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And If Bell's Inequality Were Not Violated

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Abstract

It briefly recalls the theory of Bell's inequality and some experimental measures. Then measurements are processed on one hand according to a property of the wave function, on the other hand according to the sum definition. The results of such processed measures are apparently not the same, so Bell's inequality would not be violated. It is a use of the wave function which implies the violation of the inequality, as it can be seen on the last flowcharts.

Keywords

Bell's Theorem, Bell's Inequality Violation, Entangled Photons, Quantum Sum, Experimental Measures, Wave Function

1. Introduction

Some quantum phenomena can cause astonishment. The violation of Bell's inequality is a part of the surprises raised by Quantum Mechanics [1] [2]. This violation means that it would be necessary to give up at least one of the following three assumptions: the principle of locality (two photons can not influence each other at a distance greater than the speed of light), the assumption of causality (to each effect at least one cause) and the assumption of realism (any particle has its own property). After recalling what Bell's inequality and experimental measures are, we will discuss how this inequality can be violated or not.

This paper is a criticism of Pr Alain Aspect's demonstration; it is neither a criticism of Bell's inequality theory, nor of its consequences here before.

2. Experimental Measures of Bell's Inequality

2.1. The Experimental Set-Up

Hereinafter proposed by Professor John Stewart BELL [3] the experimental set-up:

The source is a stream of 2 entangled photons:

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-the two photons leave in two opposite directions.

-"entangled" means that one photon is polarized along θ and the other photon along $(\theta+\pi)$, it means the entangled photons have the same polarity $(\pm\pi)$.

A and B are separators with switching function depending on the photon polarity (that is to say in the direction of the electric field associated with the photon).

DA, $DA^{\perp}$ DB and $DB^{\perp}$ are four independent photon detectors, or counters.

Experiments done in Paris, Innsbruck and Genève [4] with two-channel polarizers look like **Figure 1**.

2.2. Measures of Entangled Photons

-if the polarity of the photon is rather parallel to α, the photon is leaving to DA detector,

-otherwise, if the polarity of the photon is rather perpendicular to α, the photon is leaving to $DA^{\perp}$ detector

-if the polarity of the other photon is rather parallel to β, the photon is leaving to DB

-otherwise, if the polarity of the other photon is rather perpendicular to β, the photon is leaving to $DB^{\perp}$

When a photon is detected the measurement is conventionally +1, and when it is not detected the measurement is conventionally −1.

Given:

$a_i = DA$ measurement (at the i$^{\text{th}}$ throw)

$\overline{a}_i = DA^{\perp}$ measurement (at the i$^{\text{th}}$ throw); $\overline{a}_i$ could also be written $a_i^{\perp}$

$b_i = DB$ measurement (at the i$^{\text{th}}$ throw)

$\overline{b}_i = DB^{\perp}$ measurement (at the i$^{\text{th}}$ throw); $\overline{b}_i$ could also be written $b_i^{\perp}$

$\overline{a}_i$ is the complement of a_i [5]: when the photon is detected by DA, it is not detected by $DA^{\perp}$ (and conversely)

$\overline{b}_i$ is the complement of b_i: when the photon is detected by DB, it is not detected by $DB^{\perp}$ (and conversely)

At each entangled photon, by construction:

$$a_i = -\overline{a}_i \tag{1}$$

which means $1 = -(-1)$ or $-1 = -(1)$, either the photon is detected by DA, or by $DA^{\perp}$ and

$$b_i = -\overline{b}_i \tag{2}$$

which means $1 = -(-1)$ or $-1 = -(1)$, either the other photon is detected by DB, or by $DB^{\perp}$.

2.3. Experimental Measures

The measures are of the form:

1) If the two polarizers are oriented in the same direction, the two photons always behave the same way (transmitted or absorbed depending on the angle of the polarizer with the polarization).

2) If the two polarisers are inclined at an angle of 30˚ with respect to each other, then the two photons have exactly the same behavior in 3/4 cases and in opposite fourth cases.

3) If the two polarisers are inclined at an angle of 60˚ with respect to each other, then the two photons have the same behavior in exactly 1/4 cases and in opposed 3/4 cases [6].

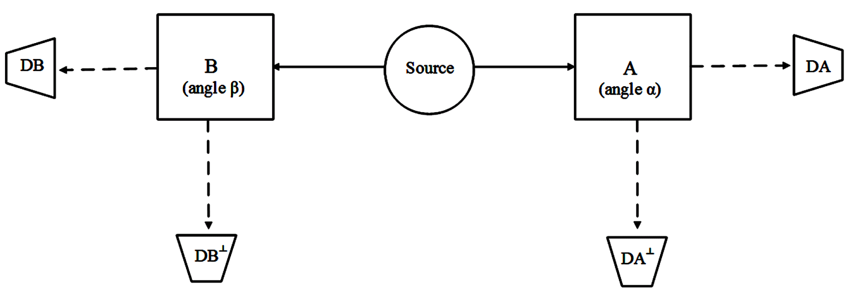

Figure 1. Bell's inequality set-up.

2.4. The Quantum Sum s_i

Let us define the first quantum sum s_i: "each pair of particles carries with it sufficient information to calculate the following number" [7]

$$s_i = \left(a_i + \bar{a}_i\right)\cdot b_i + \left(a_i - \bar{a}_i\right)\cdot \bar{b}_i \tag{3}$$

Previous quantum sum can also be written as:

$$s_i = a_i \cdot b_i + \bar{a}_i \cdot b_i + a_i \cdot \bar{b}_i - \bar{a}_i \cdot \bar{b}_i \tag{4}$$

Remark: another way to write the quantum sum is:

$$s_i' = a_i \cdot b_i' - \bar{a}_i \cdot b_i' + a_i \cdot \bar{b}_i' + \bar{a}_i \cdot \bar{b}_i' \tag{5}$$

where b_i has thus been noted $\bar{b}_i'$ (and consequently $\bar{b}_i$ has been noted b_i')

It is only a convention to call a detector DB or $DB^{\perp}$, and so the measures b_i or $\bar{b}_i'$. In practice, it will change the sign of the result s_i', not its absolute value:

$$|s_i| = |s_i'| \tag{6}$$

2.5. Bell's Inequality Definition

Let us remind what Bell's inequality definition is:

"Bell's theorem [or Bell's inequality] is not defined according to a clear statement that would be found in a reference article" [8]; so we will hereafter take the definitions given by Professor SCARANI:

"This is the statement of the Bell theorem: if our hypothesis is correct, the average value of s_i must be between −2 and +2. That's all..." [7].

Then Bell's therorem (or inequality) is mathematically transcribed to:

$$-2 \le S \le +2 \tag{7}$$

with S the average value of s_i

2.6. Average Quantum Sum S_1

The sum used in the updated Bell's inequality definition is: "making measurements on a large number of pairs, it can measure the average value of s_i" [7].

Let us call "the average value of s_i": S_1

$$S_1 = Av\cdot\left[s_i\right] \tag{8}$$

S_1 the average value of s_i is "the algebraic sum of the four average values corresponding to the measures" [7], which is mathematically transcripted (see Appendix) by:

$$S_1 = Av\cdot\left[a_i\cdot b_i\right] + Av\cdot\left[\bar{a}_i\cdot b_i\right] + Av\cdot\left[a_i\cdot\bar{b}_i\right] - Av\cdot\left[\bar{a}_i\cdot\bar{b}_i\right] \tag{9}$$

3. Treatment of the Measures

3.1. Treatment from a Wave Function Property

Pr. Valerio Scarani [9] [10], popularizing Pr Aspect's demonstration, proceeds starting from the wave function and according to quantum calculation ending up to a property [7]:

$$S_2(\alpha,\beta) = E(\alpha,\beta) + E(\alpha+\pi/2,\beta) + E(\alpha,\beta+\pi/2) - E(\alpha+\pi/2,\beta+\pi/2) \tag{10}$$

with

$$E(\alpha,\beta) = -\cos(\alpha,\beta) \tag{11}$$

Important remark:

This sum S_2 looks like the average sum S_1 but it is not the same:

✧ S_1 is function of experimental measures (a_i, b_i) [cf. Equation (9)]; (a_i, b_i) are a couple of numbers:

$$\left(a_i, b_i\right) = \left(\pm 1, \pm 1\right) \tag{12}$$

✧ S_2 is function of experimental conditions (α, β) [cf. Equation (10)]; (α, β) are a couple of angles:

$$(\alpha, \beta) \in [0; 2\pi]^2 \tag{13}$$

Starting again with Equation (10) and Equation (11),

$$S_2(\alpha, \beta) = -\cos(\alpha + \beta) - \cos(\alpha + \pi/2 + \beta) - \cos(\alpha + \beta + \pi/2) + \cos(\alpha + \pi/2 + \beta + \pi/2) \tag{14}$$

$$S_2(\alpha, \beta) = -\cos(\alpha + \beta) - \sin(\alpha + \beta) - \sin(\alpha + \beta) - \cos(\alpha + \beta) \tag{15}$$

$$S_2(\alpha, \beta) = -2\left[\cos(\alpha + \beta) + \sin(\alpha + \beta)\right] \tag{16}$$

For example, for $\alpha = 0^\circ$ and $\beta = 30^\circ$:

$$S_2(\alpha, \beta) = -2[0.87 + 0.5] = -2.7 \tag{17}$$

$$S_2\left(\alpha = 0^\circ, \beta = 30^\circ\right) \le -2 \tag{18}$$

It would be in this case $\left(\alpha = 0^\circ, \beta = 30^\circ\right)$, according to Equation (10) and Equation (11), violation of Bell's inequality.

Remark: the concern is that this result is the consequence of theoretical calculations dealing with angles α and β of the test conditions; it is not the result of measures a_i and b_i experimentally found (cf **Table A2** in Appendix).

3.2. Quantum Sum from the Experimental Measures in a Particular Case

In the previous case where $\left(\alpha = 0^\circ, \beta = 30^\circ\right)$, returning to the experimental measures [cf **Table A2** in Appendix], we get **Table 1**.

According to the sum definition [cf. Equation (8)],

$$\text{for } \left(\alpha = 0^\circ, \beta = 30^\circ\right),\ S_1 = Av \cdot [s_i] = -8/8 = -1 \tag{19}$$

or according to the sum property [cf. Equation (9)],

$$\text{for } \left(\alpha = 0^\circ, \beta = 30^\circ\right),\ S_1 = (+4/8) + (-4/8) + (-4/8) - (+4/8) = -8/8 = -1 \tag{20}$$

id est:

$$-2 \le S_1\left(\alpha = 0^\circ, \beta = 30^\circ\right) \le +2 \tag{21}$$

For $\left(\alpha = 0^\circ, \beta = 30^\circ\right)$, the average sum S_1 does *not* violate Bell's inequality [as defined in Equation (7)].

Table 1. Experimental measures with an angle of 30° processed.

# i	Angle α	Angle β	$a_i \cdot b_i$	$\bar{a}_i \cdot b_i$	$a_i \cdot \bar{b}_i$	$\bar{a}_i \cdot \bar{b}_i$	s_i
3	0	30°	1	−1	−1	1	−2
4	0	30°	1	-1	−1	1	−2
5	0	30°	1	-1	−1	1	−2
6	0	30°	−1	1	1	−1	2
7	0	30°	1	−1	−1	1	−2
8	0	30°	1	−1	−1	1	−2
9	0	30°	1	−1	−1	1	−2
10	0	30°	−1	1	1	−1	2
Average			**+4/8**	**−4/8**	**−4/8**	**+4/8**	**−8/8**

3.3. Quantum Sum from the Experimental Measures in the General Case

Let us process the three tables (**Tables A1-A3**) from Appendix into **Table 2**.

We can verify that for all experimental measures cf. Equation (9):

$$S_1 = Av\cdot[a_i \cdot b_i] + Av\cdot[\overline{a}_i \cdot b_i] + Av\cdot[a_i \cdot \overline{b}_i] - Av\cdot[\overline{a}_i \cdot \overline{b}_i]$$

$$S_1 = (+2/18) + (-2/18) + (-2/18) - (+2/18) = -4/18 = -0.22 \quad (22)$$

If we processed from the definition for all experimental measures cf. Equation (8):

$$S_1 = Av\cdot[s_i]$$

$$S_1 = -4/18 = -0.22 \quad (23)$$

so processing all experimental measures

$$-2 \le S_1 \le +2 \quad (24)$$

With all the measures, the average sum S_1 does *not* violate Bell's inequality [as defined in Equation (7)].

So, in the particular case or in general, using the definition S_1, Bell's inequality is *never* violated.

3.4. Synthesis about the Two Arguments

Paragraphs 3.1 and 3.2 can be boiled out to the flowchart of **Figure 2**.

And from a more general point of view, the synthesis flowchart will be **Figure 3**.

Table 2. Experimental measures processed.

# i	Angle α	Angle β	$a_i \cdot b_i$	$\overline{a}_i \cdot b_i$	$a_i \cdot \overline{b}_i$	$\overline{a}_i \cdot \overline{b}_i$	s_i
1	0	0°	1	−1	−1	1	−2
2	0	0°	1	−1	−1	1	−2
3	0	30°	1	−1	−1	1	−2
4	0	30°	1	−1	−1	1	−2
5	0	30°	1	−1	−1	1	−2
6	0	30°	−1	1	1	−1	2
7	0	30°	1	−1	−1	1	−2
8	0	30°	1	−1	−1	1	−2
9	0	30°	1	−1	−1	1	−2
10	0	30°	−1	1	1	−1	2
11	0	30°	−1	1	1	−1	2
12	0	60°	−1	1	1	−1	2
13	0	60°	−1	1	1	−1	2
14	0	60°	1	−1	−1	1	−2
15	0	60°	−1	1	1	−1	2
16	0	60°	−1	1	1	−1	2
17	0	60°	−1	1	1	−1	2
18	0	60°	1	−1	−1	1	−2
Average			**+2/18**	**−2/18**	**−2/18**	**+2/18**	**−4/18**

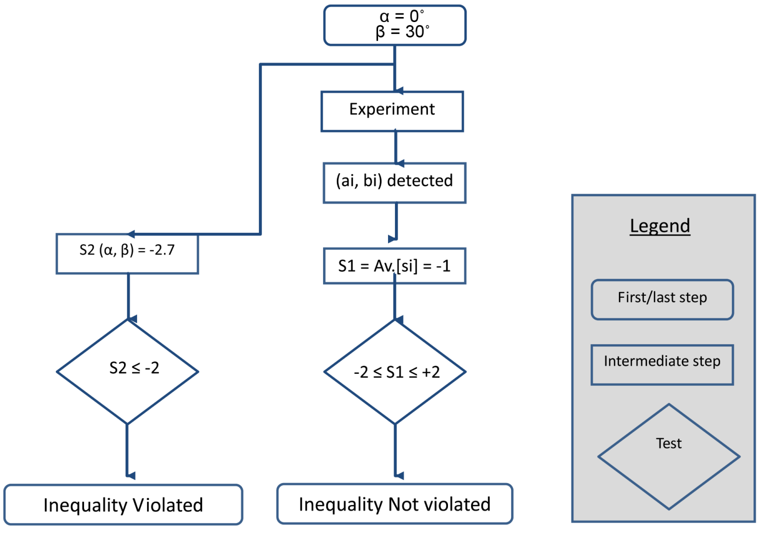

Figure 2. Flowchart of the 2 arguments.

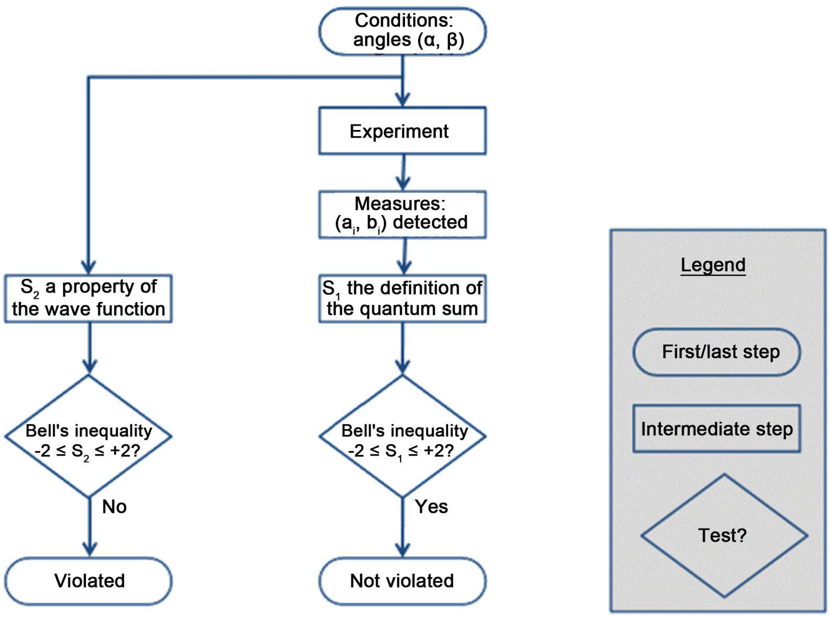

Figure 3. Synthetic flowchart.

3.5. Application to the Aspect's Experiment of 1983

In present paper, analysis has been done mainly from Dr. Scarani presentation, according to the property

$$S_2(\alpha,\beta) = E(\alpha,\beta) + E(\alpha+\pi/2,\beta) + E(\alpha,\beta+\pi/2) - E(\alpha+\pi/2,\beta+\pi/2)$$

[cf Equation10] with $E(\alpha,\beta) = -\cos(\alpha+\beta)$ [cf. Equation (11)]

Then the extremum are for values as $\pi/6$ or 30° [cf. Equation (18)], and $\pi/3$ or 60°.

It is interesting to note than in Aspect's experiment of 1983 [3], the extremum were for values as $\pi/8$ (or 22.5°) and $3\pi/8$ (or 67.5°). The reason comes the experiment was different (in BCHSH experiment the difference between two angles are not $\pi/2$ but are $\pi/4$), and so property was different:

$$S_3(\theta) = 3\cos(2\theta) - \cos(6\theta) \tag{25}$$

with

$$\theta = (a, b) \tag{26}$$

Let us correlate 1983 notation with present notation (see **Table 3**):

$$s_i = A(\lambda, a)\cdot B(\lambda, b) - A(\lambda, a)\cdot B(\lambda, b') + A(\lambda, a')\cdot B(\lambda, b) + A(\lambda, a')\cdot B(\lambda, b') \tag{27}$$

which is equivalent to

$$s_i = \overline{a}_i \cdot b_i - \overline{a}_i \cdot \overline{b}_i + a_i \cdot b_i + a_i \cdot \overline{b}_i \tag{28}$$

Equation (28) is exactly the same than the Equation (4) for the definition of Bell's inequality.

So we can use the same flowchart (**Figure 4**) to explain the difference of results between the definition of Bell's inequality and the property of a wave function: Starting from the experiment and the definition of S_1, Bell's inequality is not violated.

4. Conclusion

After recalling what the Bell's inequality and the experimental measures are, the violation of this inequality appears to have been obtained by processing from a wave function property and the experimental conditions, but

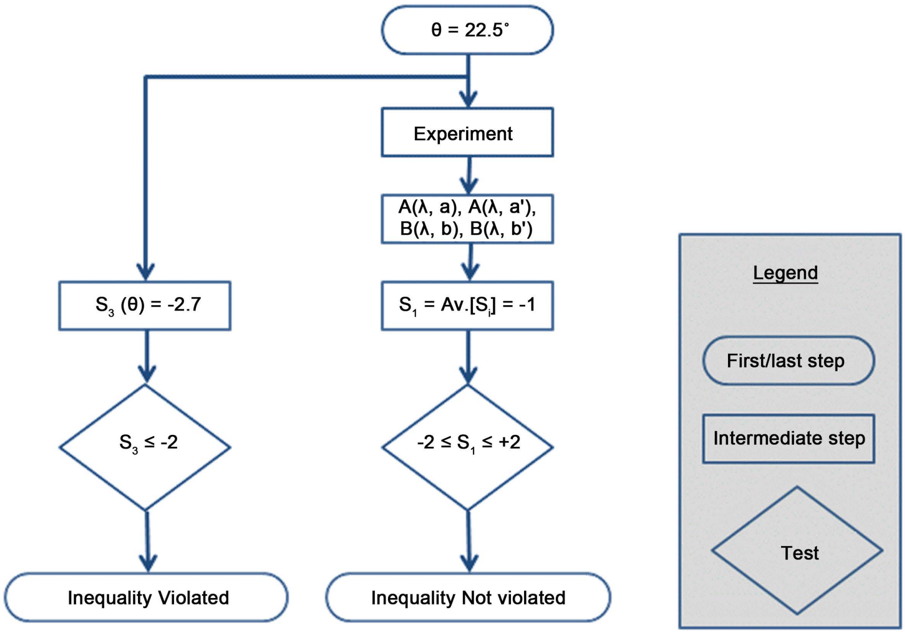

Figure 4. 1983 demonstration flowchart.

Table 3. Notation comparison.

	1983 notation	Present notation
Separator angles	a, b	α, β
Angle $\theta = (a, b)$	$\theta = b - a$	$\theta = \beta - \alpha$
Property	$S_3(\theta) = 3\cos(2\theta) - \cos(6\theta)$	$S_3(\theta) = 3\cos(2\theta) - \cos(6\theta)$
Polarisation measure from A	$A(\lambda, a)$	$\overline{a}_i$
Polarisation measure from A	$A(\lambda, a')$	a_i
Polarisation measure from B	$B(\lambda, b)$	b_i
Polarisation measure from B	$B(\lambda, b')$	$\overline{b}_i$
Definition	See Equation (27)	See Equation (28)

have obscured the measures themselves. By treating these measures from the definition of S (average sum of s_i), Bell's inequality appears to be respected. The ways of the two arguments are summarized in flowcharts.

This article is limited to the demonstration of the violation of Bell's inequality. It is neither a criticism of Bell's inequality theory, nor of its consequences.

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Appendix

A1. Bell's Inequality

Let us remind the evolution of Bell's inequality.

The inequality that Bell derived can be written as:

$$\rho(a,c)-\rho(a,b)-\rho(b,c)\leq 1 \tag{29}$$

where ρ is the correlation between measurements of the spins of the pair of particles and a, b and c refer to three arbitrary settings of the two analysers [11].

Generalizing Bell's original inequality, it has been introduced the CHSH inequality, without any assumption of perfect correlations (or anti-correlations) at equal settings

$$\rho(a,b)+\rho(a,b')+\rho(a',b)-\rho(a',b')\leq 2 \tag{30}$$

where ρ denotes correlation in the quantum physicist's sense: "the expected value of the product of the two binary (+/−1 valued) outcomes... the CHSH inequality reduces to the original Bell inequality" [11].

A2. Average Quantum Sum S_1

Let us call "the average value of s_i": S_1

$$S_1 = Av\cdot[s_i] \qquad \text{cf. Equation (8)}$$

$$S_1=\frac{1}{n}\sum_{i=1}^{i=n}s_i \tag{31}$$

S_1 the average value of s_i is "the algebraic sum of the four average values corresponding to the measures" [7], which is mathematically transcripted by:

$$S_1 = Av\cdot[a_i\cdot b_i]+Av\cdot[\bar{a}_i\cdot b_i]+Av\cdot[a_i\cdot \bar{b}_i]-Av\cdot[\bar{a}_i\cdot \bar{b}_i] \qquad \text{cf. Equation (9)}$$

Demonstration:

$$S_1=\frac{1}{n}\sum_{i=1}^{i=n}s_i \qquad \text{cf. Equation (31)}$$

so

$$S_1=\frac{1}{n}\sum_{i=1}^{i=n}\left[(a_i+\bar{a}_i)\cdot b_i\right]+\left[(a_i-\bar{a}_i)\cdot \bar{b}_i\right] \tag{32}$$

$$S_1=\frac{1}{n}\sum_{i=1}^{i=n}(a_i\cdot b_i)+\frac{1}{n}\sum_{i=1}^{i=n}(\bar{a}_i\cdot b_i)+\frac{1}{n}\sum_{i=1}^{i=n}(a_i\cdot \bar{b}_i)+\frac{1}{n}\sum_{i=1}^{i=n}(-\bar{a}_i\cdot \bar{b}_i) \tag{33}$$

and so

$$S_1 = Av\cdot[a_i\cdot b_i]+Av\cdot[\bar{a}_i\cdot b_i]+Av\cdot[a_i\cdot \bar{b}_i]-Av\cdot[\bar{a}_i\cdot \bar{b}_i] \qquad \text{cf. Equation (9)}$$

So S_1, average value of s_i, is effectively the algebraic [quantum] sum of the four average values. It is an equivalent definition.

A3. Experimental Measures

Let us translate these synthetic measures through examples.

a) If the two polarizers are oriented in the same direction, the two photons always behave the same way (transmitted or absorbed depending on the angle of the polarizer with the polarization): $\{a_i=b_i\}$ in all cases (**Table A1**).

b) If the two polarisers are inclined at an angle of 30˚ with respect to each other, then the two photons have exactly the same behavior in 3/4 cases and in opposite fourth cases: $\{a_i=b_i\}$ in 3/4 cases (**Table A2**).

c) If the two polarisers are inclined at an angle of 60˚ with respect to each other, then the two photons have the same behavior in exactly 1/4 cases and in opposed 3/4 cases: $\{a_i=b_i\}$ in 1/4 cases (**Table A3**).

Table A1. Experimental measures with an angle of 0°.

# i	Angle α	Angle β	a_i	$\bar{a}_i$	b_i	$\bar{b}_i$
1	0	0°	1	−1	1	−1
2	0	0°	−1	1	−1	1

Table A2. Experimental measures with an angle of 30°.

# i	Angle α	Angle β	a_i	$\bar{a}_i$	b_i	$\bar{b}_i$
3	0	30°	1	−1	1	−1
4	0	30°	1	−1	1	−1
5	0	30°	1	−1	1	−1
6	0	30°	1	−1	**−1**	**1**
7	0	30°	−1	1	−1	1
8	0	30°	−1	1	−1	1
9	0	30°	−1	1	−1	1
10	0	30°	−1	1	**1**	**−1**

Table A3. Experimental measures with an angle of 60°.

# i	Angle α	Angle β	a_i	$\bar{a}_i$	b_i	$\bar{b}_i$
11	0	60°	1	−1	**−1**	**1**
12	0	60°	1	−1	**−1**	**1**
13	0	60°	1	−1	**−1**	**1**
14	0	60°	1	−1	1	−1
15	0	60° °	−1	1	**1**	**−1**
16	0	60°	−1	1	**1**	**−1**
17	0	60°	−1	1	**1**	**−1**
18	0	60°	−1	1	−1	1

More generally, the probability that the behavior of photons to be identical is

$$\text{Probability}\left(a_i = b_i\right) = \cos^2\left(\beta - \alpha\right) \tag{34}$$

with $\left(\beta - \alpha\right)$ the relative angle of the two polarizers.

a) When $\alpha = \beta = 0^\circ$,

$$\text{Probability}\left(a_i = b_i\right) = \cos^2\left(0^\circ - 0^\circ\right) = 1 = 100\% \tag{35}$$

b) When $\alpha = 0^\circ$ and $\beta = 30^\circ$,

$$\text{Probability}\left(a_i = b_i\right) = \cos^2\left(0^\circ - 30^\circ\right) = \left(\sqrt{3}/2\right)^2 = 3/4 = 75\% \tag{36}$$

c) When $\alpha = 0^\circ$ and $\beta = 60^\circ$,

$$\text{Probability}\left(a_i = b_i\right) = \cos^2\left(0^\circ - 60^\circ\right) = \left(1/2\right)^2 = 25\% \tag{37}$$