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Electron-Proton Pairing at High Temperatures, Solar Flares, and the FIP Effect

L. K. Pande[1,2]

[1]Theory Group, School of Environmental Sciences, Jawaharlal Nehru University, New Delhi, India
[2]C1/1206, Vasant Kunj, New Delhi, India
Email: lalit_pande@hotmail.com

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Abstract

We analyze the line data from solar flares to present evidence for the emission spectrum of the recently discussed electron-proton pairs at high temperatures. We also point out that since the pairing phenomenon provides an additional source for these lines—the conventional source being the highly ionized high-Z atoms already existing in the solar atmosphere, we have a plausible explanation of the FIP effect.

Keywords

Electron-Proton Pairing at High Temperatures, Emission Spectra from Solar Flares, FIP Effect

1. Introduction

Some time back, a finite-temperature Schrodinger equation was obtained to describe the pairing of an electron and a proton in a medium of such particles at finite temperature [1]. This work was a follow-up of an earlier paper where the Coulomb potential was temperature-generalized for the first time [2]. In these papers the approach followed was somewhat intuitive, leaving open the question of a derivation from first principles. Hence, beginning with the Bethe-Salpeter equation in vacuum, such a derivation was given in [3]. In an approximation scheme, which should work at low particle densities and high temperatures, this equation gives a temperature-dependent bound state spectrum for the electron-proton pair. In the temperature range around 10^6 K, the deepest of the states in the spectrum have binding energies in the keV range and can withstand the background thermal agitation of the medium. The transitions from the short-lived excited states to the deepest ones in this spectrum lead to spectral lines in the soft X-ray region. An application of the approach to the flaring regions of the Sun therefore leads to the prediction of such lines in the flare spectra. In the present note, we report on this matter:

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the calculated lines at a certain temperature from three Balmer-like series are essentially all seen in the flare data. We also point out that, since many of these lines are identifiable with the lines which in the conventional approach are presumed to originate from the low first ionization potential (FIP) elements, our approach seems to provide an explanation of the FIP effect which has been extensively discussed in the literature, e.g., in [4] and [5]. The details are given in the following.

2. The Finite-Temperature Schrodinger Equation (FTSE) and Its Solutions

The equation for the bound state at temperature T, or the FTSE, is given by

$$\left(W-\frac{p^2}{2\mu}\right)\psi(\boldsymbol{p})=\frac{\alpha}{4\pi^2}Q(W,p)\int\frac{\mathrm{d}q}{(\boldsymbol{p}-\boldsymbol{q})^2}\psi(\boldsymbol{q}), \tag{1}$$

where W is the binding energy,

$$\alpha=\frac{e^2}{4\pi\hbar c}\simeq\frac{1}{137},\quad \mu=\frac{m_a m_b}{m_a m_b} \tag{2}$$

with m_a and m_b as the electron and the proton mass, respectively. The function $Q(W, p)$ is given by

$$\begin{aligned} Q(W,p) &= \tanh\left[\frac{\beta}{2}\left(\mu_a W-\varepsilon_p^a\right)\right]+\tanh\left[\frac{\beta}{2}\left(\mu_b W-\varepsilon_p^b\right)\right] \\ &= \frac{a-\tanh\left(\beta p^2/4m_a\right)}{1-a\tanh\left(\beta p^2/4m_a\right)}+\frac{b-\tanh\left(\beta p^2/4m_b\right)}{1-b\tanh\left(\beta p^2/4m_b\right)} \end{aligned} \tag{3}$$

where

$$\begin{aligned} &a=\tanh\left(\beta\mu_a W/2\right),\quad b=\tanh\left(\beta\mu_b W/2\right) \\ &\mu_a=m_a/\left(m_a+m_b\right),\quad \mu_b=m_b/\left(m_a+m_b\right), \\ &\varepsilon_p^a=p^2/2m_a,\ \varepsilon_p^b=p^2/2m_b\quad \beta=1/kT \end{aligned} \tag{4}$$

and k is the Boltzmann constant. Using the approximation

$$\tanh\left(\frac{\beta p^2}{4m_{a,b}}\right)\simeq\frac{\beta p^2}{4m_{a,b}} \tag{5}$$

we can write

$$Q(W,p)\simeq\frac{d_1-d_2p^2}{1-d_3p^2}\equiv\tilde{Q}(W,p) \tag{6}$$

where

$$d_1=a+b,\quad d_2=\frac{1+ab}{4\mu kT},\quad d_3=\frac{a/m_a+b/m_b}{4kT}. \tag{7}$$

Equation (7) can be solved in this approximation to give [1]

$$\begin{aligned} &\psi(\boldsymbol{p})=g_l(\rho)Y_l^m(\theta,\eta);\quad \rho=|\boldsymbol{p}|/p_0,\ \ p_0^2=-2\mu W, \\ &g_l(\rho)=\cos^4(\varphi/2)\sum_{k=l}^{\infty}a_k P_{k,l}^{(2)}(\cos\varphi);\quad \varphi=2\arctan(\rho) \end{aligned} \tag{8}$$

where l is the angular momentum quantum number, m the azimuthal quantum number and $P_{k,l}^{(2)}(\cos\varphi)$ are given in terms of Gegenbauer polynomials $C_{k-l}^{l+1}(\cos\varphi)$:

$$P_{k,l}^{(2)}(\cos\varphi)=\frac{\sin^l(\varphi)}{k+1}C_{k-l}^{l+1}(\cos\varphi); \tag{9}$$

the coefficients a_k in Equation (8) obey a difference equation given by

$$b_{k+1}+b_k\frac{2(k+1)}{(k+l+1)}\frac{\left[(k+1)c_1-c_2\right]}{\left[(k+1)c_3-c_4\right]}+b_{k-1}\frac{(k-l)}{(k+l)}=0;\quad b_k=0\ \text{ for }\ k<l \tag{10}$$

where we have put

$$a_k\frac{(k+l+1)}{(k+1)^3}\left[(k+1)c_3-c_4\right]=b_k. \tag{11}$$

The parameters c_i are given by

$$\begin{aligned} c_1&=\left(1-d_3p_0^2\right),\quad c_2=\left(\frac{\lambda}{2p_0}\right)\left(1-d_2p_0^2\right)\\ c_3&=\left(1+d_3p_0^2\right),\quad c_4=\left(\frac{\lambda}{2p_0}\right)\left(d_1+d_2p_0^2\right);\quad \lambda=-\alpha\mu \end{aligned} \tag{12}$$

Equation (10) is a second order difference equation and will in general have two solutions; the dominant and the dominated. The eigenvalues of the equation are those values of p_0^2 (or W) for which the dominant solution vanishes (for a review see [6] [7]), and they can be computed easily by the Hill determinant method; see [8]-[10] and [1]. The generic computer program for such computations is already given in [9], and can be adapted to the present situation without any difficulty. On substituting the numerical values for m_a (electron mass), m_b (proton mass) and α (the fine structure constant), we can use the program to solve for W at any given value of T. What emerges is a discrete spectrum for W: for each l, we get an increasing set of W values which can be labeled by a serial number n (which plays the role of the principal quantum number in the case of the hydrogen atom). The results are illustrated in **Table 1**. The typical dimensions of the Hill determinant for stability of levels up to the third decimal place (in eV) are $k = 400$ for $n = 1$, $l = 0$; $k = 16{,}000$ for $n = 10$, l = 0, $k = 80{,}000$ for $n = 20$, $l = 0$; and so on.

It is now important to make sure that the solutions obtained are indeed consistent with the approximation given in Equation (5). This is done by verifying that the expectation values of $\left\langle \beta p^2/4m_a\right\rangle$ over the desired solution $\psi(\boldsymbol{p})$ corresponding to any $W_{n,l}$ are quite small, say < 0.01. If not so, the solution must be discarded. It may be noted that since m_a is much smaller than m_b, it is sufficient to check that $\left\langle \beta p^2/4m_a\right\rangle\equiv\left\langle x_a\right\rangle$ is quite small. To do this, we have to first calculate. $\left\langle x_a\right\rangle$. The details are given in **Appendix A**. We quote here only the result:

$$\left\langle x_a\right\rangle=\frac{\mu_b\left|W_{n,l}\right|}{2kT}\frac{S_2}{S_1} \tag{13}$$

where

$$S_1=\sum_{k=l}^{M}\left[\frac{\Gamma(k+l+2)}{(k-l)!(k+1)}\frac{a_ka_k}{(k+1)^2}+\frac{\Gamma(k+l+3)}{(k-l)!(k+1)}\frac{a_ka_{k+1}}{(k+1)(k+2)^2}\right] \tag{14}$$

$$S_2=\sum_{k=l}^{M}\left[\frac{\Gamma(k+l+2)}{(k-l)!(k+1)}\frac{a_ka_k}{(k+1)^2}-\frac{\Gamma(k+l+3)}{(k-l)!(k+1)}\frac{a_ka_{k+1}}{(k+1)(k+2)^2}\right] \tag{15}$$

Note that M in the summations S_1 and S_2 denotes the highest k value for which $a_k \neq 0$. $\left|W_{n,l}\right|$ is the magnitude of the binding energy at serial number n and angular momentum l, corresponding to which $\left\langle x_a\right\rangle$ is being calculated. Note further that $\left|W_{n,l}\right|$ is already calculated from Equation (10) by the Hill determinant method, which also determines the value of M. This method, however, does not require an explicit knowledge of the coefficients a_k. The latter are therefore yet to be determined. This can be done conveniently by using the backward Miller algorithm [11] [12] which is reviewed in **Appendix A**, where we also record the steps that need to be taken to calculate a_k and whence $\left\langle x_a\right\rangle$ can be calculated. We then find that the smallness criterion is satisfied

Table 1. Energy eigenvalues $|W_{n,l}|$ of Equation (10) at temperature $T = 4.26 \times 10^6$ K for $1 \le n \le 30$ and $l = 0$, 1, and 2 obtained via the Hill determinant method. Note that energy level ($n = 1$, $l = 0$) will not be physically realizable at this temperature (see the text for details). Typical dimensions of the Hill determinant for stability up to the third place of decimal are: $k = 400$ ($n = 1$, $l = 0$); $k = 16{,}000$ for ($n = 10$, $l = 0$).

n	Binding energy $\lvert W_{n,l} \rvert$ (eV)		
	$\lvert W_{n,0} \rvert$	$\lvert W_{n,1} \rvert$	$\lvert W_{n,2} \rvert$
1	5003.429		
2	929.218	934.085	
3	360.299	360.398	360.421
4	197.600	197.607	197.609
5	125.586	125.587	125.587
6	86.994	86.994	96.994
7	63.845	63.845	63.845
8	48.856	48.856	48.856
9	38.591	38.591	38.591
10	31.254	31.254	31.254
11	25.827	25.827	25.827
12	21.700	21.700	21.700
13	18.489	18.489	18.489
14	15.942	15.942	15.942
15	13.887	13.887	13.887
16	12.205	12.205	12.205
17	10.811	10.811	10.811
18	9.643	9.643	9.643
19	8.655	8.655	8.655
20	7.811	7.811	7.811
21	7.085	7.085	7.085
22	6.455	6.455	6.455
23	5.906	5.906	5.906
24	5.424	5.424	5.424
25	4.999	4.999	4.999
26	4.622	4.622	4.622
27	4.285	4.285	4.285
28	3.985	3.985	3.985
29	3.715	3.715	3.715
30	3.471	3.471	3.471
∞	0.000	0.000	0.000

for any given $W_{n,l}$ only if the temperature T is above a certain value T_n, and furthermore, $T_1 > T_2 > T_3$…etc. The relevant results are displayed in **Table 2**. We see from this table that the solution corresponding to $W_{1,0}$ is acceptable only for $T > T_1 \approx 10^7$ K, that corresponding to W_{20} or W_{21} only for $T > T_2 \approx 2.6\times 10^6$ K, and so on. Clearly, all the solutions corresponding to $W_{n,l}$ for $n > 2$ are now acceptable if $T > T_2$. We must also keep in mind that a bound state with binding energy W will survive in a medium at temperature T only if $|W|$ is appreciably greater than ($3kT/2$), the latter being the average kinetic energy of thermal motion per particle in the medium. Thus, for instance, at a temperature $T = 4.26 \times 10^6$ K, the deepest possible states that can form in our spectrum are at the level $n = 2$, *i.e.*, $W_{2,0}$ and $W_{2,1}$. We note that for these levels $|W| > (3kT/2)$. So the states at these levels will not only form, but also survive in the hot medium. Of course the states with $n > 2$ will increasingly have $|W| < (3kT/2)$ and will therefore behave as very short-lived excited states. The transitions from these $n > 2$ excited states to the $n = 2$ states should lead, in view of $\Delta l = \pm 1$ selection rule, to the following three Balmer- like series of spectral lines:

$$\begin{aligned} &A:\left(W_{3,0},W_{4,0},W_{5,0},\cdots\right)\rightarrow W_{2,1}\\ &B:\left(W_{3,1},W_{4,1},W_{5,1},\cdots\right)\rightarrow W_{2,0}\\ &C:\left(W_{3,2},W_{4,2},W_{5,2},\cdots\right)\rightarrow W_{2,1}\end{aligned} \tag{16}$$

The wavelength λ of the spectral line corresponding to any transition involving the energy difference $|\Delta W|$ in eV is given by

$$\lambda = \frac{hc}{|\Delta W|} = \frac{12398.4927}{|\Delta W|}\ \dot{\text{A}}. \tag{17}$$

Substituting for any allowed $|\Delta W|$ from the above series, and using numbers from **Table 1**, we can check that the resulting wavelengths fall in the soft X-ray region.

3. Application to Solar Flares

Let us now apply our approach to Solar flares. The latter are appropriate for such an application as they occur in a medium with low particle densities ($\approx 10^{12}$/cm^3) at temperatures around a few million degrees Kelvin and are known to emit soft X-ray lines. Furthermore, since the flaring phenomenon is a prolonged affair, we may expect that between the initial (growing) phase and the final (decaying) phase, there should be a period over which the flare burns at a reasonably constant temperature (with fluctuations of, say, not more than ± 5000 K). If we knew this temperature, we could calculate the spectral lines of the kind given by Equation (13) and then look for them in the soft X-ray line data from flares. Of course this temperature is not quite known to us. So what we could do is to calculate our spectral lines at different temperatures between 10^6 K and 10^7 K, to see if there is any temperature in this range at which our calculated lines are reproduced in the flare data. As will be seen in the following, we find that there indeed is such a temperature given by $T = 4.26 \times 10^6$ K. The binding energy spectrum $|W_{n,l}|$ at this temperature is given in **Table 1**. From here, the wavelengths of spectral lines corresponding to series A, B, and C, given by Equation (13), can easily be calculated. They are given in **Table 3**. The notation ($n',l' \rightarrow n,l$) means that the transition is from the level $W_{n',l'}$ to $W_{n,l}$. These transitions are referred to in columns 2 and 6 and the corresponding calculated wavelengths are given in columns 3 and 7.

Table 2. Temperatures for decreasing values of $\langle x_a\rangle = \langle p^2/4m_a kT\rangle$ for several energy levels calculated via Miller's algorithm.

	$T(W_{n,l})$: Temperature for energy level $\lvert W_{n,l}\rvert$ (10^6 K)								
$\langle x_a\rangle$	$T(W_{1,0})$	$T(W_{2,1})$	$T(W_{2,0})$	$T(W_{3,1})$	$T(W_{3,0})$	$T(W_{4,1})$	$T(W_{4,0})$	$T(W_{5,1})$	$T(W_{5,0})$
≈0.20	4.0	1.05	0.875	0.436	0.38	0.229	0.209	0.141	0.126
≈0.10	5.25	1.38	1.22	0.575	0.501	0.302	0.275	0.186	0.174
≈0.01	10.9	2.63	2.63	1.20	1.20	0.650	0.650	0.398	0.398
≈0.0	20	5	5.0	2.0	2.0	1.0	1.0	0.630	0.630

Table 3. Calculated wavelength $\lambda(\dot{A})$ at temperature T = 4.26×10^6 K for the emission lines comprising the generalized Balmer series A, B, and C which characterize our pair spectrum. Except for a very small difference in wavelength for the first set of lines at (3.0 → 2.1) and (3.2 → 2.1), the series A and C are degenerate. Therefore for the rest of the transitions in these series, only those from the former are given in the table. The observed wavelengths (good to within $\pm 0.02\ \dot{A}$) are from the data on Solar flares by Phillips *et al.* [12]; those marked with (+) are from the data of McKenzie *et al.* [14].

S. No.	Transitions (Series B)	Calculated $\lambda\ (\dot{A})$	Observed $\lambda\ (\dot{A})$ to within $\pm 0.02\ \dot{A}$	Convention-al source	Transitions (Series A & C)	Calculated $\lambda\ (\dot{A})$	Observed $\lambda\ (\dot{A})$ to within $\pm 0.02\ \dot{A}$	Conventional source
1	3.1 → 2.0	21.797	21.798+	O VII	3.0 → 2.1 3.2 → 2.1	21.608 21.613	21.602+ 21.602+	O VII
2	4.1 → 2.0	16.947	16.956+	?	4.0 → 2.1	16.835	16.821	?
3	5.1 → 2.0	15.428	15.428	?	5.0 → 2.1	15.335	-	-
4	6.1 → 2.0	14.721	14.737	Fe XIX	6.0 → 2.1	14.636	-	-
5	7.1 → 2.0	14.327	14.311	?	7.0 → 2.1	14.247	14.258	Fe XVIII
6	8.1 → 2.0	14.083	14.076	Ni XIX	8.0 → 2.1	14.006	14.017	?
7	9.1 → 2.0	13.921	13.934	?	9.0 → 2.1	13.845	13.842	?
8	10.1 → 2.0	13.807	13.796	Fe XIX	10.0 → 2.1	13.733	13.738	Fe XIX
9	11.1 → 2.0	13.724	13.719	Ne VIII	11.0 → 2.1	13.651	13.645	Ne VIII
10	12.1 → 2.0	13.662	13.669	Fe XIX	12.0 → 2.1	13.589	-	-
11	13.1 → 2.0	13.614	13.630	?	13.0 → 2.1	13.541	13.551	Ne IX
12	14.1 → 2.0	13.576	-	-	14.0 → 2.1	13.504	13.504	Fe XIX
13	15.1 → 2.0	13.545	13.551	Ne IX	15.0 → 2.1	13.474	13.463	Fe XIX
14	16.1 → 2.0	13.520	13.520	Fe XIX	16.0 → 2.1	13.449	13.446	Ne IX
15	17.1 → 2.0	13.500	13.504	Fe XIX	17.0 → 2.1	13.429	13.426	Fe XIX (?)
16	18.1 → 2.0	13.483	13.463	Fe XIX	18.0 → 2.1	13.412	13.402	Fe XIX (?)
17	19.1 → 2.0	13.468	13.463	Fe XIX	19.0 → 2.1	13.397	13.4-2	Fe XIX (?)
18	20.1 → 2.0	13.456	13.446	Ne IX	20.0 → 2.1	13.385	13.375	Fe XVIII
19	21.1 → 2.0	13.445	13.446	Ne IX	21.0 → 2.1	13.375	13.375	Fe XVIII
20	22.1 → 2.0	13.436	13.446	Ne IX	22.0 → 2.1	13.366	13.354	Fe XVIII
21	23.1 → 2.0	13.428	13.426	Fe XIX (?)	23.0 → 2.1	13.358	13.354	Fe XVIII
22	24.1 → 2.0	13.421	13.426	Fe XIX (?)	24.0 → 2.1	13.351	13.354	Fe XVIII
23	25.1 → 2.0	13.415	13.402	Fe XIX (?)	25.0 → 2.1	13.345	13.354	Fe XVIII
24	26.1 → 2.0	13.410	13.402	Fe XIX (?)	26.0 → 2.1	13.339	13.322	Fe XVIII
25	27.1 → 2.0	13.405	13.402	Fe XIX (?)	27.0 → 2.1	13.335	13.322	Fe XVIII
26	28.1 → 2.0	13.400	13.402	Fe XIX (?)	28.0 → 2.1	13.330	13.322	Fe XVIII
27	29.1 → 2.0	13.396	13.402	Fe XIX (?)	29.0 → 2.1	13.326	13.322	Fe XVIII
28	30.1 → 2.0	13.393	13.402	Fe XIX (?)	30.0 → 2.1	13.323	13.322	Fe XVIII
-	-	-	-	-	-	-	-	-
29	∞ → 2.0	13.343	13.354	Fe XVIII	∞ → 2.1	13.273	13.279	Fe XIX

We now turn to the relevant experimental information. The data between 5 and 20 Å were obtained by Phillips *et al.* [13] from a flare on August 25, 1982 and from another flare on November 5, 1980. The work of Acton *et al.* [14] covers the range between 11 and 94 Å. These observations were made on an M-class flare on July 13, 1982. They covered a much larger range of wavelengths than covered by Phillips *et al.*, but missed out bunches of lines, especially between 12.82 and 13.45 Å. McKenzie *et al.* [15] also provide useful data between 8 and 22 Å. In the range common to all these three groups, the data of [13] are the most exhaustive. As to the accuracy of the wavelengths quoted by these authors, Acton *et al.* estimate it to be not better than ± 0.02 Å. The other authors do not give direct estimates of their own, but their discussions suggest that it would not be safe to violate the limits set by Acton *et al.* Keeping this in mind, we now compare the emission lines from these data with those calculated from our pair spectrum at $T = 4.26 \times 10^6$ K. Any line from the data which is within ± 0.02 Å of a calculated line is identified with the latter. We may mention here that a change of up to ± 5000 K does not make any change in our calculated wavelengths up to the third decimal place.

The experimental results in **Table 3** (columns 4 and 8) are taken from Phillips *et al.* [13], except in the range not covered by them, *i.e.*, for $\lambda(3.1 \to 2.0)$, $\lambda(3.0 \to 2.1)$ and $\lambda(3.2 \to 2.1)$, which are from McKenzie *et al.* (the corresponding numbers from Acton *et al.* are 21.80 Å and 21.60 Å, not different from those of McKenzie *et al.* within the accuracy limit of ± 0.02 Å). An exceptional situation exists in the case of $\lambda(4.1 \to 2.0)$: the line close to its calculated value, 16.947 Å, is not seen by Phillips *et al.*, though it is within their range. The experimental value 16.956 Å is from McKenzie *et al.*

A look at **Table 3** will show that except for the single line at $\lambda(14.1 \to 2.0) = 13.576$ Å [the observed wavelength closest to which is $\lambda = 13.551$ Å, so that $\Delta\lambda = 0.025$ which is just outside the accuracy limit of ± 0.02 Å], series B is observed in its entirety. The single missing line could also account for the line at $\lambda(12.0 \to 2.1) = 13.589$ Å in Series A. For the latter series, however, two additional lines $\lambda(5.0 \to 2.1)$ and $\lambda(6.0 \to 2.1)$ are not observed. Other than this, all the lines in this series are also observed. Series C is degenerate with Series A except for the first line $\lambda(3.2 \to 2.1) = 21.613$ Å, the counterpart of the line $\lambda(3.0 \to 2.1) = 21.608$ Å. These two lines are nearly degenerate and correspond to the observed line at 21.602 Å [13] or 21.60 Å [12].

4. Possible Connection with the FIP Effect

It may be noted that 29 distinct lines from the Solar flare data have been used up in the above as evidence for our generalized Balmer series. The data of course contain hundreds of lines over a wide range in the X-ray region. Our object is not to suggest that they all originate from our pairing mechanism. In fact, following the pioneering works of Grotrian and Edlen, we take it for granted that these lines follow from the so-called forbidden transitions in highly ionized high-Z atoms, the presence of which in stellar plasmas is a natural consequence of their having been formed in the interior of stars. The sources of a great many of these lines have accordingly been identified and the effort in that direction continues. Nevertheless, we may mention that, of the 29 lines from the data which we matched with our calculated lines, eight remain unidentified (marked [?] in **Table 3**) and two have questionable identification (marked [?] after the possible identification).

The conventional approach is thus not all-encompassing. In fact, the lack of proper identification in this approach of as many as eight (possibly 10) lines out of 29 does suggest that these lines may have a different origin. But what evidence could there be for such an additional origin for the other 19 lines which have been identified in the conventional approach? Interestingly enough, some evidence is indeed there. As already noted, it comes from what is called the FIP effect [4] [5]. The effect is widely observed, but we shall confine ourselves here to the part which relates to the emission lines from the Solar flares. According to conventional wisdom underlying the classical stellar atmospheres theory, the relative abundances of various elements in a star are not expected to show any variation in its upper layers (unlike in the interior where thermonuclear-process gradients exist). The relative abundance pattern for elements from the Solar photosphere should thus not be different from that obtained from the Solar flares or from the Solar coronal active regions. It turns out that this is not so. Elements with low FIP appear to be relatively more abundant in regions with $T \geq 10^6$ K than in the photosphere ($T \approx 6000$ K). How does one infer this? One compares the relative intensity pattern of emission lines for the same elements from a) the high temperature sources like the flares and the coronal active regions and b) the photosphere. One finds that the intensities of lines corresponding to the low FIP elements from the former are anomalously enhanced. This may be interpreted to mean that these low FIP elements are relatively more abundant in the high temperature sources, which is in disagreement with the classical stellar atmospheres theory. Let us now look at the situation from the point of view of this study. We observe that of the 19 lines predicted by us, which are also identifiable as lines from known elements, almost all can be traced to a single low FIP element Fe (see **Table 3**) in its ionized state given by Fe XIX. In the light of the FIP effect, these are thus the lines which show anomalous enhancement. Clearly, such enhancement can now be attributed to an additional mechanism for their origin viz. the pairing mechanism considered herein. Note that such enhancement will not be possible for any lines originating from the photosphere which exists at a temperature of about 6000 K at which the pairing does not take place. This opens up the possibility of leaving undisturbed the generally accepted (and empirically corroborated) picture of the uniformity of relative abundances of elements in the upper reaches of the Sun.

It may also be pointed out that the detailed theoretical calculations [16] of the intensity ratios of lines, specifically from Fe XIX, do not seem to match with the observed data from the flares. This mismatch between theory and observation may be another pointer to the existence of an additional mechanism for the origin of the above lines.

5. Conclusions

In the context of the present study, we note that the generally accepted and empirically corroborated picture of the uniformity of elements in the upper reaches of the Sun, e.g., the photosphere and the Solar flares and coronal regions, would imply that the relative intensities of spectral lines from various elements would not show any significant variation from one region to another. It turns out that this is not so. The reason is: while 19 of the 29 lines in the data analyzed here and attributed to our pairing mechanism at $T = 4.26 \times 10^6$ K can also be identified with those from a single low FIP elements Fe XIX, their intensities in the flares region are found to be anomalously enhanced as compared with the intensities of lines from the photosphere. It then makes sense to conclude that this enhancement is due to the existence of another mechanism operative in the region of flares, but not in the photosphere. As has been argued above, our pairing mechanism takes place at temperatures exceeding about a million K and not at temperatures around 5000 K that characterize the photosphere. We further note that 8 (possibly 10) of the 29 lines in the data analyzed above are not identifiable (or have questionable identification) on the basis of transitions from Fe XIX or any other elements. This lends support to the view that they may well be due to the additional pairing mechanism presented here.

Finally, we should note that while a recent review [17] gives several references to the *experimental* data on solar flares observed since 1996, the problem of the anomalous enhancement of the abundance of elements dealt with here continues to be unaddressed. To deal with these data via the approach presented here is our next task.

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References

[1] Malik, G.P., Pande, L.K. and Varma, V.S. (1991) *Astrophysical Journal*, **379**, 788-795. http://dx.doi.org/10.1086/170554

[2] Malik, G.P., Pande, L.K. and Varma, V.S. (1989) *Pramana—Journal of Physics*, **32**, L89-L94.

[3] Pande, L.K. (1991) *Pramana—Journal of Physics*, **37**, 39-45.

[4] Haisch, B., Julia, L., Saba, R. and Meyer, J.-P. (1996) In: Bowyer, S. and Malina, R.F., Eds., *Astrophysics in the Extreme Ultraviolet*, Kluwer Academic Publishers, Netherland, 511-518.

[5] Laming, J.M., Drake, J.J. and Widing, K.G. (1995) *Astrophysical Journal*, **443**, 416-422. http://dx.doi.org/10.1086/175534

[6] Hautot, A. (1986) *Physical Review D*, **33**, 437-443. http://dx.doi.org/10.1103/PhysRevD.33.437

[7] Znojil, M. (1986) *Physical Review D*, **34**, 1224-1228. http://dx.doi.org/10.1103/PhysRevD.34.1224

[8] Biswas, S.N., Datta, K., Saxena, R.P., Srivastava, P.K. and Varma, V.S. (1971) *Physical Review D*, **4**, 3617-3620. http://dx.doi.org/10.1103/PhysRevD.4.3617

[9] Biswas, S.N., Datta, K., Saxena, R.P., Srivastava, P.K. and Varma, V.S. (1973) *Journal of Mathematical Physics*, **14**, 1190-1195. http://dx.doi.org/10.1063/1.1666462
http://scitation.aip.org/content/aip/journal/jmp/14/9/10.1063/1.1666462

[10] Killingback, J.F. (1983) Microcomputer Quantum Mechanics. Adam Hilger, Bristol.

[11] Gautschi, W. (1967) *SIAM Review*, **9**, 24-82. http://dx.doi.org/10.1137/1009002

[12] Malik, G.P., Malik, U. and Varma, V.S. (1993) *Astrophysics and Space Science*, **199**, 299-315. http://dx.doi.org/10.1007/BF00613204

[13] Phillips, K.J.H., Fawcett, B.C., Kent, B.J., Gabriel, A.H., Leibbacher, J.W., *et al.* (1982) *Astrophysical Journal*, **256**, 774-787. http://dx.doi.org/10.1086/159950

[14] Acton, L.W., Bruner, M.E., Brown, W.A., Fawcett, B.C., Schweizer, W. and Speer, R.J. (1985) *Astrophysical Journal*, **291**, 865-878. http://dx.doi.org/10.1086/163125

[15] McKenzie, D.L., Landecker, P.B., Broussard, R.M., Rugge, H.R., Young, R.M., Feldman, U., *et al.* (1980) *Astrophysical Journal*, **241**, 409-416. http://dx.doi.org/10.1086/158354

[16] Bhatia, A.K., Fawcett, B.C., Lemen, J.R., Mason, H.E. and Phillips, K.J.N. (1989) *Monthly Notices of Royal Astronomical Society*, **240**, 421-444. http://dx.doi.org/10.1093/mnras/240.2.421

[17] Laming, J.M. (2015) *Living Rev. Solar Physics*, **12**, 2. http://dx.doi.org/10.1007/lrsp-2015-2

Appendix A

We evaluate here the expectation value $\langle x_a \rangle$, given by

$$\langle x_a \rangle = \left\langle \frac{\beta p^2}{4m_a} \right\rangle = \frac{\beta}{4m_a} \int \psi^*(p) p^2 \psi(p) \mathrm{d}p \Big/ \int \psi^*(p) \psi(p) \mathrm{d}p \tag{A1}$$

where, in view of Equations (8) in the text,

$$\psi(p) = \psi(p,\theta,\eta) = \cos^4\left(\frac{\varphi}{2}\right) \sum_{k=l}^{\infty} a_k P_{k,l}^{(2)}(\cos\varphi) Y_l^m(\theta,\eta), \tag{A2}$$

and $P_{k,l}^{(2)}(\cos\varphi)$ is given by Equation (9).

Upon using

$$\rho = \frac{|p|}{p_0},\ \rho' = \frac{|p'|}{p_0},\ p_0^2 = -2\mu W > 0,\ \lambda = -\alpha\mu,\ \cos\varphi = x,$$

we obtain, after going through some elementary algebra,

$$\int \psi^*(p)\psi(p)\mathrm{d}p = \frac{p_0^3}{16} \int_{-1}^{1} \mathrm{d}x \left(1-x^2\right)^{l+1/2} (1+x) \sum_{k=l}^{\infty} \sum_{k'=l}^{\infty} \frac{a_k a_{k'}}{(k+1)(k'+1)} C_{k-l}^{l+1}(x) C_{k'-l}^{l+1}(x). \tag{A3}$$

With the help of the standard result

$$\int_{-1}^{1} \mathrm{d}x \left(1-x^2\right)^{\upsilon-1/2} C_n^{\upsilon}(x) C_m^{\upsilon}(x) = \frac{\pi 2^{1-2\upsilon} \Gamma(2\upsilon+1)}{n!(n+\upsilon)\left[\Gamma(\upsilon)\right]^2} \delta_{n,m} \tag{A4}$$

and the recurrence relation

$$x C_{k'-l}^{l+1}(x) = \frac{1}{2(k'+1)} \left[(k'-l+1) C_{k'-l+1}^{l+1}(x) + (k'+l+1) C_{k'-l-1}^{l+1}(x) \right], \tag{A5}$$

we then get

$$\int \psi^*(p)\psi(p)\mathrm{d}p = \frac{p_0^3}{16} \frac{\pi 2^{-1-2l}}{\left[\Gamma(l+1)\right]^2} S_1, \tag{A6}$$

where

$$S_1 = \sum_{k=l}^{M} \left[\frac{\Gamma(k+l+2)|a_k|^2}{(k-l)!(k+1)^3} + \frac{\Gamma(k+l+3) a_k a_{k+1}}{(k-l)!(k+1)^2 (k+2)^2} \right]. \tag{A7}$$

Proceeding in a similar manner, we also obtain

$$\begin{aligned} \int \psi^*(p) p^2 \psi(p)\mathrm{d}p &= \frac{p_0^5}{16} \int_{-1}^{1} \mathrm{d}x \left(1-x^2\right)^{l+1/2} (1-x) \sum_{k=l}^{\infty} \sum_{k'=l}^{\infty} \frac{a_k a_{k'}}{(k+1)(k'+1)} C_{k-l}^{l+1}(x) C_{k'-l}^{l+1}(x) \\ &= \frac{p_0^5}{16} \frac{\pi 2^{-1-2l}}{\left[\Gamma(l+1)\right]^2} S_2, \end{aligned} \tag{A8}$$

where

$$S_2 = \sum_{k=l}^{M} \left[\frac{\Gamma(k+l+2)|a_k|^2}{(k-l)!(k+1)^3} - \frac{\Gamma(k+l+3) a_k a_{k+1}}{(k-l)!(k+1)^2 (k+2)^2} \right]. \tag{A9}$$

We thus obtain [12]

$$\langle x_a \rangle = \frac{\beta p_0^2}{4m_a} \frac{S_2}{S_1} = \frac{\beta \mu_b |W_{n,l}|}{2} \frac{S_2}{S_1} \tag{A10}$$

Note that M is the summations in S_1 and S_2 denotes the highest k value for which $a_k \neq 0$, and $\left|W_{n,l}\right|$ is the magnitude of the binding energy corresponding to principal quantum number n and angular momentum l for which $\langle x_a \rangle$ is being calculated. Note further that $\left|W_{n,l}\right|$ is already calculated vide Equation (10) by the Hill determinant method, which also determines the value of M. The method does not require an explicit knowledge of coefficients a_k, which can be determined by using the backward Miller algorithm [12]. To this end, we define

$$r_{k-1} = \frac{b_k}{b_{k-1}}, \tag{A11}$$

whence Equation (10) can be written as

$$r_k r_{k-1} + A_k r_{k-1} + B_k = 0, \tag{A12}$$

where

$$A_k = \frac{2(k+1)\left[c_1(k+1) - c_2\right]}{(k+l+1)\left[c_3(k+1) - c_4\right]}, \ B_k = \frac{k-l}{k+l}. \tag{A13}$$

We thus have

$$r_{k-1} = \frac{-B_k}{(A_k + r_k)}, \tag{A14a}$$

$$b_k = r_{k-1} b_{k-1}; \quad l+1 \le k \le M. \tag{A14b}$$

These equations have to be supplemented by

$$r_M = 0, \quad b_{l-1} = 0. \tag{A15}$$

Equation (A14a) then determines $r_{M-1}, r_{M-2}, \cdots, r_{l+1}, r_l$, and given b_l (which can be taken as unity because the ratio S_2/S_1 above will be independent of this choice), $b_{l+1}, b_{l+2}, \cdots, b_M$ are determined via Equation (A14b). Equation (11) then enables the coefficients $a_{l+1}, a_{l+2}, \cdots, a_M$ to be calculated.

Thus knowing $W_{n,l}$ corresponding to any value of T as calculated from Equation (10), and the corresponding values of l and M, we can calculate the coefficients a_k and, finally, using Equation (A10), the value of $\langle x_a \rangle$.