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Couette Flow Problem for an Unsteady MHD Fourth-Grade Fluid with Hall Currents

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Abstract

In this work, we analyze Couette flow problem for an unsteady magnetohydrodynamic (MHD) fourth-grade fluid in presence of pressure gradient and Hall currents. The existing literature on the topic shows that the effect of Hall current on Couette flow of an unsteady MHD fourth-grade fluid with pressure gradient has not been investigated so far. The arising non-linear problem is solved by the homotopy analysis method (HAM) and the convergence of the obtained complex series solution is carefully analyzed. The influence of pressure number, Hartmann number, Hall parameter and fourth-grade material parameters on the unsteady velocity is discussed through plots and on local skin-friction coefficient discussed through numerical values presented in tabular form.

Keywords

Couette Flow; Hall Currents; Unsteady; Fourth-Grade Fluid; HAM

1. Introduction

In fluid mechanics, everyone is familiar with Couette flow problem, the flow between two parallel plates in which bottom plate is fixed and upper plate is initially at rest and is suddenly set into motion in its own plane with a constant velocity, is termed as Couette flow [1] [2]. Bhaskara and Bathaiah [3] have analyzed Couette flow problem with Hall effects for flow through a porous straight channel. Ganapathy [4] wrote a note on the oscillatory Couette flow in a rotating system. Erdogan [5] solved unsteady Couette flow for viscous fluid by Laplace transform method. Stokes and Couette flows due to an oscillating wall are discussed by Khaled and Vafai [6]. Hayat *et al*. [7] used Laplace transform method to determine the analytic solutions of Couette flows of a second grade fluid. Oscillatory Couette flow is studied by Singh [8]. Guria [9] discussed Couette flow problem for rotating and oscillatory flow. Couette flow of an unsteady third-grade fluid with variable magnetic field is

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investigated by Hayat and Kara [10], here fluid is in an annular region between two coaxial cylinders. The axial Couette flow problem of an electrically conducting fluid in an annulus is examined by Hayat *et al.* [11]. Das *et al.* [12] studied unsteady Couette flow problem in a rotating system. Recently, Zaman *et al.* [13] presented solution for unsteady Couette flow problem for the Eyring-Powell model. When a strong magnetic field is applied in an ionized gas of low density, the conductivity normal to the magnetic field is decreased by free spiraling of electrons and ions about the magnetic lines of force before suffering collisions. This phenomenon is known as Hall effect and a current induced in a direction normal to the electric and magnetic fields is called Hall current [15]. The study of the Effects of Hall current on flow of non-Newtonian fluids [15]-[23] is important because of its applications in power generators and pumps, Hall accelerators, refrigeration coils, electric transformers, in flight MHD, electronic system cooling, cool combustors, fiber and granular insulation, oil extraction, thermal energy storage and flow through filtering devices.

In order to understand the interaction of electric, magnetic, and hydrodynamic forces in the unsteady fourth-grade fluid, we considered a simple flow problem, known as the Couette flow. The effects of pressure gradient and Hall current on the flow are also taken into account. The complex analytic solution for non-linear problem is found by using the homotopy analysis method (HAM) [24]-[31]. This solution is valid for all values of the time in the whole spatial domain $0 \le \eta < 1$. The convergence of the analytic solution is ensured with the help of h-curve. The effects of pressure number, Hartmann number, Hall parameter, second-grade parameter, third-grade and fourth grade parameters on the unsteady velocity are illustrated through plots. Also the effects of the pertinent parameters on the local skin friction coefficient at the surface of the wall are presented numerically in tabular form.

2. Formulation of the Problem and Its Analytic Solution

Consider the unsteady flow of an electrically conducting incompressible fourth-grade fluid between two parallel flat plates, subjected to a uniform transverse magnetic field. We assume that the bottom plate is fixed and the top plate is stationary when $t < 0$ and at $t = 0$, the top plate starts moving impulsively in its own plane with a constant velocity U and a pressure gradient is also applied. The flow here is maintained by the motion of the top plate. The Cauchy stress tensor $\boldsymbol{T}$ for a fourth-grade fluid is given as [32]

$$
\begin{aligned}
\boldsymbol{T} = {} & -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2 + \beta_1\boldsymbol{A}_3 + \beta_2\left(\boldsymbol{A}_1\boldsymbol{A}_2 + \boldsymbol{A}_2\boldsymbol{A}_1\right) + \beta_3\left(\mathrm{tr}\boldsymbol{A}_1^2\right)\boldsymbol{A}_1 + \gamma_1\boldsymbol{A}_4 + \gamma_2\left(\boldsymbol{A}_3\boldsymbol{A}_1 + \boldsymbol{A}_1\boldsymbol{A}_3\right) + \gamma_3\boldsymbol{A}_2^2 \\
& + \gamma_4\left(\boldsymbol{A}_2\boldsymbol{A}_1^2 + \boldsymbol{A}_1^2\boldsymbol{A}_2\right) + \gamma_5\left(\mathrm{tr}\boldsymbol{A}_2\right)\boldsymbol{A}_2 + \gamma_6\left(\mathrm{tr}\boldsymbol{A}_2\right)\boldsymbol{A}_1^2 + \left[\gamma_7\left(\mathrm{tr}\boldsymbol{A}_3\right) + \gamma_8\left(\mathrm{tr}\boldsymbol{A}_2\boldsymbol{A}_1\right)\right]\boldsymbol{A}_1,
\end{aligned}
\tag{1}
$$

where p is the scalar pressure, $\boldsymbol{I}$ is the identity tensor, μ is the coefficient of viscosity, α_i, β_j, γ_k $\left((i=1,2),(j=1,2,3)\right.$ and $\left.(k=1,2,\cdots,8)\right)$ are the material parameters of fourth-grade fluid, and $\boldsymbol{A}_i$ $(i=1,2,3,4)$ are the first four Rivlin-Ericksen tensors defined by [32]

$$
\boldsymbol{A}_1 = \left(\mathrm{grad}\,\boldsymbol{V}\right) + \left(\mathrm{grad}\,\boldsymbol{V}\right)^{\mathrm{T}},
\tag{2}
$$

$$
\boldsymbol{A}_{n+1} = \frac{\mathrm{d}\boldsymbol{A}_n}{\mathrm{d}t} + \boldsymbol{A}_n\left(\mathrm{grad}\,\boldsymbol{V}\right) + \left(\mathrm{grad}\,\boldsymbol{V}\right)^{\mathrm{T}}\boldsymbol{A}_n, \; n = 1,2,3.
\tag{3}
$$

The equations governing the magnetohydrodynamic flow with Hall effect are:

$$
\begin{aligned}
\frac{\partial u}{\partial t} = {} & -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2\left(1+i\in\right)}{\rho\left(1+\in^2\right)}u + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial y^2\partial t} + \frac{\beta_1}{\rho}\frac{\partial^4 u}{\partial y^2\partial t^2} + \frac{6\left(\beta_2+\beta_3\right)}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} \\
& + \frac{\gamma_1}{\rho}\frac{\partial^5 u}{\partial y^2\partial t^3} + \frac{2\left(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8\right)}{\rho}\left[2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\frac{\partial^2 u}{\partial y\partial t} + \left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^3 u}{\partial y^2\partial t}\right],
\end{aligned}
\tag{4}
$$

$$
0 = -\frac{1}{\rho}\frac{\partial p}{\partial y},
\tag{5}
$$

The boundary and initial conditions are

$$
u(y,t) = 0, \text{ at } y = 0, \text{ for } t > 0, u(y,t) = u_w = U, \text{ at } y = h, \text{ for } t \ge 0, u(y,t) = 0, \text{ at } t = 0, \text{ for } 0 \le y < h, \tag{6}
$$

where $u(y,t)$ is the velocity component in the x-direction, t is time, ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity of the fluid, $\boldsymbol{B}_0$ is the applied magnetic field, $\in (= w_e \tau_e)$ is the Hall parameter, w_e and τ_e are the cyclotron frequency and collision time of the electrons respectively, U is the velocity of the upper plate and h is the distance between two parallel plates and it will be considered as a length scale of the flow. Equation (5) shows that p is independent of y. In order to non-dimensionalize the problem let us introduce the similarity transformations

$$u = U f(\eta,\xi), \eta = \frac{y}{h}, \xi = \frac{\nu t}{h^2}, \tag{7}$$

where $f(\eta,\xi)$ is the dimensionless velocity function, η is the dimensionless distance from the bottom fixed plate and ξ is the dimensionless time. Equations (4) and (6) become

$$f'' + \alpha \frac{\partial f''}{\partial \xi} + \beta \frac{\partial^2 f''}{\partial \xi^2} + \zeta f'^2 f'' + \Gamma_1 \frac{\partial^3 f''}{\partial \xi^3} + 4\Gamma_2 f f'' \frac{\partial f'}{\partial \xi} + 2\Gamma_2 f'^2 \frac{\partial f''}{\partial \xi} - \frac{N(1+i\in)}{(1+\in^2)} f - \frac{\partial f}{\partial \xi} - P = 0, \tag{8}$$

$$f(0,\xi) = 0, \ f(1,\xi) = 1, \text{ for } \xi > 0, \ f(\eta,0) = 0, \text{ for } 0 \le \eta < 1, \tag{9}$$

where prime denotes differentiation with respect to η, $\alpha \left(= \alpha_1/\rho h^2\right)$ is dimensionless second-grade parameter, $\beta \left(= \beta_1 \nu/\rho h^4\right)$, $\zeta \left(= 6(\beta_2 + \beta_3)U^2/\rho \nu h^2\right)$ are dimensionless third-grade parameters, $\Gamma_1 \left(= \gamma_1 \nu^2/\rho h^6\right)$, $\Gamma_2 \left(= (3\gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + 3\gamma_7 + \gamma_8)U^2/\rho h^4\right)$ are dimensionless fourth-grade parameters, $P\left(= h^2/\rho U \nu \times \mathrm{d}p/\mathrm{d}x\right)$ is the dimensionless pressure number and $N\left(= \sigma B_0^2 h^2/\rho \nu\right)$ is the dimensionless modified Hartmann number [33]. The local skin friction coefficient or fractional drag coefficient on the surface of the moving wall is

$$C_f = \frac{2\tau_{xy}\big|_{y=h}}{\rho u_w^2}, \tag{10}$$

Now using Equations (1)-(3) and (7) the Equation (10) can be written in dimensionless variables as

$$\frac{1}{2}R_e \times C_f = f'(1,\xi) + \alpha \frac{\partial}{\partial \xi} f'(1,\xi) + \beta \frac{\partial^2 f'(1,\xi)}{\partial \xi^2} + \frac{1}{3}\zeta\left(f'(1,\xi)\right)^3$$
$$+ \Gamma_1 \frac{\partial^3 f'(1,\xi)}{\partial \xi^3} + 2\Gamma_2\left(f'(1,\xi)\right)^2 \frac{\partial f'(1,\xi)}{\partial \xi}, \tag{11}$$

where $R_e \left(= U h/\nu\right)$ is the Reynolds number

The boundary conditions (9) lead us to take base functions for the velocity $f(\eta,\xi)$ as

$$\left\{\eta^n \xi^j, n \ge 0, j \ge 0\right\}, \tag{12}$$

The velocity $f(\eta,\xi)$ can be expressed in terms of base functions as

$$f(\eta,\xi) = \sum_{n=0}^{\infty}\sum_{j=0}^{\infty} \delta_n^j \eta^n \xi^j, \tag{13}$$

To start with the homotopy analysis method, due to the boundary conditions (9) it is reasonable to choose the initial guess approximation

$$f_0(\eta,\xi) = \eta(1 + \xi - \eta\xi), \xi > 0, \tag{14}$$

and the auxiliary linear operator

$$\mathbf{L}(f) = \frac{\partial^3 f(\eta,\xi)}{\partial \eta^3}, \tag{15}$$

with the property:

$$\mathbf{L}\left[C_1 + C_2\eta + C_3\eta^2\right] = 0, \tag{16}$$

where C_1, C_2 and C_3 are arbitrary constants. Following the HAM and trying higher iterations with the unique and proper assignment of the results converge to the exact solution:

$$f(\eta,\xi) \approx f_0(\eta,\xi) + f_1(\eta,\xi) + f_2(\eta,\xi) + \cdots + f_m(\eta,\xi),\tag{17}$$

At $\in = 0.1$, $N = 0.1$, $\alpha = 0.1$, $\beta = 0.1$, $\zeta = 0.1$, $\Gamma_1 = 0.1$, $\Gamma_2 = 0.1$, $P = 0.1$ using the symbolic computation software such as MATLAB, MAPLE, MATHEMATICA to successively obtain

$$f_1(\eta,\xi) = \eta(1 + \xi - \eta\xi) + \left(\frac{1363}{3030} + \frac{i}{606}\right)\hbar\eta - \frac{7\hbar\eta^2}{20} - \left(\frac{37}{202} + \frac{i}{606}\right)\hbar\eta^3 + \frac{\hbar\eta^4}{12} + \left(\frac{2083}{1515} + \frac{i}{1212}\right)\hbar\eta\xi$$

$$- \frac{19\hbar\eta^2\xi}{10} + \left(\frac{261}{505} - \frac{i}{606}\right)\hbar\eta^3\xi + \left(\frac{261}{505} - \frac{i}{606}\right)\hbar\eta^3\xi + \left(\frac{5}{606} + \frac{i}{1212}\right)\hbar\eta^4\xi + \frac{4\hbar\eta\xi^2}{15} - \frac{4\hbar\eta^2\xi^2}{5} \quad (18)$$

$$+ \frac{14\hbar\eta^3\xi^2}{15} - \frac{2\hbar\eta^4\xi^2}{5} + \frac{\hbar\eta\xi^3}{30} - \frac{\hbar\eta^2\xi^3}{10} + \frac{2\hbar\eta^3\xi^3}{15} - \frac{\hbar\eta^4\xi^3}{15},$$

similarly $f_2(\eta,\xi)$, $f_3(\eta,\xi)$, $f_4(\eta,\xi)$ and so on are calculated. The obtained values of f_0, f_1, f_2, ..., lead us to take

$$f_m(\eta,\xi) = \sum_{n=0}^{2m+23m+4-n} \sum_{j=0} \delta_{m,n}^j \eta^n \xi^j.\tag{19}$$

The total complex analytic solution in compact form is

$$f(\eta,\xi) = \sum_{m=0}^{\infty} f_m(\eta,\xi) = \lim_{M\to\infty}\left(\sum_{m=0}^{M}\sum_{j=0}^{3m+4}\delta_{m,0}^j\xi^j\right) + \lim_{M\to\infty}\left[\sum_{n=1}^{M+1}\eta^n\left(\sum_{m=n-1}^{M}\sum_{j=0}^{3m+4-n}\delta_{m,n}^j\xi^j\right)\right],\tag{20}$$

where from initial guess in Equation (14) we obtain

$$\delta_{0,1}^0 = 1, \delta_{0,1}^1 = 1, \delta_{0,2}^1 = -1, \text{ all other } \delta_{0,n}^j = 0, (j = 0,1,2,3,4), (n = 0,1,2),\tag{21}$$

all other unknown constants can be determined by utilizing first nine given in Equation (21) by using the recurrence relations, which we calculated but it is not possible to write here due to their length. We know that the auxiliary parameter $\hbar$ gives the convergence region and rate of approximation for the homotopy analysis method. From $\hbar$-curve in **Figure 1** we note that the range for the admissible value for $\hbar$ is $-0.7 < \hbar < 0$. Our calculations depict that the series of the dimensionless velocity in Equation (20) converges in the whole region of η and ξ for $\hbar = -0.5$.

3. Graphs, Tables and Discussion

The discussion of emerging parameters on the dimensionless velocity $f(\eta,\xi)$ is as follows:

Figures 2 to **10** are plotted in absence of Hall currents and in **Figure 11** Hall current is taken into account. **Figure 2** displays the velocity f for various values of η. This figure describes that as we move from fixed
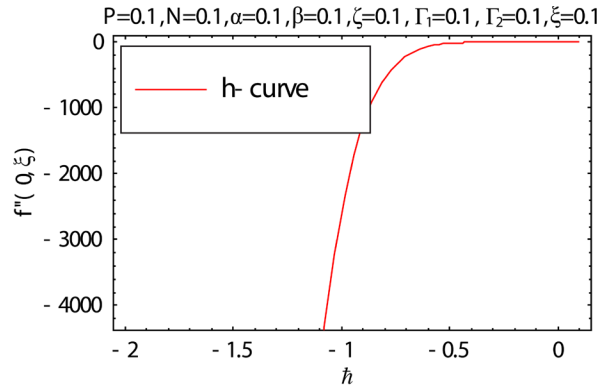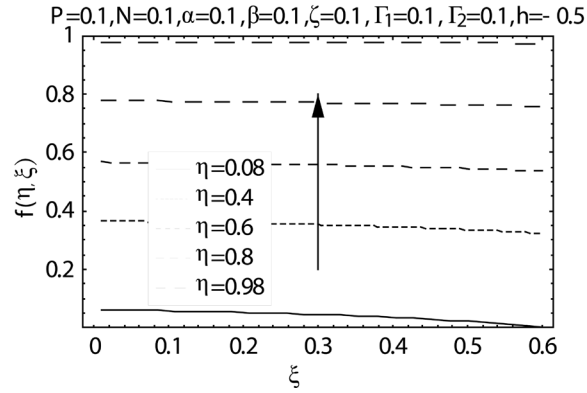

Figure 1. $\hbar$-curve for $f(\eta, \xi)$.

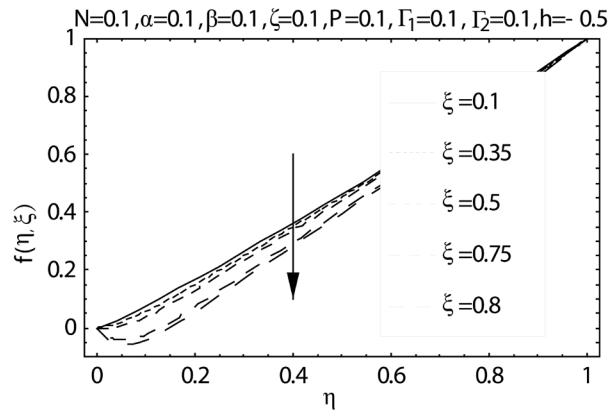Figure 2. Influence of η on $f(\eta, \xi)$.
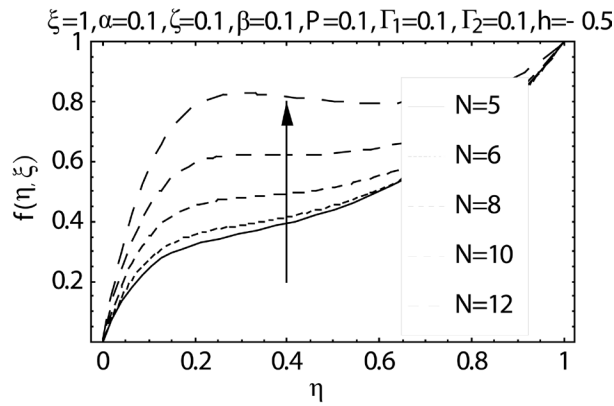

Figure 3. Influence of ξ on $f(\eta, \xi)$.

Figure 4. Influence of N on $f(\eta, \xi)$.

bottom plate to towards the moving top plate the velocity increases for all values of the time, even the fluid close to the upper plate moves with the same velocity as of the upper plate and the fluid close to the bottom plate has nearly zero velocity. **Figure 3** presents the velocity profile f for various values of ξ. This figure shows that with the passage of time the velocity of the fluid decreases as we go in the increasing direction of η. **Figure 4** elucidates the variation of Hartmann number on the velocity. It is found that the velocity increases with an increase in N and the boundary layer thickness decreases. This means that the magnetic force provides a mechanism to the control of boundary layer thickness. **Figure 5** illustrates the influence of second-grade parameter on the velocity profile f. It is evident from the figure that an increase in α results in the increase of the velocity, here boundary layer thickness decreases and shear thinning is observed. In **Figures 6** and **7** the velocity
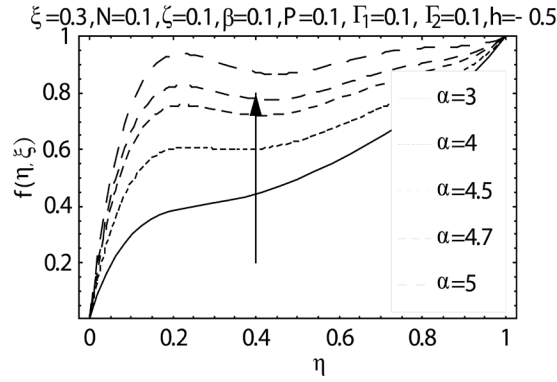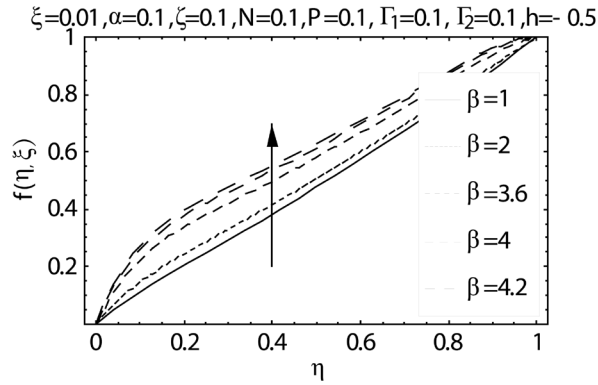
Figure 5. Influence of α on $f(\eta, \xi)$.
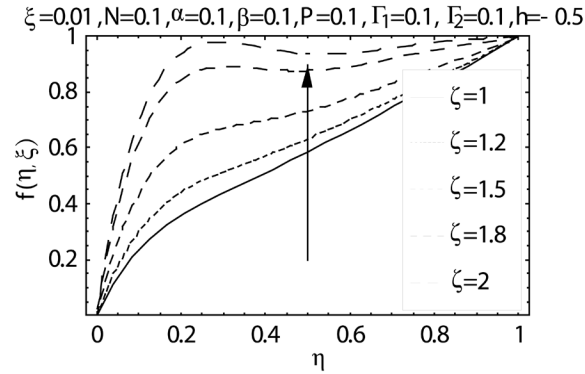

Figure 6. Influence of β on $f(\eta, \xi)$.

Figure 7. Influence of ζ on $f(\eta, \xi)$.

distribution is presented for the various values of third-grade parameters β and ζ. It is observed that the velocity increases by increasing the influence of β and ζ. **Figure 8** depicts the variation of the pressure number on the velocity. It is observed that the velocity increases with an increase in P, which is consistent with what we expected. In **Figures 9** and **10** it is observed that the velocity f has opposite behavior for fourth-grade parameters Γ_1 and Γ_2. For Γ_1 velocity increases and for Γ_2 velocity decreases as we move from fixed bottom plate to towards the moving upper plate. With the inclusion of Hall term velocity field becomes complex, so we plot absolute value of the velocity profile f in **Figure 11**. We observe that with increase in Hall parameter $\in$ absolute value of the velocity increases and boundary layer thickness decreases.

It is observed from **Table 1** that with increase in Hartmann number N absolute value of the skin friction coefficient $R_e \times C_f$ increases for all values of the time ξ and pressure P, also with increase in Hall current $\in$ absolute value of the skin friction coefficient decreases. Increase in dimensionless time ξ leads a reduction
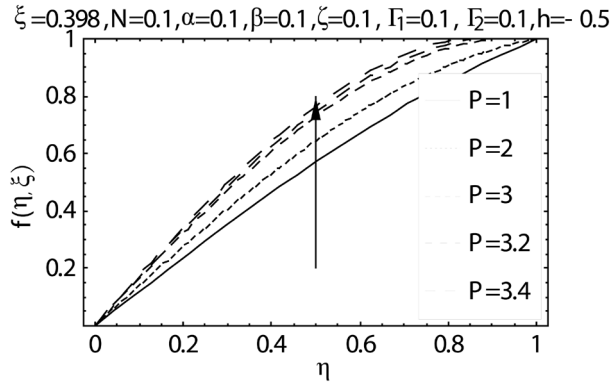
$\xi=0.398, N=0.1, \alpha=0.1, \beta=0.1, \zeta=0.1, \Gamma_1=0.1, \Gamma_2=0.1, h=-0.5$

Figure 8. Influence of P on $f(\eta, \xi)$.

$\xi=0.8, N=0.1, \zeta=0.1, \alpha=0.1, \beta=0.1, P=0.1, \Gamma_2=0.1, h=-0.5$

Figure 9. Influence of Γ_1 on $f(\eta, \xi)$.

$\xi=1, N=0.1, \zeta=0.1, \alpha=0.1, \beta=0.1, P=0.1, \Gamma_1=0.1, h=-0.5$
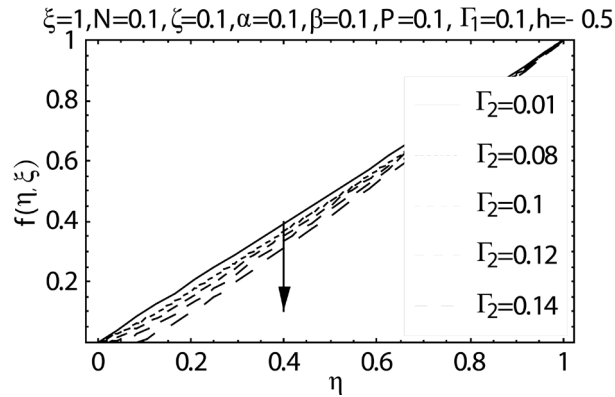
Figure 10. Influence of Γ_2 on $f(\eta, \xi)$.

in the absolute value of the skin friction coefficient. Increase in pressure number P, increases the shear stress at the moving wall. **Table 2** illustrates that increase in the fourth-grade material parameters Γ_1 and Γ_2 give a reduction in the value of the shear stress at the moving wall.

4. Conclusion

The Couette flow between two parallel plates filled with MHD unsteady fourth-grade fluid is studied analytical-ly. The effects of the pressure and Hall current are also incorporated. A non-linear fourth-grade model for the fluid is used. The model is invoked into the governing equations and the resulting one dimensional equation for unsteady MHD flow is derived. This equation is solved by HAM in general to study the sensitivity of the flow to
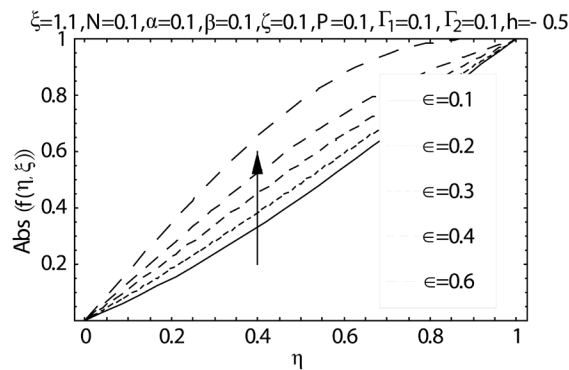
Figure 11. Influence of ∈ on *Abs*(*f*(*η*, *ξ*)).

Table 1. Absolute values of the skin friction coefficient $R_e \times C_f$ with $\alpha = 0.1$, $\beta = 0.1$, $\zeta = 0.1$, $\Gamma_1 = 0.1$, $\Gamma_2 = 0.1$, $\hbar = -0.5$.

ξ	P	∈ = 0.1	∈ = 0.1	∈ = 0.1	$N = 0.1$	$N = 0.1$	$N = 0.1$
		$N = 0.1$	$N = 0.3$	$N = 0.5$	∈ = 0.2	∈ = 0.3	∈ = 0.5
0.1	0.1	2.35909	2.47416	2.58937	2.35745	2.35492	2.34816
0.3	0.1	2.17475	2.28871	2.40271	2.17313	2.17062	2.16393
0.5	0.1	1.98266	2.09463	2.20655	1.98106	1.9786	1.97203
0.1	0.2	2.44703	2.56261	2.67833	2.44539	2.44284	2.43605
0.1	0.3	2.53576	2.65186	2.76812	2.5341	2.53155	2.52472
0.1	0.5	2.71564	2.83286	2.95026	2.71397	2.71139	2.7045

Table 2. Absolute values of the skin friction coefficient $R_e \times C_f$ with $\alpha = 0.1$, $\beta = 0.1$, $\zeta = 0.1$, $\xi = 0.1$, $P = 0.1$, $\hbar = -0.5$.

Γ_1	Γ_2	∈ = 0.1	∈ = 0.1	∈ = 0.1	$N = 0.1$	$N = 0.1$	$N = 0.1$
		$N = 0.1$	$N = 0.3$	$N = 0.5$	∈ = 0.2	∈ = 0.3	∈ = 0.5
0.1	0.1	2.35909	2.47416	2.58937	2.35745	2.35492	2.34816
0.3	0.1	2.29954	2.41317	2.52694	2.29792	2.29542	2.2887
0.5	0.1	2.24003	2.35223	2.46455	2.23843	2.23596	2.22938
0.1	0.2	2.20476	2.32664	2.44886	2.20303	2.20035	2.19321
0.1	0.3	1.63943	1.76654	1.8943	1.63763	1.63486	1.62744
0.1	0.5	1.40772	1.27198	1.13504	1.40971	1.4128	1.42096

the parameters that are used in the fourth-grade model. The various dimensionless parameters seem to affect the velocity a lot. The velocity profile and local skin friction coefficient are greatly influenced by the Hall parameter, fourth-grade fluid parameters, pressure and Hartmann numbers.

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