time of job J_j scheduled on machine M_i is

$$p_{ijr}^A = p_{ij} \max\{f_{ij}(r), b\}, \quad i=1,2,\ldots,m;\ r, j=1,2,\ldots,n, \tag{1}$$

where $p_{ij} \ge 0$ denotes the normal (basic) processing time of job J_j $(j=1,2,\ldots,n)$ on machine M_i, r is the position of a sequence, b is a truncation parameter with $0<b<1$, $f_{ij}(r)$ is the general case of positional learning for job J_j on machine M_i, special $f_{ij}(r)=r^{a_{ij}}$ is the polynomial learning index for job J_j on machine M_i $(a_{ij}<0)$, $f_{ij}(r)=b_{ij}^{r-1}$ is the exponential learning index for job J_j on machine M_i $(0<b_{ij}<1)$.

Let C_{ij} and $W_{ij}=C_{ij}-p_{ij}$ be the completion and waiting time for job J_j on machine M_i respectively. The goal is to determine the jobs assigned to corresponding each machine and the corresponding optimal schedule so that the following objective functions is to be minimized: the total machine load $\sum_{i=1}^m C_{\max}^i$, the total completion (waiting) times $\sum_{i=1}^m \sum_{j=1}^{n_i} C_{ij} \left(\sum_{i=1}^m \sum_{j=1}^{n_i} W_{ij}\right)$, the total absolute differences in completion (waiting) times $\sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |C_{ik}-C_{ij}| \left(\sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |W_{ik}-W_{ij}|\right)$, where $C_{\max}^i$ denotes the makespan of machine M_i. Using the three-field notation [18] the problems can be denoted as $Rm|Y|Z$, where Y denote the model (1), $Z \in \left\{\sum_{i=1}^m C_{\max}^i, \sum_{i=1}^m \sum_{j=1}^{n_i} C_{ij}, \sum_{i=1}^m \sum_{j=1}^{n_i} W_{ij}, \sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |C_{ik}-C_{ij}|, \sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |W_{ik}-W_{ij}|\right\}$.

3. Main Results

Let p_{ij} denote the actual processing time of a job when it is scheduled in position j on machine M_i, then $f_{i[j]}(j)$, $J_{i[j]}$, $C_{i[j]}$, $W_{i[j]}$ are defined similarly.

Lemma 1. For a given permutation $\pi_i = \left(J_{i[1]}, J_{i[2]}, \ldots, J_{i[n_i]}\right)$ on machine M_i,

$$\sum_{i=1}^m C_{\max}^i = \sum_{i=1}^m \sum_{j=1}^{n_i} p_{i[j]} \max\left\{f_{i[j]}(j), b\right\}$$

$$\sum_{i=1}^m \sum_{j=1}^{n_i} C_{ij} = \sum_{i=1}^m \sum_{j=1}^{n_i} (n_i - j + 1)\, p_{i[j]} \max\left\{f_{i[j]}(j), b\right\}$$

$$\sum_{i=1}^m \sum_{j=1}^{n_i} W_{ij} = \sum_{i=1}^m \sum_{j=1}^{n_i} (n_i - j)\, p_{i[j]} \max\left\{f_{i[j]}(j), b\right\}$$

$$\sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |C_{ik}-C_{ij}| = \sum_{i=1}^m \sum_{j=1}^{n_i} (j-1)(n_i - j + 1) p_{i[j]} \max\left\{f_{i[j]}(j), b\right\} \quad \text{(Kanet [19])}$$

$$\sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |W_{ik}-W_{ij}| = \sum_{i=1}^m \sum_{j=1}^{n_i} j\,(n_i - j)\, p_{i[j]} \max\left\{f_{i[j]}(j), b\right\} \quad \text{(Bagchi [20]).}$$

If the vector $(n_1, n_2, \ldots, n_m)$ is given, let X_{jir} be a $0/1$ variable such that $X_{jir}=1$ if job $J_j\,(j=1,2,\ldots,n)$ is assigned at position $r\,(r=1,2,\ldots,n_i)$ on machine $M_i\,(i=1,2,\ldots,m)$, and $X_{jir}=0$, otherwise. Then, the problem $Rm|Y|Z$ (where $\sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |C_{ik}-C_{ij}|, \sum_{i=1}^m \sum_{k=1}^{n_i} \sum_{j=k}^{n_i} |W_{ik}-W_{ij}|\}$) can be solved by the following assignment problem:

$$\min\ Z=\sum_{i=1}^{m}\sum_{r=1}^{n_i}\sum_{j=1}^{n}\lambda_{ir}p_{ij}\max\left\{f_{ij}(r),b\right\}X_{jir} \tag{2}$$

s.t.

$$\sum_{i=1}^{m}\sum_{r=1}^{n_i}X_{jir}=1,\ j=1,2,\ldots,n, \tag{3}$$

$$\sum_{j=1}^{n}X_{jir}=1,\ i=1,2,\ldots,m,\ r=1,2,\ldots,n_i, \tag{4}$$

$$X_{jir}=0\quad\text{or } 1,\quad j=1,2,\ldots,n,\ i=1,2,\ldots,m,\ r=1,2,\ldots,n_i, \tag{5}$$

where $\lambda_{ir}=1$ for $\sum_{i=1}^{m}C_{\max}^{i}$, $\lambda_{ir}=(n_i-r+1)$ for $\sum_{i=1}^{m}\sum_{k=1}^{n_i}C_{ik}$, $\lambda_{ir}=(n_i-r)$ for $\sum_{i=1}^{m}\sum_{k=1}^{n_i}W_{ik}$, $\lambda_{ir}=(r-1)(n_i-r+1)$ for $\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|C_{ik}-C_{ij}\right|$, $\lambda_{ir}=r(n_i-r)$ for $\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|W_{ik}-W_{ij}\right|$.

Now, the question is how many vectors $(n_1,n_2,\ldots,n_m)$ exist. Obviously n_i may be 0, 1, 2, $\cdots$, n $(i=1,2,\ldots,m)$. So if the numbers of jobs assigned to the first $m-1$ machines is given, the number of jobs assigned to the last machine is then determined uniquely ($\sum_{i=1}^{m}n_i=n$). Therefore, the upper bound of $(n_1,n_2,\ldots,n_m)$ is $(n+1)^{m-1}$. Based on the above analysis, we have the following result.

Theorem 1. For a given constant m, $Rm\left|Y\right|Z$ can be solved in $O\left(n^{m+2}\right)$ time, where

$$Z\in\left\{\sum_{i=1}^{m}C_{\max}^{i},\sum_{i=1}^{m}\sum_{j=1}^{n_i}C_{ij},\sum_{i=1}^{m}\sum_{j=1}^{n_i}W_{ij},\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|C_{ik}-C_{ij}\right|,\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|W_{ik}-W_{ij}\right|\right\}.$$

Proof. As discussed above, to solve the problem $Rm\left|Y\right|Z$, polynomial number (*i.e.*, $(n+1)^{m-1}$) of assignment problems need to be solved. Each assignment problem is solved in $O\left(n^3\right)$ time (by using the Hungarian method). Hence, the time complexity of the problem $Rm\left|Y\right|Z$ can be solved in $O\left(n^{m+2}\right)$ time, where

$$Z\in\left\{\sum_{i=1}^{m}C_{\max}^{i},\sum_{i=1}^{m}\sum_{j=1}^{n_i}C_{ij},\sum_{i=1}^{m}\sum_{j=1}^{n_i}W_{ij},\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|C_{ik}-C_{ij}\right|,\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|W_{ik}-W_{ij}\right|\right\}.$$

Note that if the number of machines m is fixed, then the problem $Rm\left|Y\right|Z$ can be solved in polynomial time. Based on the above analysis, we can determine the optimal solution for the problem $Rm\left|Y\right|Z$ via the following algorithm:

Algorithm 1

Step 1. For each possible vector $(n_1,n_2,\ldots,n_m)$, solve the assignment problem (2)-(5). Then, obtain the optimal schedule and the corresponding objective function Z.

Step 2. The optimal solution for the problem is the one with the minimum value of the objective function Z, where $Z\in\left\{\sum_{i=1}^{m}C_{\max}^{i},\sum_{i=1}^{m}\sum_{j=1}^{n_i}C_{ij},\sum_{i=1}^{m}\sum_{j=1}^{n_i}W_{ij},\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|C_{ik}-C_{ij}\right|,\sum_{i=1}^{m}\sum_{k=1}^{n_i}\sum_{j=k}^{n_i}\left|W_{ik}-W_{ij}\right|\right\}$.

The following example illustrates the working of Algorithm 1 to find the optimal solution for the problem $Rm\left|Y\right|\sum_{i=1}^{m}\sum_{j=1}^{n_i}C_{ij}$.

Example 1. There are $n=5$ jobs and $f_{ij}(r)=r^{a_{ij}}$, The number of machines is $m=2$ and $p_{11}=15$, $p_{12}=11$, $p_{13}=14$, $p_{14}=3$, $p_{15}=9$, $p_{21}=12$, $p_{22}=10$, $p_{23}=9$, $p_{24}=16$, $p_{25}=8$, $a_{11}=-0.23$, $a_{12}=-0.32$, $a_{13}=-0.25$, $a_{14}=-0.35$, $a_{15}=-0.26$, $a_{21}=-0.32$, $a_{22}=-0.21$, $a_{23}=-0.31$, $a_{24}=-0.24$, $a_{25}=-0.29$, $b=0.7$ are given.

Solution. When $n_1=0,\ n_2=5$, the positional weights on machine M_2 are $\theta_{21}=5$, $\theta_{22}=4$, $\theta_{23}=3$, $\theta_{24}=2$, $\theta_{25}=1$. Then values $\theta_{ir}p_{ij}\max\left\{r^{a_{ij}},b\right\}$ are given in **Table 1** (the bold value is the optimal solution of the assignment problem (2)-(5)).We solve the assignment problem (2)-(5) to $\sum_{i=1}^{m}\sum_{j=1}^{n_i}C_{ij}=339.65119$.

When $n_1=1,\ n_2=4$, the positional weights on machine M_1 and M_2 are $\theta_{11}=1$, $\theta_{21}=4$, $\theta_{22}=3$, $\theta_{23}=2$, $\theta_{24}=1$. Then values $\theta_{ir}p_{ij}\max\left\{r^{a_{ij}},b\right\}$ are given in **Table 2**. We solve the assignment problem

Table 1. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1. for $n_1 = 0, n_2 = 5$.

$ij \backslash ir$	θ_{21}	θ_{22}	θ_{23}	θ_{24}	θ_{25}
J_{21}	60	38.45135	25.32933	16.80000	**8.40000**
J_{22}	50	34.58149	23.81912	**14.94849**	7.13208
J_{23}	45	29.03910	**19.20685**	12.60000	6.30000
J_{24}	**90**	54.19170	36.87501	22.94328	11.20000
J_{25}	40	**27.09585**	18.43750	11.20000	5.60000

Table 2. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1 for $n_1 = 1, n_2 = 4$.

$ij \backslash ir$	θ_{11}	θ_{21}	θ_{22}	θ_{23}	θ_{24}
J_{i1}	15	48	28.83852	16.88622	**8.40000**
J_{i2}	11	40	25.93612	**15.87942**	7.13208
J_{i3}	14	36	**21.77933**	12.80457	6.30000
J_{i4}	**3**	64	40.64377	24.58334	11.20000
J_{i5}	9	**32**	19.62965	11.63469	5.60000

(2)-(5) to obtain that the optimal schedule on machine M_1 is $[J_4]$, and on machine M_2 is $[J_5, J_3, J_2, J_1]$. The objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 81.05875$.

When $n_1 = 2, n_2 = 3$, the positional weights on machine M_1 and M_2 are $\theta_{11} = 2$, $\theta_{12} = 1$, $\theta_{21} = 3$, $\theta_{22} = 2$, $\theta_{23} = 1$. Then values $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ are given in **Table 3**. We solve the assignment problem (2)-(5) to obtain that the optimal schedule on machine M_1 is $[J_4, J_2]$, and on machine M_2 is $[J_5, J_3, J_1]$. The objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 61.77443$.

When $n_1 = 3, n_2 = 2$, the positional weights on machine M_1 and M_2 are $\theta_{11} = 3$, $\theta_{12} = 2$, $\theta_{13} = 1$, $\theta_{21} = 2$, $\theta_{22} = 1$. Then values $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ are given in **Table 4**. We solve the assignment problem (2)-(5) to obtain that the optimal schedule on machine M_1 is $[J_4, J_5, J_2]$, and on machine M_2 is $[J_3, J_1]$. The objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 59.38394$.

When $n_1 = 4, n_2 = 1$, the positional weights on machine M_1 and M_2 are $\theta_{11} = 4$, $\theta_{12} = 3$, $\theta_{13} = 2$, $\theta_{14} = 1$, $\theta_{21} = 1$. Then values $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ are given in **Table 5**. We solve the assignment problem (2)-(5) to obtain that the optimal schedule on machine M_1 is $[J_4, J_5, J_2, J_1]$, and on machine M_2 is $[J_3]$. The objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 69.93119$.

When $n_1 = 5, n_2 = 0$, the positional weights on machine M_1 and are $\theta_{11} = 5$, $\theta_{12} = 4$, $\theta_{13} = 3$, $\theta_{14} = 2$, $\theta_{15} = 1$. Then values $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ are given in **Table 6**. We solve the assignment problem (2)-(5) to obtain that the optimal schedule on machine M_1 is $[J_4, J_5, J_2, J_3, J_1]$. The objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 98.58071$.

Table 3. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1 for $n_1 = 2, n_2 = 3$.

$ij \setminus ir$	θ_{11}	θ_{21}	θ_{22}	θ_{23}	θ_{24}
J_{i1}	30	12.78952	36	19.22568	**8.44311**
J_{i2}	22	**8.81177**	30	17.29074	7.93971
J_{i3}	28	11.77255	27	**14.51955**	6.40228
J_{i4}	**6**	2.35375	48	27.09585	12.29167
J_{i5}	18	7.51579	**24**	13.08643	5.81735

Table 4. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1 for $n_1 = 3, n_2 = 2$.

$ij \setminus ir$	θ_{11}	θ_{21}	θ_{22}	θ_{23}	θ_{24}
J_{i1}	45	25.57905	11.65074	24	**9.61284**
J_{i2}	33	17.62354	**7.739517**	20	8.64537
J_{i3}	42	23.54510	10.63770	**18**	7.25978
J_{i4}	**9**	4.70751	2.10000	32	13.54792
J_{i5}	27	**15.03158**	6.76380	16	6.54322

Table 5. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1 for $n_1 = 4, n_2 = 1$.

$ij \setminus ir$	θ_{11}	θ_{21}	θ_{22}	θ_{23}	θ_{24}
J_{i1}	60	38.36857	23.30147	**10.90479**	12
J_{i2}	44	26.43531	**15.47903**	7.70000	10
J_{i3}	56	35.31765	21.2754	9.89950	**9**
J_{i4}	**12**	7.06126	4.20000	2.10000	16
J_{i5}	36	**22.54737**	13.52761	6.30000	8

Table 6. The $\theta_{ir} p_{ij} \max\left\{r^{a_{ij}}, b\right\}$ values of Example 1 for $n_1 = 5, n_2 = 0$.

$ij \setminus ir$	θ_{21}	θ_{22}	θ_{23}	θ_{24}	θ_{25}
J_{11}	75	51.15809	34.95221	21.80959	**10.50000**
J_{12}	55	35.24707	**23.21855**	15.40000	7.70000
J_{13}	70	47.09020	31.91310	**19.79899**	9.80000
J_{14}	**15**	9.41501	6.30000	4.20000	2.10000
J_{15}	45	**30.06317**	20.29141	12.60000	6.30000

Hence, the optimal schedule on machine M_1 is $[J_4, J_5, J_2]$, and on machine M_2 is $[J_3, J_1]$. The optimal objective function is $\sum_{i=1}^{m}\sum_{j=1}^{n_i} C_{ij} = 59.38394$.

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