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Effect of Rotation and Initial Magnetic Field in Fibre-Reinforced Anisotropic Elastic Media

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Abstract

In this work, we study an analytical procedure for evaluation of the displacement and stresses in fibre-reinforced anisotropic elastic media under effects of rotation and initial magnetic field, and due to the application of the rotation and initial magnetic field. Effects of rotation and initial magnetic field are analyzed theoretically and computed numerically. Numerical results have been given and illustrated graphically. Comparison was made with the results obtained in the presence of rotation and initial magnetic field in fibre-reinforced anisotropic and isotropic elastic media. The results indicate the effect of rotation and initial magnetic field.

Keywords

Fibre-Reinforced Medium, Harmonic Vibrations, Initial Magnetic Field, Rotation, Anisotropic

1. Introduction

The linear theory of elasticity of paramount importance in the stress analysis of steel is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. The problem of rotating disks or cylinders has its application in high-speed cameras, steam and gas turbines, planetary landings and in many other domains. Various authors have formulated these generalized theories on different grounds. Lord and Shulman [1] have developed a theory on the basis of a modified heat conduction law which involves heat-flux rate. Green and Lindsay [2] have developed a theory by including temperature-rate among the constitutive variables. Lebon [3] has formulated a theory by considering heat-flux as an independent variable. Also some problems in thermoelastic rotating media are due to Roychoudhuri and Debnath [4] [5]. These problems are based on more realistic elastic model since earth, moon and other planets have angular velocity. Abd-Alla *et al*. [6] study effects of the rotation on an

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infinite generalized magneto-thermoelastic diffusion body with a spherical cavity. Effects of rotation and initial stress on generalized-thermoelastic problem in an infinite circular cylinder are due to Abd-Alla *et al*. [7]. Bayones [8] studied effects of rotation and hydrostatic initial stress on propagation of Raylegh in waves in an elastic solide half-space under the GN theory. The solution to the problems of homogeneous isotropic rotating cylinders may be found in Love [9] and Sokolnikoff [10]. Abd-Alla and Abo-Dahab [11] and Sharma *et al*. [12] studied the effect of the time-harmonic source in a generalized thermoelasticity. Chandrasekharaiah [13], Green and Naghdi [14], and Hossen and Mallet [15] discussed the problem of thermoelasticity without energy dissipation. Abd-Alla *et al*. [16] studied M. I. Helmy's *Propagation of S-Wave in a Non-Homogeneous Anisotropic Incompressible and Initially Stressed Medium under Influence of Gravity Field*. Effects of the rotation on a non-homogeneous infinite cylinder of orthotropic material are due to Abd-Alla *et al*. [17].

Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The mechanical behavior of many fibre-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites, the fibres are usually arranged in parallel straight lines. The characteristic property of a reinforced composite is that its components act together as a single anisotropic unit as long as they remain in the elastic condition.

The idea of introducing a continuous self-reinforcement at every point of an elastic solid was discussed by Belfied *et al*. [18]. The model was later applied to the rotation of a tube as discussed by Verma and Rana [19]. The problem of surface waves in fiber-reinforced anisotropic elastic media was discussed by Sengupta and Nath [20]. The elastic moduli for fiber-reinforced materials was given by Hashin and Rosen [21]. The problem of reflection of plane waves at the free surface of a fiber-reinforced elastic half-space was discussed by Singh and Singh [22]. The dispersion of Loves waves in a self-reinforced layer over an elastic non-homogeneous half-space was studied by Pradhan *et al*. [23]. The propagation of plane waves in a fiber-reinforced media was discussed by Chattopadhyay *et al*. [24]. The problem of wave propagation in thermally conducting linear fiber-reinforced composite materials was discussed by Singh [25]. Recently, the effect of rotation on plane waves at the free surface of a fiber-reinforced thermoelastic half-space using the finite element method was studied by Othman and Abbas [26].

In this paper, we studied an analytical procedure for evaluation of the displacement, and stresses in fibre-reinforced anisotropic elastic media under effect of rotation and initial magnetic field. Using the harmonic vibrations, we found the general solution, determining the displacements and stress components. The special case was studied in isotropic generalized elastic medium with rotation and initial magnetic field. Finally, we represented this case graphically.

2. Formulation of the Problem (Figure 1)

The propagation of general surface waves is examined here for a fiber-reinforced elastic solid semi-infinite medium M covered by another fiber-reinforced elastic medium M_1 (M_1 above M and mechanical properties different from M and which is welded in contact with M to prevent any relative motion or sliding during disturbance). We consider an orthogonal Cartesian coordinate system $ox_1x_2x_3$ with origin O at the common plane boundary surface and ox_2 directed normally into M. The elastic medium is rotating uniformly with angular velocity $\mathbf{\Omega} = \Omega \underline{n}$ where $\underline{n}$ is a unite vector representing the direction of the axis of rotation $\mathbf{\Omega} \equiv (0,0,\Omega)$.

Both media are under the primary magnetic field $\boldsymbol{Ho}$ acting on Z-axis, $\boldsymbol{HO} \equiv (0,0,HO)$. The displacement equation of motion in the rotating frame has two additional terms $\underline{\Omega} \wedge (\underline{\Omega} \wedge \underline{u})$ is The centripetal acceleration due to time varying motion only, and $2\mathbf{\Omega} \wedge \boldsymbol{u}$ is the Coriolis acceleration.

The electromagnetic field is governed by Maxwell equations , under the consideration that the medium is a perfect electric conductor taking into account the absence of the displacement current (SI) (see work of Mukhopadhyay [27]):

$$\begin{aligned}
&\boldsymbol{J} = \operatorname{curl} \boldsymbol{h}, \\
&\operatorname{curl} \boldsymbol{E} = -\mu_e \frac{\partial \boldsymbol{h}}{\partial t}, \\
&\operatorname{div} \boldsymbol{h} = 0, \\
&\operatorname{div} \boldsymbol{E} = 0, \\
&\boldsymbol{E} = -\mu_e \left(\frac{\partial \boldsymbol{u}}{\partial t} \wedge \boldsymbol{H} \right).
\end{aligned} \tag{1}$$

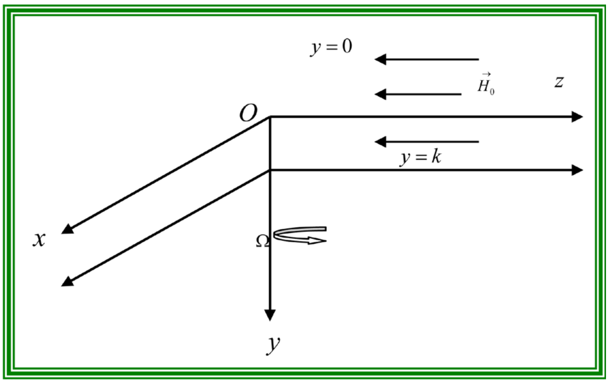

Figure 1. Schematic of the problem.

where

$$\boldsymbol{h} = \operatorname{curl}\left(\boldsymbol{u} \wedge \boldsymbol{H}_0\right), \quad \boldsymbol{f} = \mu_e\left(\boldsymbol{J} \wedge \boldsymbol{H}\right), \quad \boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h}, \quad \boldsymbol{H}_0 \equiv \left(0, 0, H_0\right) \tag{2}$$

where $\boldsymbol{h}$ is the perturbed magnetic field over the primary magnetic field vector, $\boldsymbol{E}$ is the electric intensity, $\boldsymbol{J}$ is the electric current density, μ_e is the magnetic permeability, $\boldsymbol{Ho}$ is the constant primary magnetic field vector, $\boldsymbol{u}$ the displacement vector.

The constitutive equation for the fiber reinforced linearly elastic anisotropic medium with respect to preferred direction $\boldsymbol{a}$ is Belfied *et al*. [28]

$$\begin{aligned}\tau_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T eij + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}a_i a_j\right)\\ &\quad + 2\left(\mu_L - \mu_T\right)\left(a_i a_k e_{kj} + a_j a_k e_{ki}\right) + \beta\left(a_k a_m e_{km} a_i a_j\right)\end{aligned} \tag{3}$$

where are τ_{ij} components of stress,

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

Are the components of strain, λ, μ_T are elastic parameter; α, β, $\left(\mu_L - \mu_T\right)$ are reinforced anisotropic elastic parameters; u_j are the displacement vector components and $\boldsymbol{a}\left(a_1, a_2, a_3\right)$ where $a_1^2 + a_2^2 + a_3^2 = 1$ if a $\boldsymbol{a}$ has components the are (1,0,0) so that the preferred direction is x_1 -axis, (3) simplifies as given below:

$$\begin{aligned}\tau_{11} &= \lambda\left(e_{kk}\right) + 2\mu_T e_{11} + \alpha\left(a_k a_m e_{lm} + e_{kk}a_1^2\right)\\ &\quad + 2\left(\mu_L - \mu_T\right)\left(a_1 a_k e_{k1} + a_1 a_k e_{k1}\right) + \beta\left(a_k a_m a_{km} a_1^2\right); \quad k, m = 1, 2, 3\end{aligned}$$

$$\begin{aligned}\tau_{11} &= \left(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta\right)e_{11} + \left(\lambda + \alpha\right)e_{22} + \left(\lambda + \alpha\right)e_{33},\\ \tau_{22} &= \left(\lambda + \alpha\right)e_{11} + \left(\lambda + 2\mu_T\right)e_{22} + \lambda e_{33},\\ \tau_{33} &= \left(\lambda + \alpha\right)e_{11} + \lambda e_{22} + \left(\lambda + 2\mu_T\right)e_{33},\\ \tau_{23} &= 2\mu_T e_{23},\\ \tau_{13} &= 2\mu_L e_{13},\\ \tau_{12} &= 2\mu_L e_{12}.\end{aligned} \tag{5}$$

The equations of motion are:

$$\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial \tau_{13}}{\partial x_3} + F_x = \rho\left[\frac{\partial^2 u_1}{\partial t^2} - 2\Omega \dot{u}_2 - \Omega^2 u_1\right] \tag{6}$$

$$\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} + \frac{\partial \tau_{23}}{\partial x_3} + F_y = \rho\left[\frac{\partial^2 u_2}{\partial t^2} + 2\Omega \dot{u}_1 - \Omega^2 u_2\right] \tag{7}$$

$$\frac{\partial \tau_{31}}{\partial x_1}+\frac{\partial \theta \tau_{32}}{\partial x_2}+\frac{\partial \tau_{33}}{\partial x_3}+F_z=\rho\left[\frac{\partial^2 u_3}{\partial t^2}\right] \tag{8}$$

where,

$$\boldsymbol{F}=\mu_e\left(\boldsymbol{J}\wedge H\right)$$

$$\boldsymbol{F}\equiv\left(\mu_e H_0^2\left(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_2}{\partial x_1 \partial x_2}-\frac{\partial^2 u_1}{\partial x_3^2}\right),\quad \mu_e H_0^2\left(\frac{\partial^2 u_1}{\partial x_1 \partial x_2}+\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_2}{\partial x_3^2}\right),0\right)$$

where ρ is the density of the elastic medium. Using (5)-(8) and assuming all derivatives with respect to x_3 vanish, the equations of motion become

$$\begin{aligned}&\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_e H_0^2\right)\frac{\partial^2 u_1}{\partial x_1^2}+\left(\lambda+\alpha+\mu_L+\mu_e H_0^2\right)\frac{\partial^2 u_2}{\partial x_1 \partial x_2}+\mu_L\frac{\partial^2 u_1}{\partial x_2^2}\\&=\rho\left(\frac{\partial^2 u_1}{\partial t^2}-2\Omega\frac{\partial u_2}{\partial t}-\Omega^2 u_1\right)\end{aligned} \tag{9}$$

$$\begin{aligned}&\mu_L\frac{\partial^2 u_2}{\partial x_1^2}+\left(\lambda+2\mu_T+\mu_e H_0^2\right)\frac{\partial^2 u_2}{\partial x_2^2}+\left(\mu_L+\lambda+\alpha+\mu_e H_0^2\right)\frac{\partial^2 u_1}{\partial x_1 \partial x_2}\\&=\rho\left(\frac{\partial^2 u_2}{\partial t^2}+2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2\right)\end{aligned} \tag{10}$$

$$\left(\mu_L-\mu_T\right)\frac{\partial^2 u_3}{\partial x_1^2}+\mu_T\left(\frac{\partial^2 u_3}{\partial x_1^2}+\frac{\partial^2 u_3}{\partial x_2^2}\right)=\rho\frac{\partial^2 u_3}{\partial t^2} \tag{11}$$

To examine dilatational and rotational disturbances, we introduce two displacement potentials ϕ and φ by the relations:

$$u_1=\frac{\partial \phi}{\partial x_1}+\frac{\partial \varphi}{\partial x_2},\quad u_2=\frac{\partial \phi}{\partial x_2}-\frac{\partial \varphi}{\partial x_1} \tag{12}$$

The component u_3 is associated with purely distortional movement. Using (12) in (9) we obtain the following equation in M satisfied by ϕ and φ as:

$$\begin{aligned}&\left(\lambda+2\alpha+4\mu_L-2\mu_T+\beta+\mu_e H_0^2\right)\frac{\partial^2 \phi}{\partial x_1^2}+\left(\lambda+\alpha+2\mu_L+\mu_e H_0^2\right)\frac{\partial^2 \phi}{\partial x_2^2}\\&=\rho\left(\frac{\partial^2 \phi}{\partial t^2}+2\Omega\frac{\partial \varphi}{\partial t}-\Omega^2 \phi\right)\end{aligned} \tag{13}$$

$$\left(\alpha+3\mu_L-2\mu_T+\beta\right)\frac{\partial^2 \varphi}{\partial x_1^2}+\mu_L\frac{\partial^2 \varphi}{\partial x_2^2}=\rho\left(\frac{\partial^2 \varphi}{\partial t^2}-2\Omega\frac{\partial \phi}{\partial t}+\Omega^2 \varphi\right) \tag{14}$$

And for medium M_1:

$$\begin{aligned}&\left(\lambda_1+2\alpha_1+4\mu_{L1}-\mu_{T1}+\beta_1+\mu_e H_0^2\right)\frac{\partial^2 \phi_1}{\partial x_1^2}+\left(\lambda_1+\alpha_1+2\mu_{L1}+\mu_e H_0^2\right)\frac{\partial^2 \phi_1}{\partial x_2^2}\\&=\rho_1\left(\frac{\partial^2 \phi}{\partial t^2}+2\Omega\frac{\partial \varphi_1}{\partial t}-\Omega^2 \phi_1\right)\end{aligned} \tag{15}$$

$$\left(\alpha_1+3\mu_{L1}-2\mu_{T1}+\beta_1\right)\frac{\partial^2\varphi_1}{\partial x_1^2}+\mu_{L1}\frac{\partial^2\varphi_1}{\partial x_2^2}=\rho\left(\frac{\partial^2\varphi_1}{\partial t^2}-2\Omega\frac{\partial\varphi_1}{\partial t}+\Omega^2\varphi_1\right) \tag{16}$$

3. Boundary Conditions

The boundary conditions for the titled problem are:

a) The component of displacement at the boundary surface between the media M and M_1 must be continues at all times and places.

$$u_1=u_1',\quad u_2=u_2',\quad u_3=u_3'\quad \text{at}\quad x_2=0$$

$$\mu_{T1}\frac{\partial u_3}{\partial x_2}=\mu_{T1}\frac{\partial u_3^{(1)}}{\partial x_2}\quad \text{at}\quad x_2=0$$

b) The stress components $\tau_{21}\tau_{22}$ and τ_{23} must be continuous a crass the interface of M and M_1 at all times and place s.

$$\tau_{21}=\tau_{21}^{(1)},\quad \tau_{22}-\tau_{22}^{(1)}=H\mathrm{e}^{i\omega(x_1-ct)},\quad \tau_{23}=\tau_{23}^{(1)}\quad \text{at}\quad x_2=0$$

where τ_{21}, τ_{22} and τ_{23} can be written in terms of ϕ and φ in medium M from (5) to (12)

$$\tau_{22}=\lambda\nabla^2\phi+\alpha\left(\frac{\partial^2\phi}{\partial x_1^2}+\frac{\partial^2\varphi}{\partial x_1\partial x_2}\right)+2\mu_T\left(\frac{\partial^2\phi}{\partial x_2^2}-\frac{\partial^2\varphi}{\partial x_1\partial x_2}\right) \tag{17}$$

$$\tau_{21}=\mu_L\left(2\frac{\partial^2\phi}{\partial x_2\partial x_1}+\frac{\partial^2\varphi}{\partial x_2^2}-\frac{\partial^2\varphi}{\partial x_1^2}\right) \tag{18}$$

$$\tau_{23}=\mu_T\frac{\partial u_3}{\partial x_2} \tag{19}$$

where ∇^2 is the two dimensional Laplaci an operator given by

$$\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}$$

Similar relations in M_1:

$$\tau_{21}'=\mu_{L1}\left(\frac{\partial^2\phi_1}{\partial x_1\partial_2}+\frac{\partial^2\varphi_1}{\partial x_2^2}-\frac{\partial^2\varphi}{\partial x_1^2}\right) \tag{20}$$

$$\tau_{22}'=\lambda_1\nabla^2\phi_1+\alpha_1\left(\frac{\partial^2\phi_1}{\partial x_1^2}+\frac{\partial^2\varphi_1}{\partial x_1\partial x_2}\right)+2\mu_{T1}\left(\frac{\partial^2\phi_1}{\partial x_2^2}+\frac{\partial^2\varphi_1}{\partial x_1\partial x_2}\right) \tag{21}$$

$$\tau_{23}'=\mu_{T1}\frac{\partial u_3'}{\partial x_2} \tag{22}$$

4. Solution of the Problem

We seek harmonic solutions for (11), (13) and (14) in the form (see Bullen [29])

$$(\phi,\varphi,u_3)=\left\{\bar{\phi}(x_2),\bar{\varphi}(x_2),u_3(x_2)\right\}\mathrm{e}^{iw(x_1-ct)} \tag{23}$$

where is a complex frequency. In M and similar relations in M_1 with the factions ϕ, φ, u_3 being replaced by ϕ_1, φ_1, u_3'. This leads us to a particular solution corresponding to group of wavelength $\frac{2\pi}{\omega}$ traveling forward with speed C. It is convenient to introduce h, r, s where

$$\left.\begin{aligned} h &= \left[\frac{\rho^2 c^2 - \mu_L}{\mu_T}\right]^{\frac{1}{2}}; \\ r &= \left[\frac{\rho c^2 - \left(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta + \mu_e H_0^2 + \Omega^2\right)}{\left(\lambda + \alpha + 2\mu_L + \mu_e H_0^2\right)}\right]^{\frac{1}{2}}; \\ s &= \left[\frac{\rho c^2 - \left(\alpha + 3\mu_L - 2\mu_T + \beta - \Omega^2\right)}{\mu_L}\right]^{\frac{1}{2}}. \end{aligned}\right\} \tag{24}$$

And similar expressions h_1, r_1 and s_1 for the medium M_1. The positive value of the square root being taken in each case.

Now substituting from (23) into (11), (13) and (14), we obtain for the medium M

$$\left.\begin{aligned} \frac{\mathrm{d}^2 \bar{u}_3(x_2)}{\mathrm{d}x_2^2} &= -\omega^2 h^2 \bar{u}_3(x_2); \\ \frac{\mathrm{d}^2 \bar{\phi}(x_2)}{dx_2^2} &= -\omega^2 r^2 \bar{\phi}(x_2); \\ \frac{\mathrm{d}^2 \bar{\varphi}(x_2)}{\mathrm{d}x_2^2} &= -\omega^2 s^2 \bar{\varphi}(x_2). \end{aligned}\right\} \tag{25}$$

Equation (25) has solutions:

$$\left.\begin{aligned} u_3 &= C \exp\left(i\omega\left(-hx_2 + x_1 - ct\right)\right); \\ \phi &= A \exp\left(i\omega\left(-rx_2 + x_1 - ct\right)\right); \\ \varphi &= B \exp\left(i\omega\left(-sx_2 + x_1 - ct\right)\right); \end{aligned}\right\} \tag{26}$$

And for the medium M_1

$$\left.\begin{aligned} u_3' &= C_1 \exp\left(i\omega\left(h_1 x_2 + x_1 - ct\right)\right) \\ \phi' &= A_1 \exp\left(i\omega\left(r_1 x_2 + x_1 - ct\right)\right) \\ \varphi' &= B_1 \exp\left(i\omega\left(s x_2 + x_1 - ct\right)\right) \end{aligned}\right\} \tag{27}$$

In the above, for the effect to be essentially a surface one ,each expression must diminish indefinitely with increasing distance from the boundary this with be the case if each expression contains an exponential factor in with the exponent is teal and negative. Hence, h, r, s and similarly h_1, r_1, s_1 are taken to be imaginary. From (12), we have

$$u_1 = A(i\omega)\mathrm{e}^{i\omega(-rx_2 + x_1 - ct)} + B(-i\omega s)\mathrm{e}^{i\omega(-sx_2 + x_1 - ct)} \tag{28}$$

$$u_2 = A(-i\omega r)\mathrm{e}^{i\omega(-rx_2 + x_1 - ct)} - B(i\omega)\mathrm{e}^{i\omega(-sx_2 + x_1 - ct)} \tag{29}$$

$$u_1' = A_1(i\omega)\mathrm{e}^{i\omega(r_1 x_2 + x_1 - ct)} + i\omega s_1 B_1 \mathrm{e}^{i\omega(s_1 x_2 + x_1 - ct)} \tag{30}$$

$$u_2' = A_1(+i\omega r_1)\mathrm{e}^{i\omega(r_1 x_2 + x_1 - ct)} - B(i\omega)\mathrm{e}^{i\omega(s_1 x_2 + x_1 - ct)} \tag{31}$$

$$\begin{aligned} \tau_{11} = &\left[\left(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta\right)\omega^2 r + \left(\lambda + \alpha\right)\omega r^2\right] A\mathrm{e}^{i\omega(rx_2 + x_1 - ct)} \\ &+ \left[\left(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta\right)\omega^2 + i\omega s\left(\lambda + \alpha\right)\right] B\mathrm{e}^{i\omega(-sx_2 + x_1 - ct)}, \end{aligned} \tag{32}$$

$$\begin{aligned} \tau_{22} = &\left[\left(\lambda + \alpha\right)\omega^2 r + \left(\lambda + 2\mu_T\right)\omega r^2\right] A\mathrm{e}^{i\omega(rx_2 + x_1 - ct)} \\ &+ \left[\left(\lambda + \alpha\right)\omega^2 + i\omega s\left(\lambda + 2\mu_T\right)\right] B\mathrm{e}^{i\omega(-sx_2 + x_1 - ct)} \end{aligned} \tag{33}$$

$$\tau_{33} = \left[(\lambda+\alpha)\omega^2 r + \lambda\omega r^2\right] A\mathrm{e}^{iw(rx_2+x_1-ct)} + \left[(\lambda+\alpha)\omega^2 + i\omega s\lambda\right] B\mathrm{e}^{iw(-sx_2+x_1-ct)} \tag{34}$$

$$\tau_{12} = \mu_L\left[\left(\omega^2+i\omega s\right) A\mathrm{e}^{i\omega(rx_2+x_1-ct)} + \left(\omega^2+i\omega s\right) B\mathrm{e}^{i\omega(-sx_2+x_1-ct)}\right] \tag{35}$$

Similar relations in M_1 with $\mu_L,\ \lambda,\ \alpha,\ \mu_T$ are replaced by $\mu_{L1},\ \lambda_1,\ \alpha_1,\ \mu_{T_1}$.

By using the boundary conditions *a* and *b*, we can determined the constants A, B, A_1 and B_1.

We can study the components of displacement and stresses in fibre-reinforced anisotropic elastic media under effect of rotation and initial magnetic field from Equations (28)-(35) by using Maple program, is clear up from **Figures 2-9**.

5. Particular Case: Isotropic Generalized Elastic Medium with Rotation and Initial Magnetic Field

In this case, substituting $\mu_L = \mu_T = \mu$ and $\beta = 0$ in Equations (28)-(35), we obtain the corresponding expressions of displacement and stress in isotropic generalized elastic medium with rotation and initial magnetic field, is clear up from **Figures 10-17**.

6. Numerical Results and Discussions

To study the surface waves in fibre-reinforced we use the following physical constants for anisotropic elastic media under the in influence of rotation and initial magnetic field, are considered [18] [19], for mediums M and M_1 respectively.

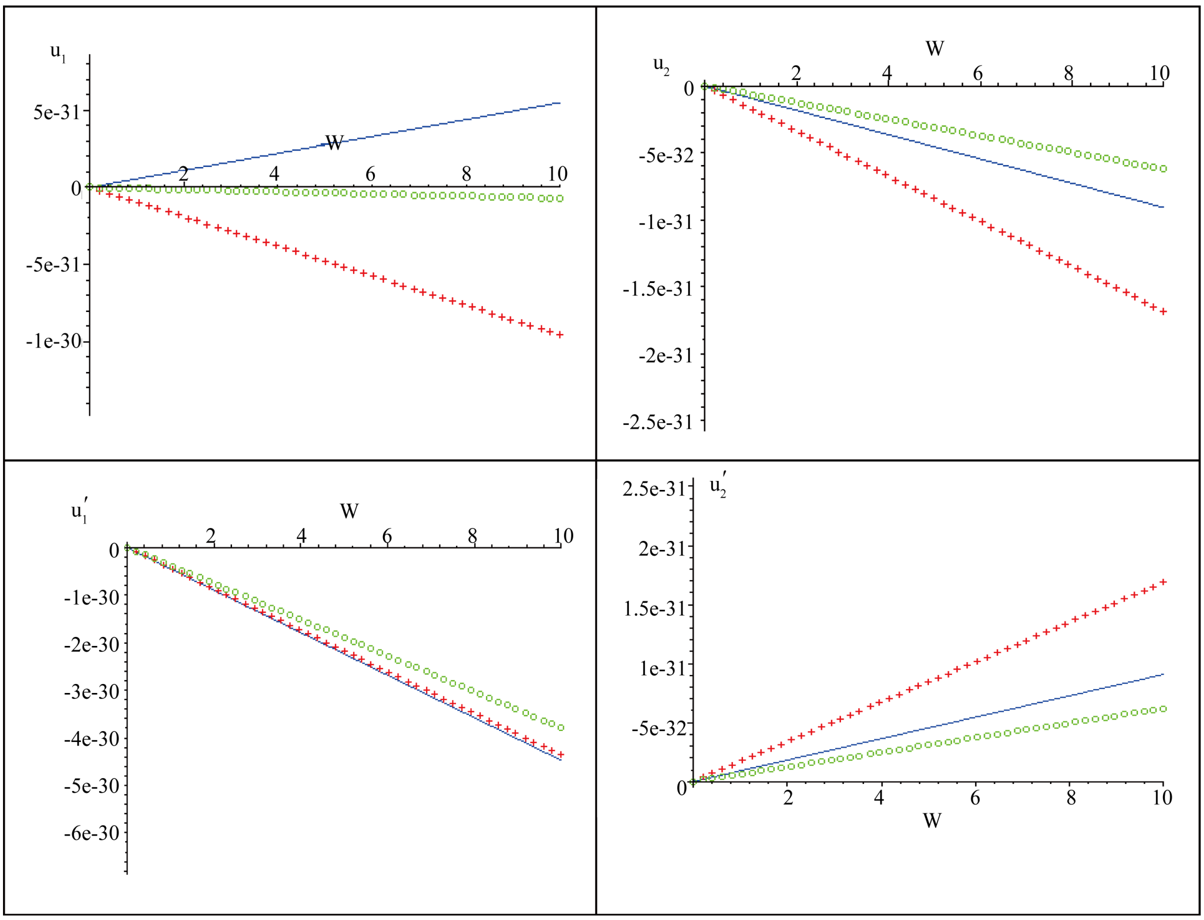

Figure 2. Effects of rotation Ω on displacements with change values of complex frequency Ω, Ω = 0.1, Ω = 0.5, Ω = 0.9.

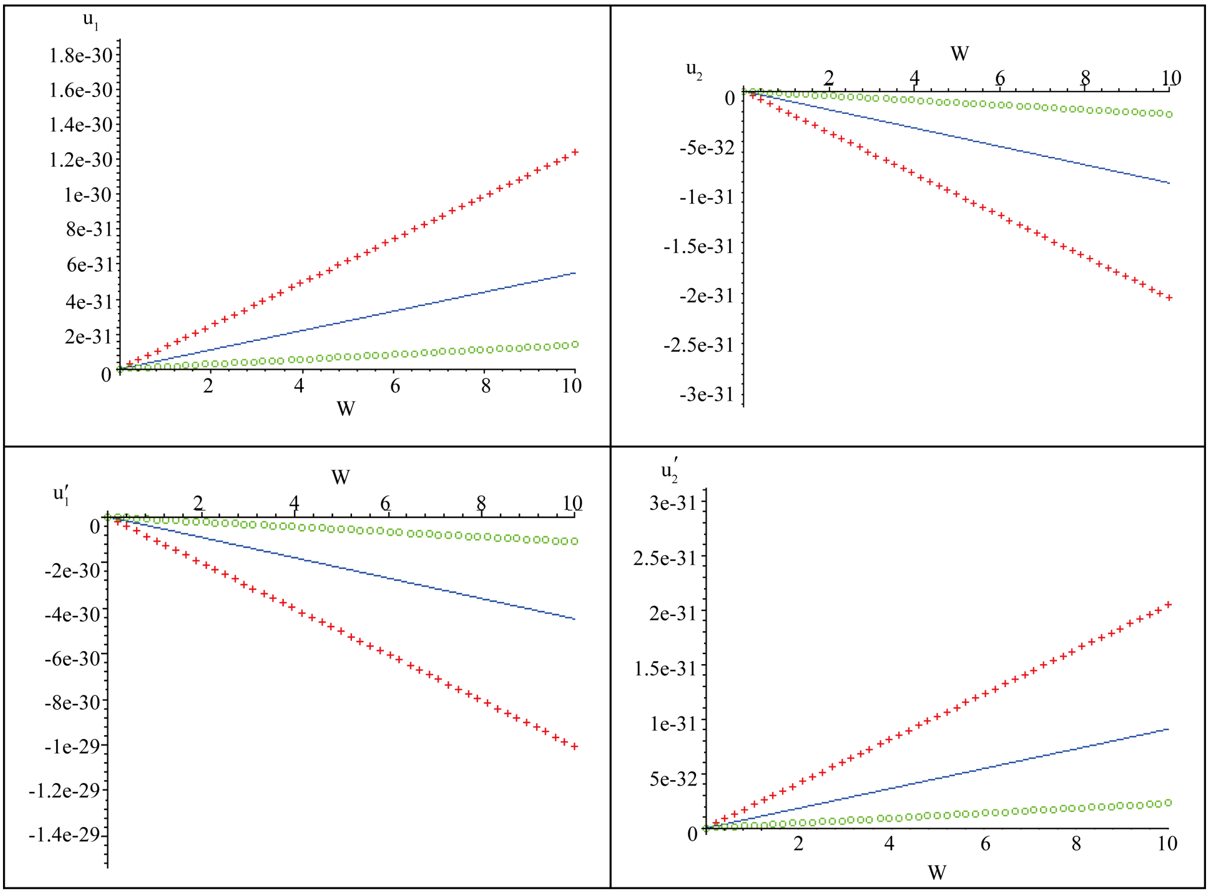

Figure 3. Effects of initial magnetic field H on displacements with change values of complex frequency Ω, $H = 0.1$, $H = 0.4$, $H = 0.9$.

$$\lambda = 7.59\times10^{9}\ \mathrm{N/m^2},\quad \mu_T = 1.89\times10^{9}\ \mathrm{N/m^2},\quad \mu_L = 2.45\times10^{9}\ \mathrm{N/m^2},$$
$$\alpha = -1.28\times10^{9}\ \mathrm{N/m^2},\quad \beta = 0.32\times10^{9}\ \mathrm{N/m^2},\quad \rho = 7800\ \mathrm{Kg/m^2},$$
$$\lambda_1 = 5.65\times10^{10}\ \mathrm{N/m^2},\quad \mu_{T1} = 2.46\times10^{10}\ \mathrm{N/m^2},\quad \mu_{L1} = 5.66\times10^{10}\ \mathrm{N/m^2},$$
$$\alpha_1 = -1.28\times10^{10}\ \mathrm{N/m^2},\quad \beta_1 = 220.90\times10^{10}\ \mathrm{N/m^2},\quad \rho = 7800\ \mathrm{Kg/m^2},$$

The numerical technique outlined above was used to obtain of the displacement, stresses in fibre-reinforced anisotropic and isotropic elastic media under effect of rotation and initial magnetic field. These distributions are shown in Figures 2-17. For the sake of brevity some computational results are being presented here.

6.1. Effect of Rotation and Initial Magnetic Field in Fibre-Reinforced Anisotropic Elastic Media

Figure 2 shows that the components of displacement in fibre-reinforced anisotropic elastic media under effect of rotation, we find that in medium M, the components of displacement u_1 and u_2 are decreasing with increasing values of the rotation Ω, put in medium M_1, u_1' decreasing and u_2' increasing with increasing values of Ω respectively.

Figure 3 shows that the components of displacement in fibre-reinforced anisotropic elastic media under effect of initial magnetic field, we find that in medium M, the components of displacement u_1 increasing and u_2 decreasing with increasing values of initial magnetic field H respectively, put u_1' decreasing and u_2' increasing with increasing values of H.

Figure 4 shows that the components of stresses in fibre-reinforced anisotropic elastic media under effect of

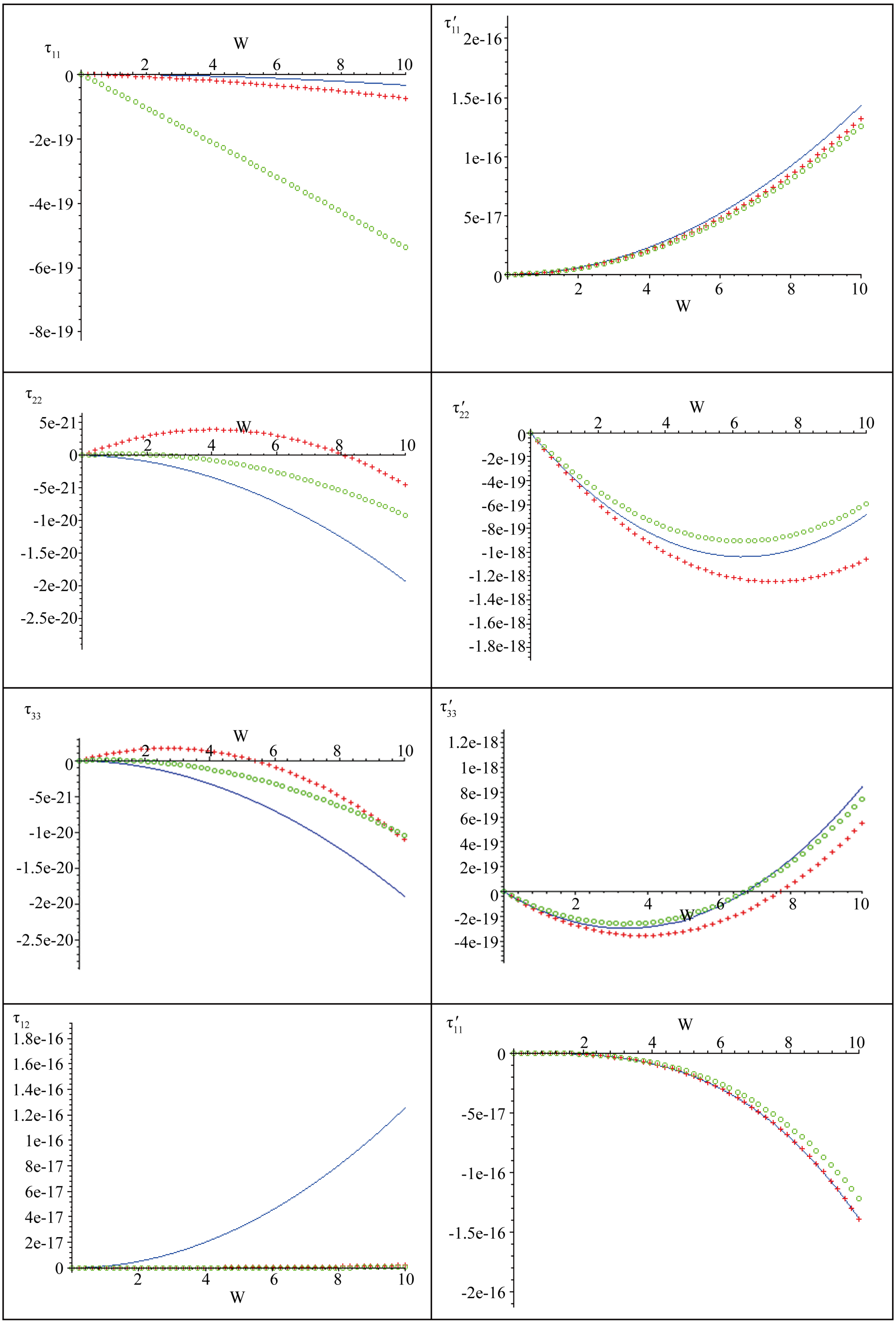

Figure 4. Effects of rotation Ω on stresses with change values of complex frequency Ω, Ω = 0.1, Ω = 0.5, Ω = 0.9.

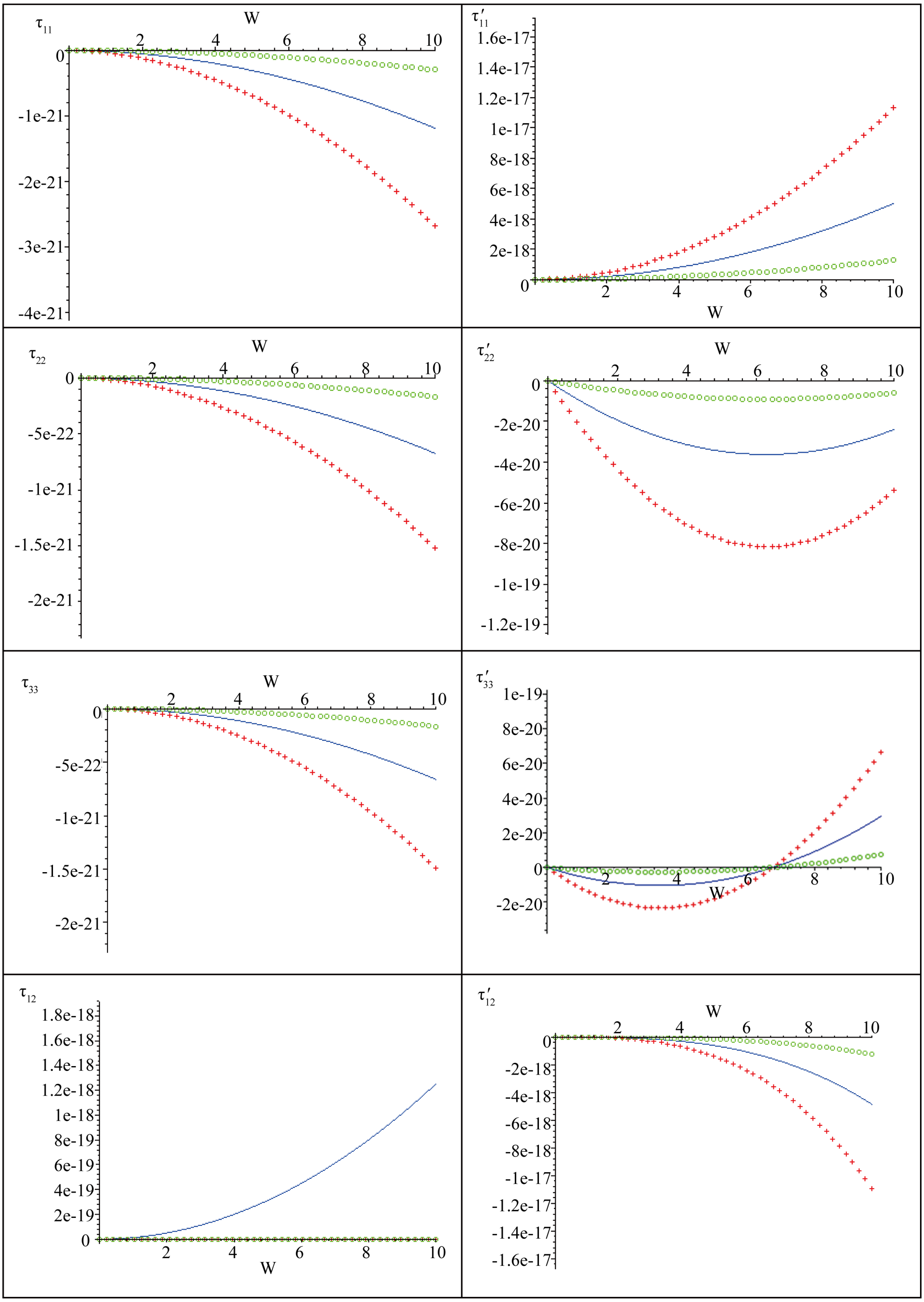

Figure 5. Effects of initial magnetic field H on stresses with change values of complex frequency Ω, $H = 0.1$, $H = 0.4$, $H = 0.9$.

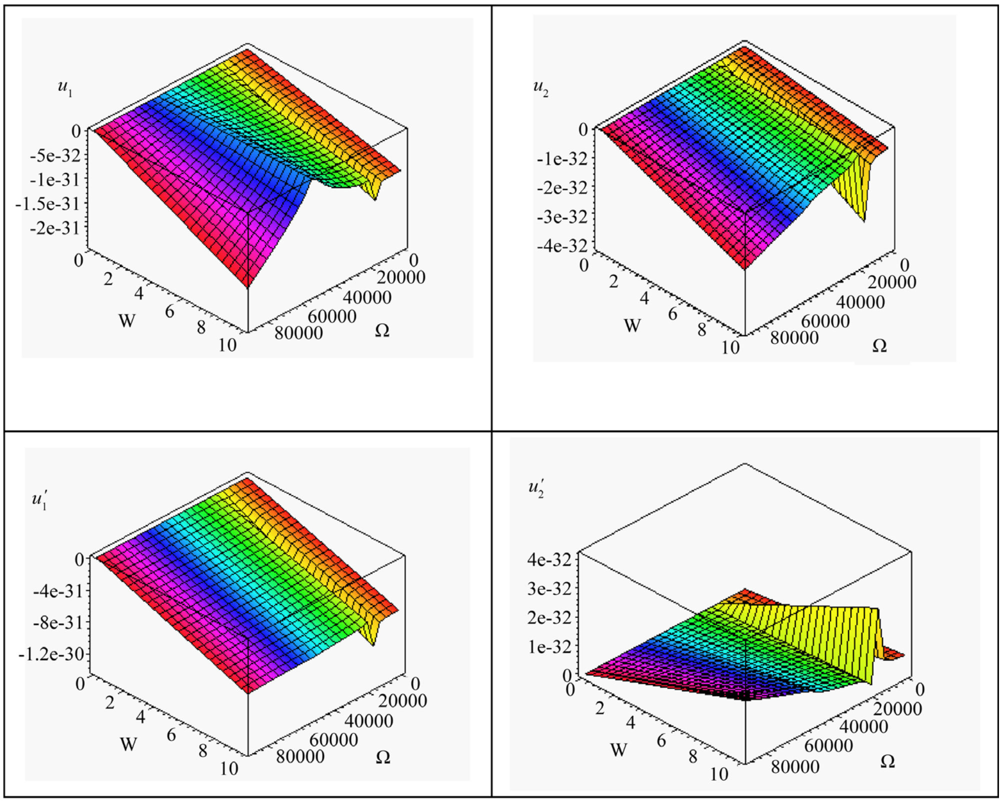

Figure 6. Displacements distribution with change values of rotation and complex frequency Ω.

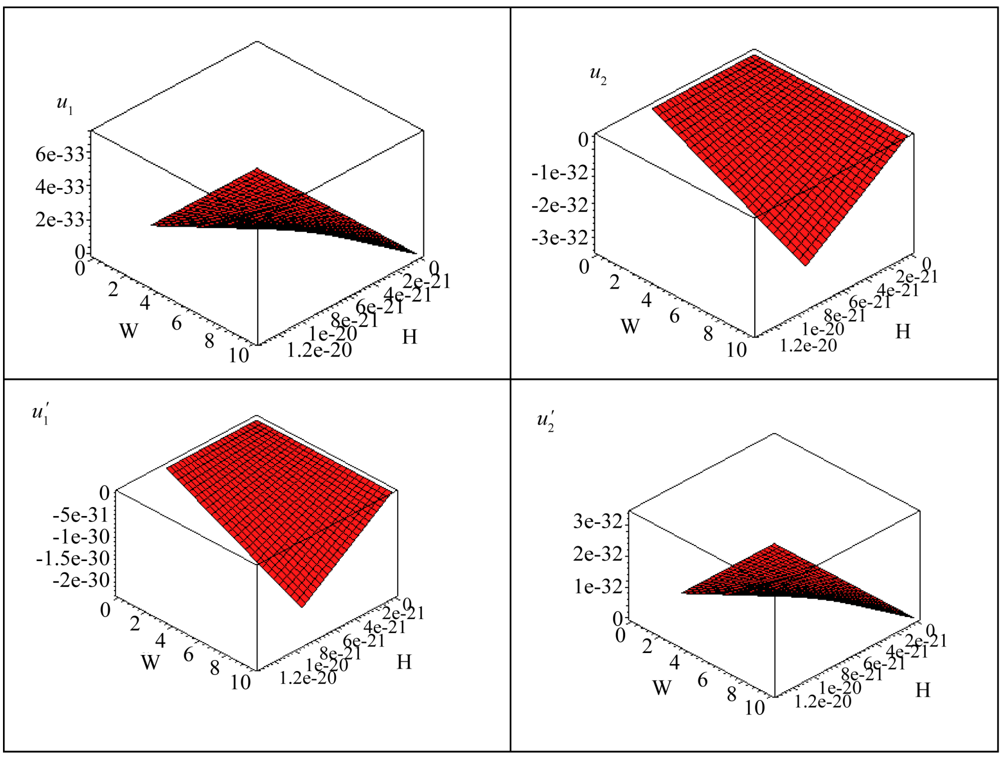

Figure 7. Displacements distribution with change values of initial magnetic field and complex frequency Ω.

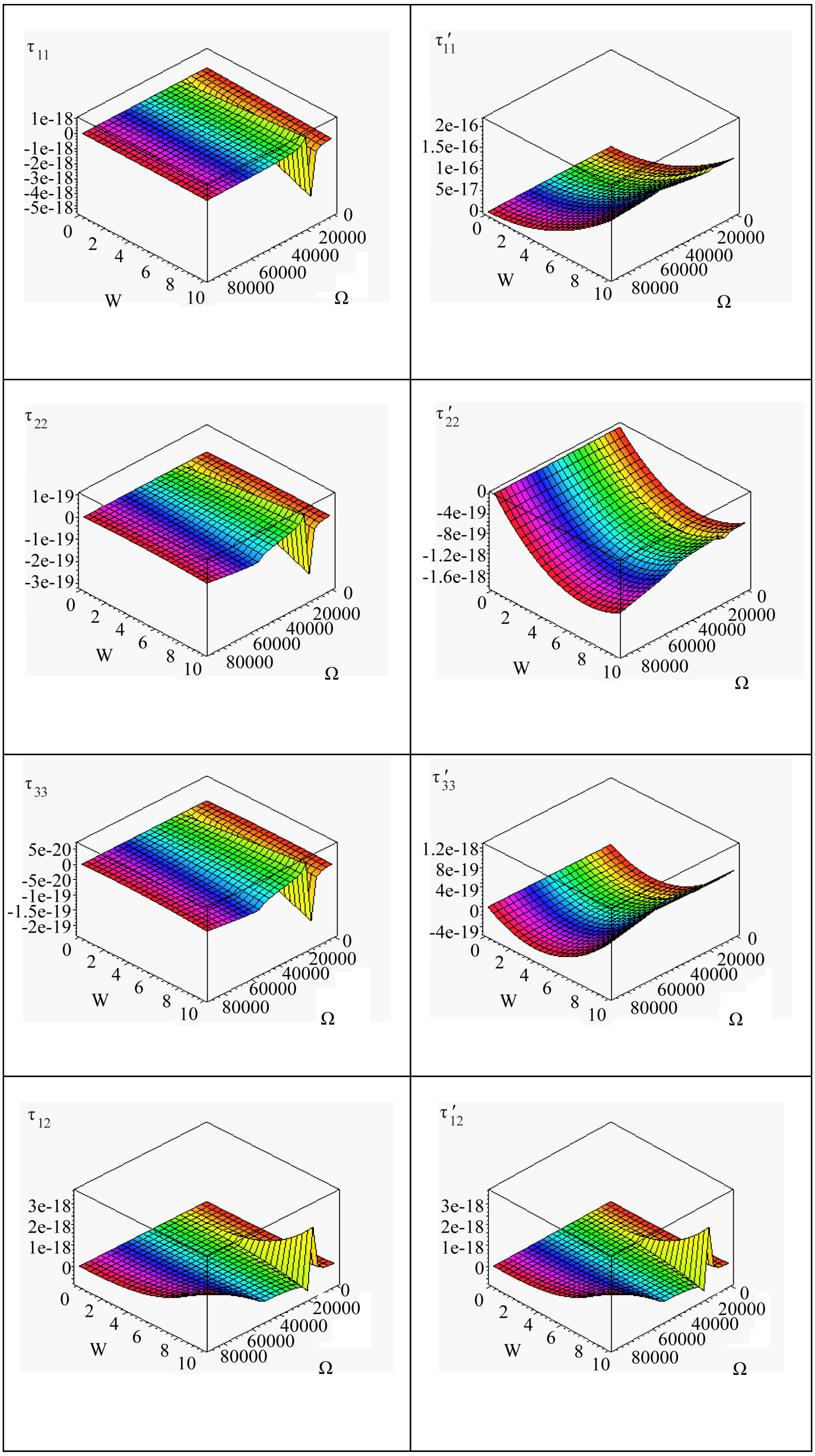

Figure 8. Stresses distribution with change values of rotation and complex frequency Ω.

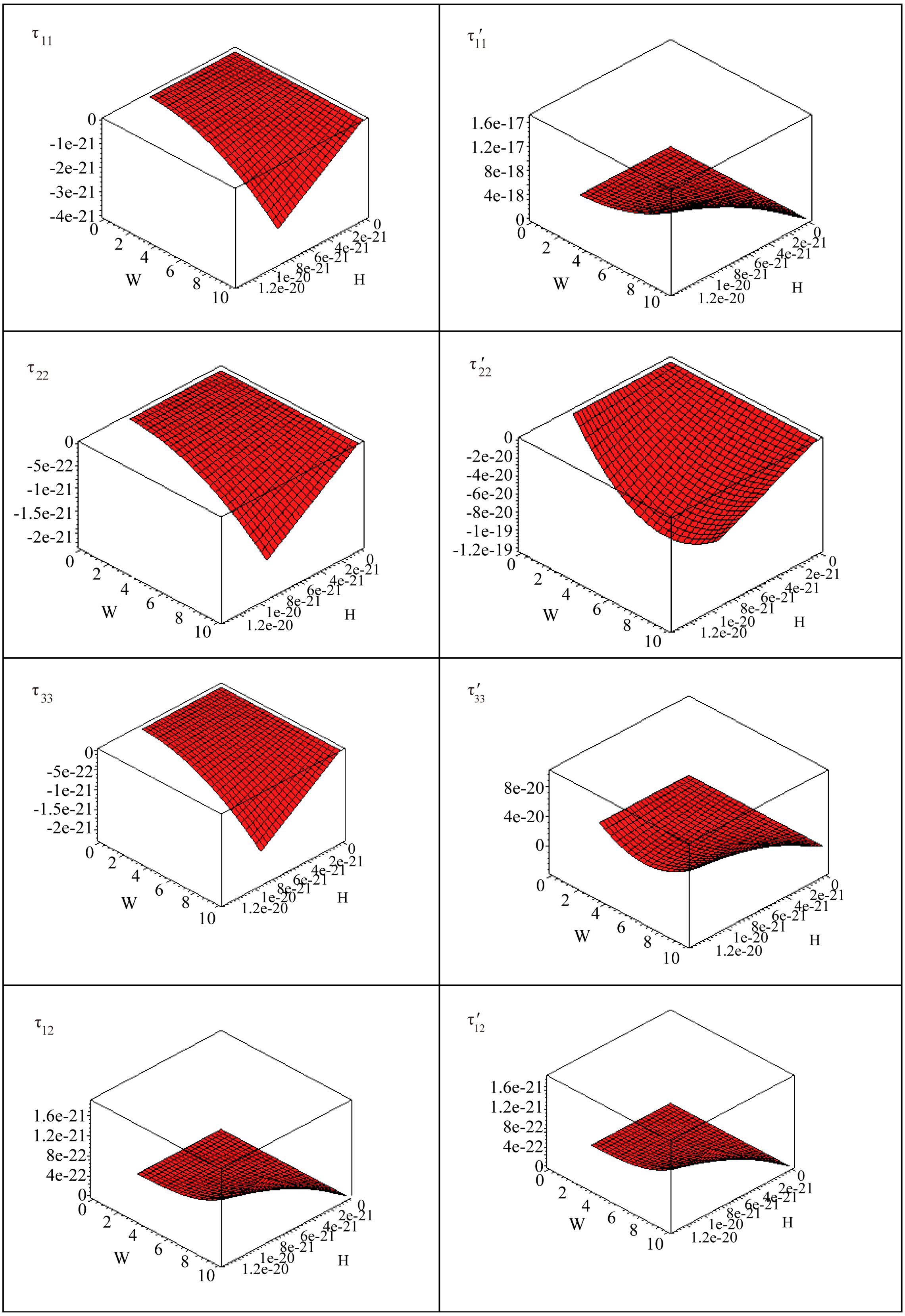

Figure 9. Stresses distribution with change values of initial magnetic field and complex frequency Ω.

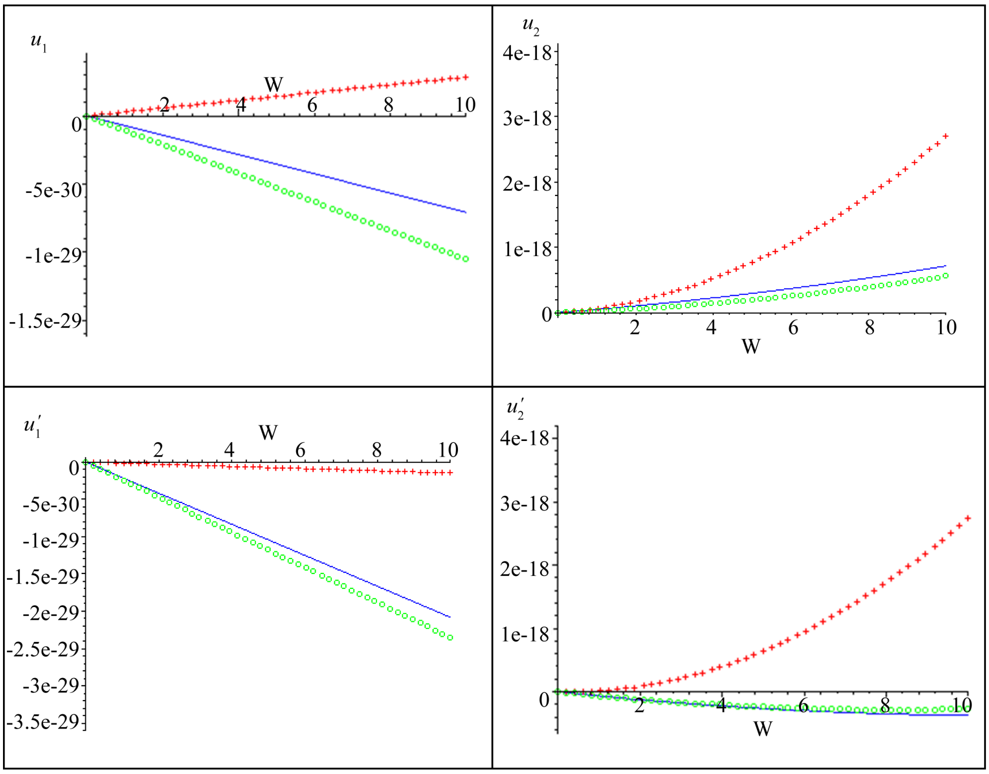

Figure 10. Effects of rotation Ω on displacements with change values of complex frequency Ω, Ω = 0.1, Ω = 0.5, Ω = 0.9.

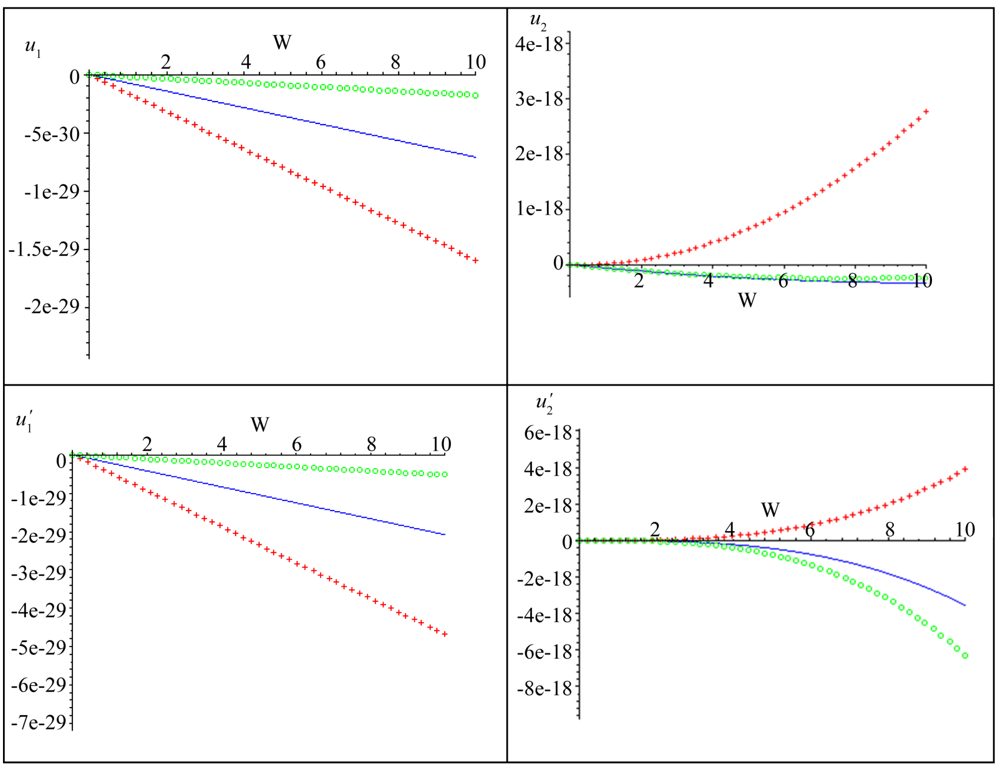

Figure 11. Effects of initial magnetic field H on displacements with change values of complex frequency Ω, $H = 0.1$, $H = 0.4$, $H = 0.9$.

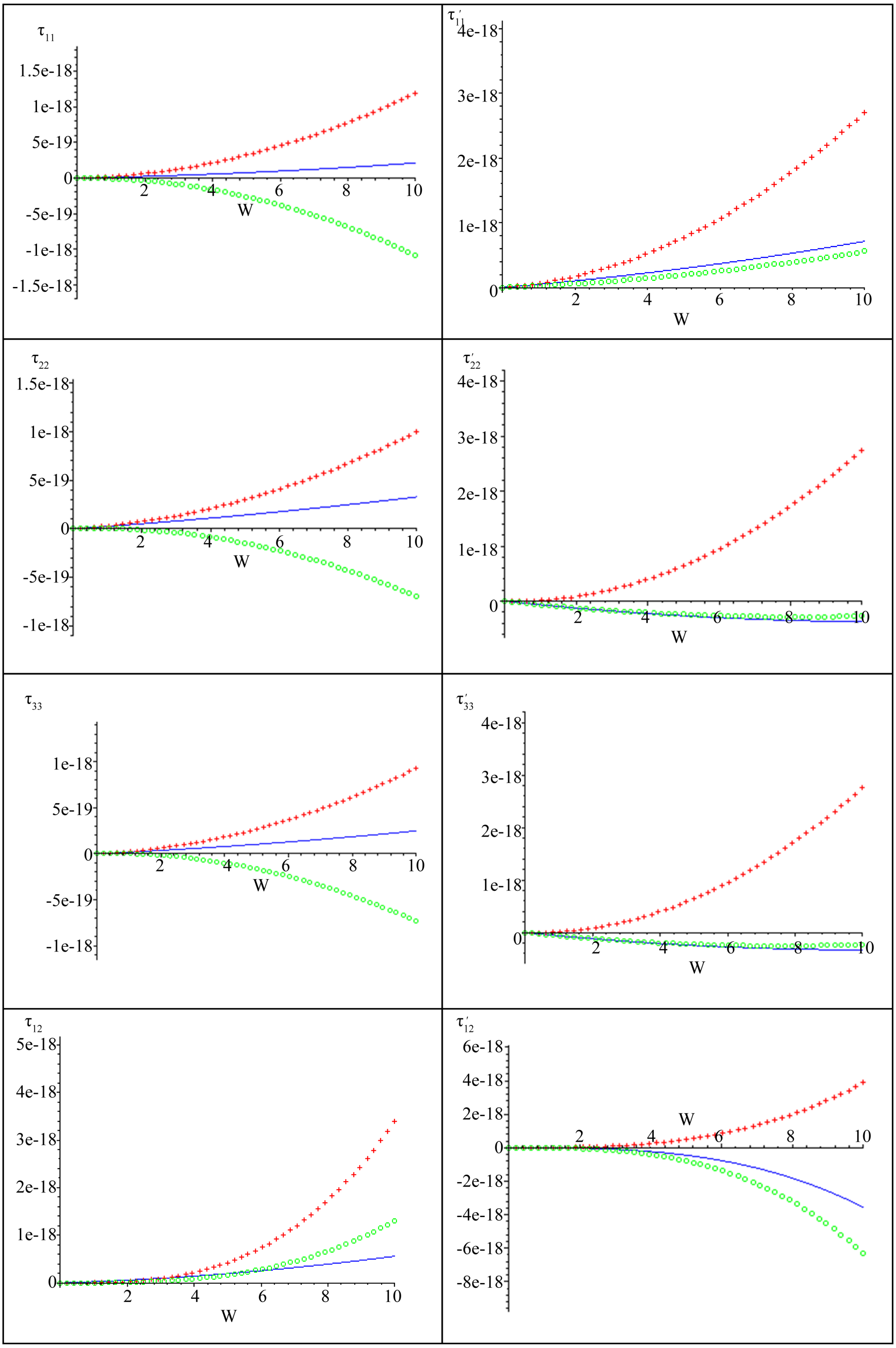

Figure 12. Effects of rotation Ω on stresses with change values of complex frequency Ω, Ω = 0.1, Ω = 0.5, Ω = 0.9.

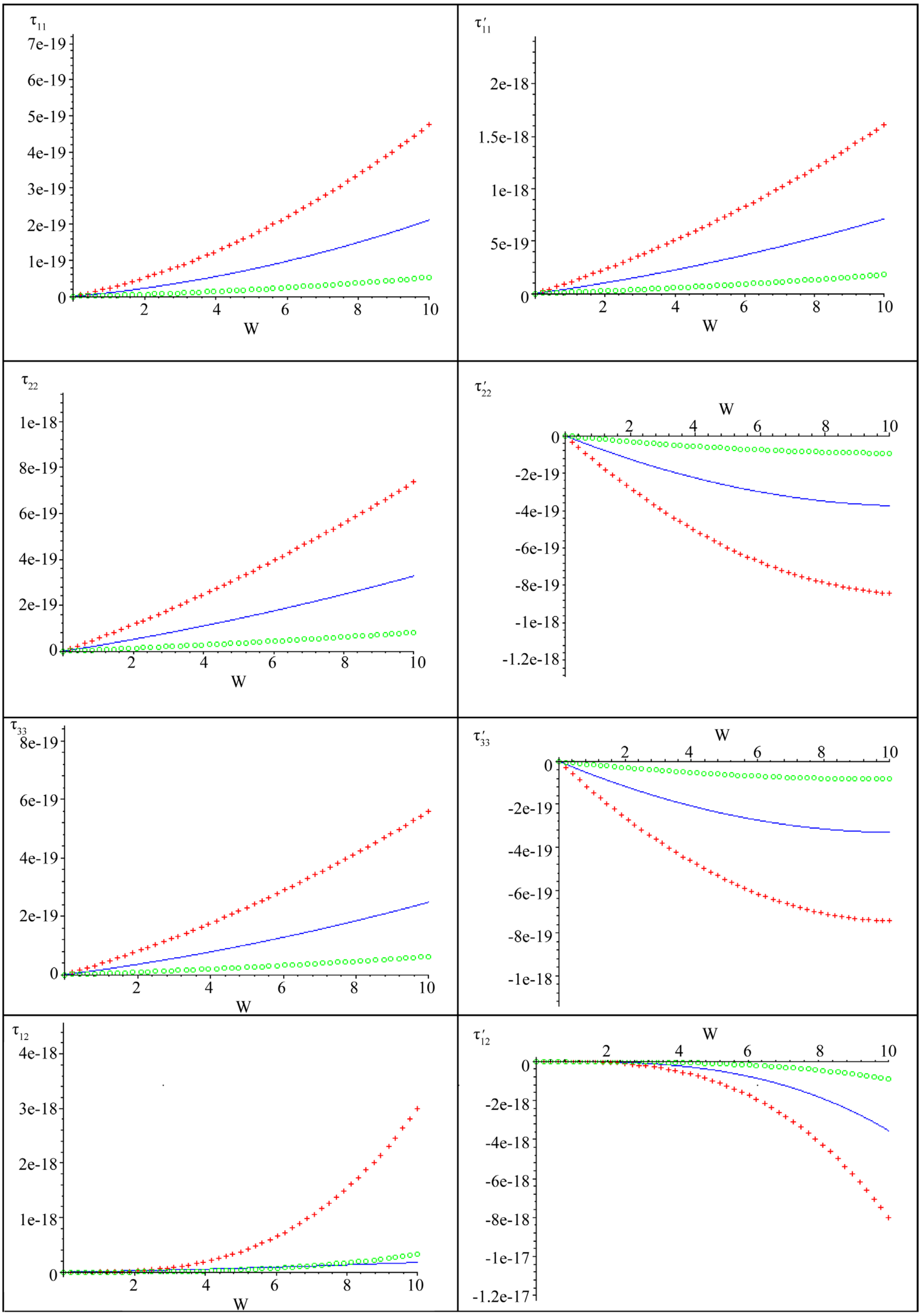

Figure 13. Effects of initial magnetic field H on stresses with change values of complex frequency Ω, $H = 0.1$, $H = 0.4$, $H = 0.9$.

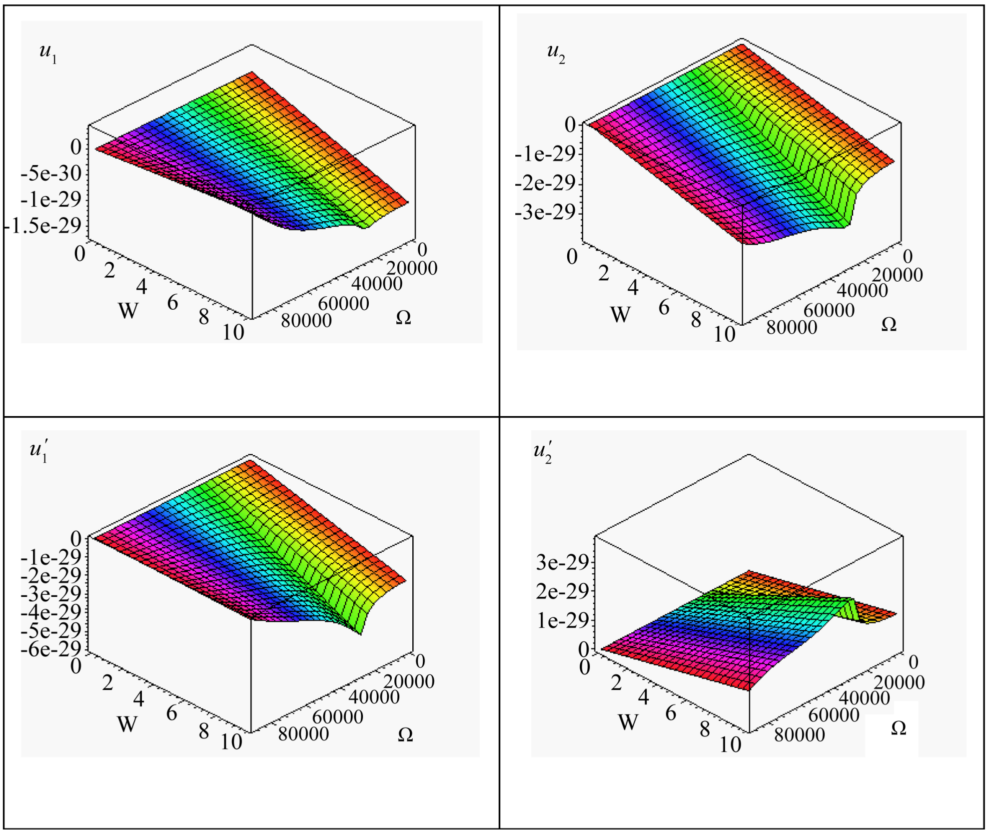

Figure 14. Displacements distribution with change values of rotation and complex frequency Ω.

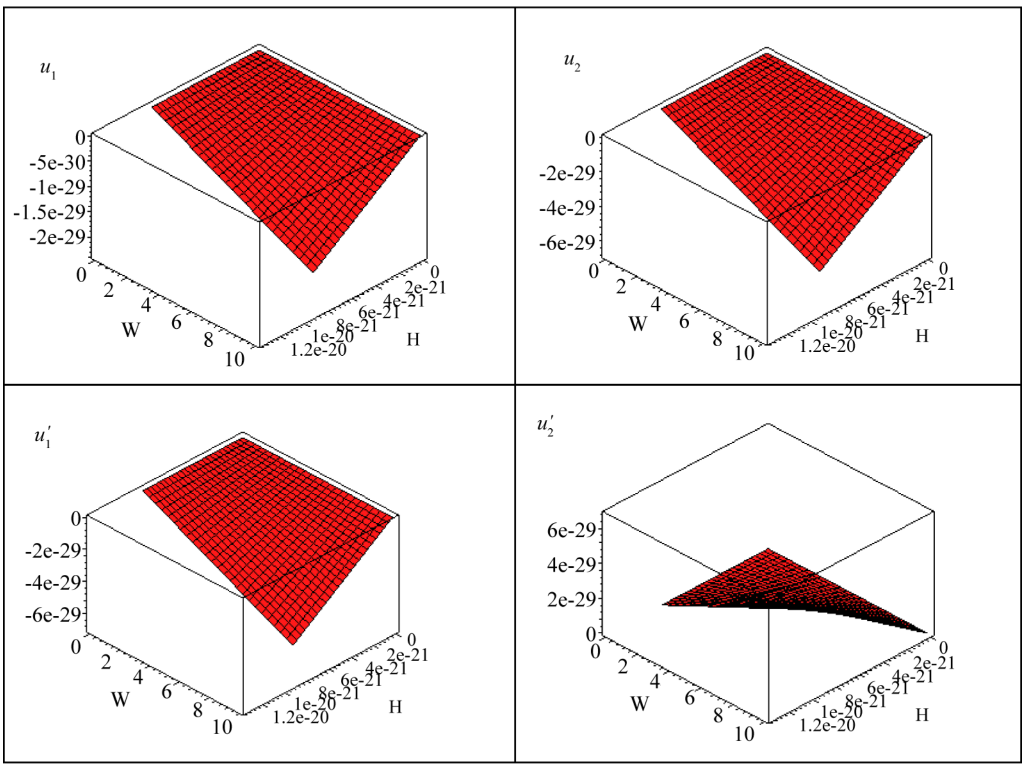

Figure 15. Displacements distribution with change values of initial magnetic field and complex frequency Ω.

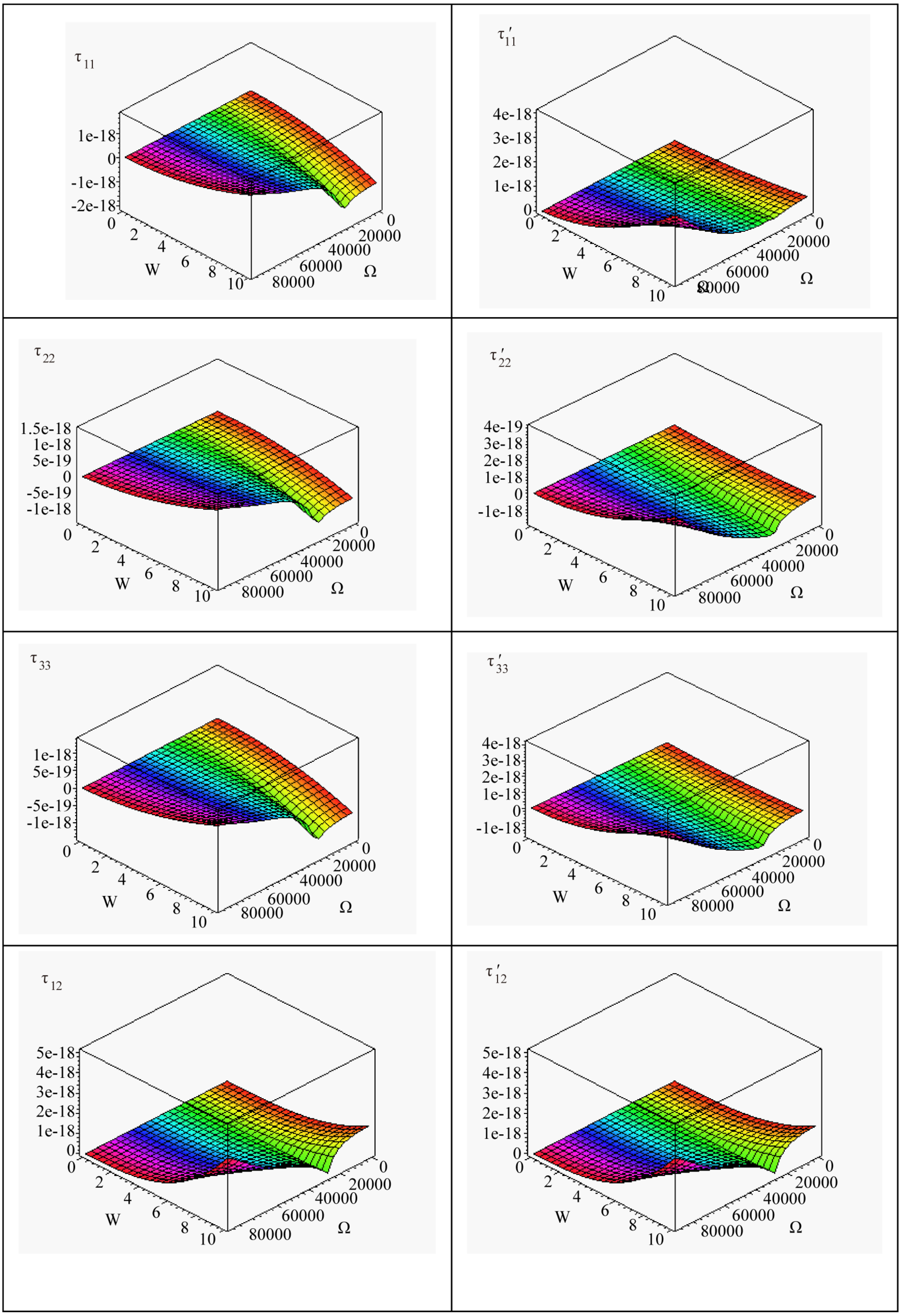

Figure 16. Stresses distribution with change values of rotation and complex frequency Ω.

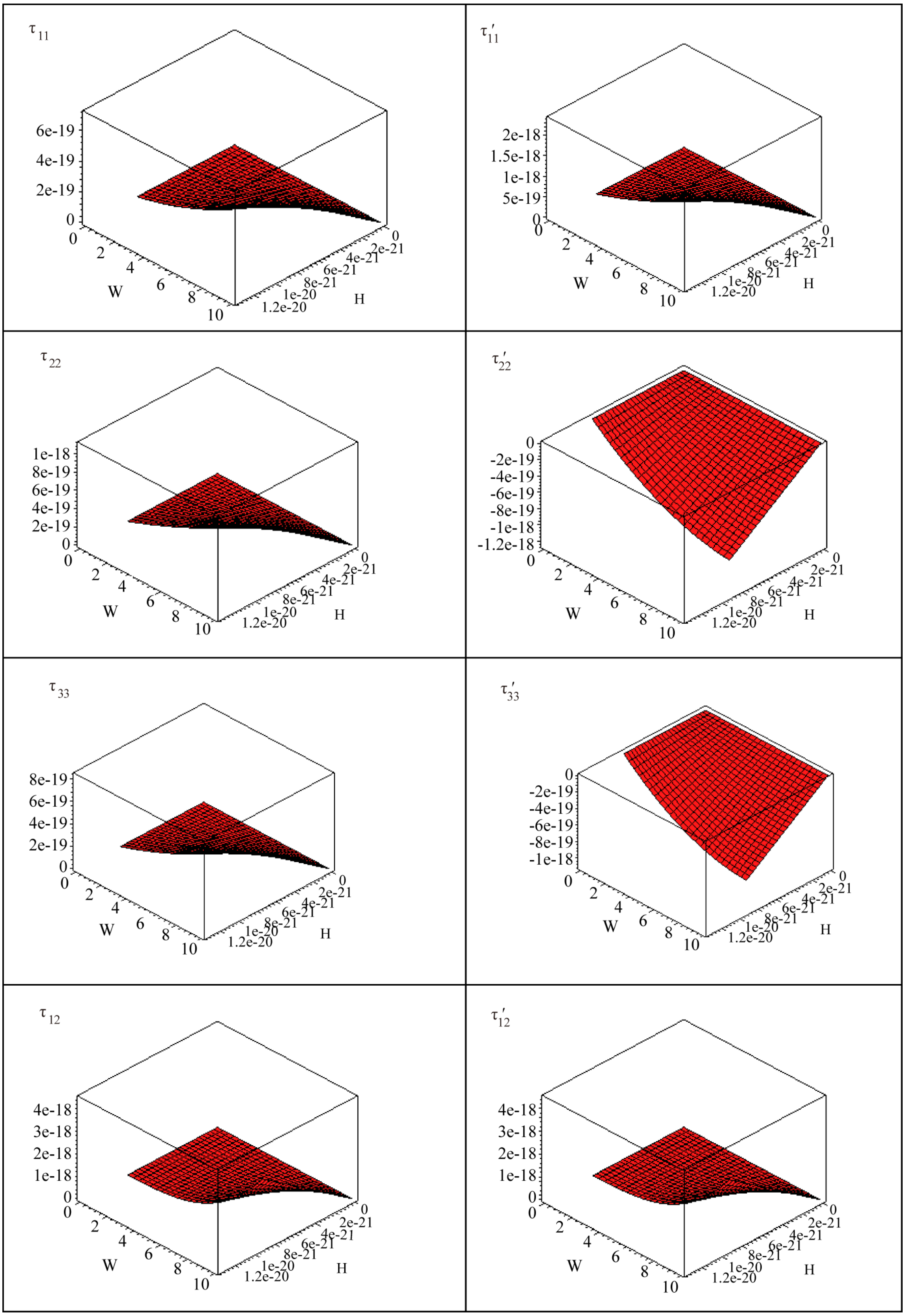

Figure 17. Stresses distribution with change values of initial magnetic field and complex frequency Ω.

rotation, we find in medium M the components of stresses τ_{11}, τ_{22} and τ_{33} are increasing with increasing values of the rotation Ω, put τ_{12} decreasing with increasing values of Ω. While, in medium M_1 the components of stresses τ'_{11}, τ'_{22}, τ'_{33} and τ'_{12} are decreasing with increasing values of Ω.

Figure 5 shows that the components of stresses in fibre-reinforced anisotropic elastic media under effect of initial magnetic field, we find in medium M the components of stresses τ_{11}, τ_{22}, τ_{33} and τ_{12} are decreasing with increasing values of initial magnetic field H. While, in medium M_1 the components of stresses τ'_{11} and τ'_{12} are decreasing with increasing values of initial magnetic field H, put τ'_{22} and τ'_{33} are increasing with increasing values of values of H.

Figure 6 shows that the displacements distribution with change values of rotation and complex frequency ω in fibre-reinforced anisotropic elastic media, we find in medium M the components of displacement u_1 and u_2 are decreasing with increasing values of the rotation Ω, put in medium M_1, u'_1 decreasing and u'_2 increasing with increasing values of Ω respectively.

Figure 7 shows that the displacements distribution with change values of initial magnetic field H and complex frequency ω in fibre-reinforced anisotropic elastic media, we find we find that that in medium M, the components of displacement u_1 increasing and u_2 decreasing with increasing values of initial magnetic field H, put in medium M_1, u'_1 decreasing and u'_2 increasing with increasing values of H, respectively.

Figure 8 shows that Stresses distribution with change values of rotation and complex frequency Ω in fibre-reinforced anisotropic elastic media under effect of rotation, we find for the medium M the components of stresses τ_{11}, τ_{22}, τ_{33} and τ_{12} are increasing with increasing values of the rotation Ω. While, for the medium M_1 the components of stresses τ'_{11} and τ'_{12} are increasing with increasing values of Ω, put the components of stresses τ'_{22} and τ'_{33} are decreasing with increasing values of Ω.

Figure 9 shows that Stresses distribution with change values of initial magnetic field and complex frequency Ω in fibre-reinforced anisotropic elastic media under effect of rotation, we find in the medium M the components of stresses τ_{11}, τ_{22} and τ_{33} are increasing with increasing values of the initial magnetic field H, put τ_{12} decreasing with increasing values of H. While, in the medium M_1 the components of stresses τ'_{11}, τ_{33} and τ'_{12} are increasing with increasing values of Ω, put τ'_{12} decreasing with increasing values of H.

6.2. Effect of Rotation and Initial Magnetic Field in Fibre-Reinforced Isotropic Elastic Media

Figure 10 shows that the components of displacement in fibre-reinforced isotropic elastic media under effect of rotation, we find in tow medium M and M_1, all components of displacement are increasing with increasing values of the rotation Ω.

Figure 11 shows that the components of displacement in fibre-reinforced anisotropic elastic media under effect of initial magnetic field, we find that in the medium M the components of displacement u_1 and u_2 are decreasing and increasing with increasing values of initial magnetic field H, put u'_1 and u'_2 are decreasing and increasing with increasing values of H, respectively.

Figure 12 shows that the components of stresses in fibre-reinforced isotropic elastic media under effect of rotation, we find in tow medium M and M_1 all components of stresses are increasing with increasing values of the rotation Ω.

Figure 13 shows that the components of stresses in fibre-reinforced isotropic elastic media under effect of initial magnetic field, we find for the medium M the components of stresses τ_{11}, τ_{22}, τ_{33} and τ_{12} are increasing with increasing values of initial magnetic field H. While, for the medium M_1 the components of stresses τ'_{11} increasing with increasing values of initial magnetic field H, put τ'_{22}, τ_{33} ans τ'_{12} are decreasing with increasing values of values of H.

Figure 14 shows that the displacements distribution with change values of rotation and complex frequency ω in fibre-reinforced isotropic elastic media, we find in tow medium M and M_1, all components of displacement are increasing with increasing values of Ω.

Figure 15 shows that the displacements distribution with change values of initial magnetic field H and complex frequency ω in fibre-reinforced isotropic elastic media, we find the components of displacement u_1, u_2 and u'_1 are decreasing with increasing values of the initial magnetic field H, put u'_2 increasing with increasing values of H.

Figure 16 shows that stresses distribution with change values of rotation and complex frequency ω in fibre-reinforced isotropic elastic media, we find in tow medium M and M_1, all components of stresses are increasing with increasing values of the rotation Ω.

Figure 17 shows that stresses distribution with change values of initial magnetic field H and complex frequency ω in fibre-reinforced isotropic elastic media, we find in medium M, all components of stresses are increasing with increasing values of H, put in medium M_1, τ'_{11} and τ'_{12} are increasing with increasing values of H and τ'_{22} and τ'_{33} are decreasing with increasing values of H.

7. Conclusions

In the light of the above analysis, the following conclusions may be made:

− Effects of rotation and initial magnetic field are cleared on the components of displacement and stresses;
− Effect of complex frequency is cleared on the components of displacement and stresses;
− There is a clear difference in the two cases, anisotropic and isotropic elastic media;
− Deformation of a body depends on the nature of the forces applied as well as the type of boundary conditions.

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