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Two Analytical Methods for Detection and Elimination of the Static Hazard in Combinational Logic Circuits

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ABSTRACT

In this paper, the authors continue the researches described in [1], that consists in a comparative study of two methods to eliminate the static hazard from logical functions, by using the form of Product of Sums (POS), static hazard "0". In the first method, it used the consensus theorem to determine the cover term that is equal with the product of the two residual implicants, and in the second method it resolved a Boolean equation system. The authors observed that in the second method the digital hazard can be earlier detected. If the Boolean equation system is incompatible (doesn't have solutions), the considered logical function doesn't have the static 1 hazard regarding the coupled variable. Using the logical computations, this method permits to determine the needed transitions to eliminate the digital hazard.

Keywords: Combinational Circuits; Static Hazard; Logic Design; Boolean Functions

1. Introduction

Under certain conditions, on the output of the logical signals may occur unwanted transitions. These transitions are known as glitches. The logic glitch is a kind of unwanted noise presenting inthe output signal that can initiate an uncontrollable process. In the next level there is an input signal [2].

We can distinguish three types of noise that is introduced in CLC (Combinational Logic Circuits), called hazards (Static, Dynamic and Function Hazards).

In the following we consider only the static hazard problem in combinational logic systems, called static hazard "0".

- Static 1 hazard, also called SOP (Sum of Products) hazard—a glitch that occurs in otherwise steady-state 1 output signal from SOP logic;
- Static 0 hazard, also called POS (Product of Sums) hazard—a glitch that occurs in otherwise steady-state 0 output signal from POS logic.

Static Hazards in Two-Level Combinational Logic Circuits (Consensus Method [2]).

We will initially define:

- Coupled variable; a variable input is complemented within a term of function and uncomplemented in another term of the same function.
- Coupled term; one of two terms containing only one coupled variable.
- Residue; the part of a coupled term that remains after removing the coupled variable.
- Hazard cover (or consensus term).

The RPI (Redundant Prime Implicant) required to eliminate the static hazards:

AND the residues of coupled p-term to obtain the SOP hazard cover,

OR the residues of coupled s-term to obtain the POS hazard cover.

POS example: any logic function can be described as:

$$y = (e_0 + x_i)(e_1 + \overline{x_i})$$

where

$$\begin{aligned} e_0 &= y(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 0, x_{i-1}, \cdots, x_0) \\ e_1 &= y(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 1, x_{i-1}, \cdots, x_0) \end{aligned} \quad (1)$$

Sometimes, the same function can be describes as:

$$y = (a + x_i)(b + \overline{x_i})c \quad (2)$$

Using the (1) form, we can say:

$$\begin{aligned} e_0 &= ac \\ e_1 &= bc \end{aligned} \quad (2.1)$$

Using the algorithm described in [3], if $c \le a+b$, the expression from (2), (2.1) doesn't present static hazard in relation with the x_i input, and if *a* = *b* = 0, then results *c* = 0.

The condition to have static hazard in relation with the x_i input, is when *a* = *b* = 0 and *c* = 1.

The consensus method [4] consists of determination of coupled terms, then by removing the coupled variables we obtain residual values.

That meaning the (2) equation can be written like:

$$y = (a + x_i)(b + \overline{x_i})c(a+b) \quad (3)$$

It can be observed that the expression of the function is multiplied by the sum of residual values, the new expression presents static hazard in relation with the x_i input.

We proposed as example the 4 inputs logic function:

$$\begin{aligned} y &= y(x_4, x_3, x_2, x_1, x_0) \\ &= R_0(1,3,6,7,9,11,14,17,19,20,25,27,28) \quad (4) \\ &+ R_\Phi(0,5,10,16,23,24,29,31) \end{aligned}$$

Using the Quine-McCluskey minimization method we obtain the equation from (5) and also the residual values determined by x_0 input:

$$\begin{aligned} y &= (x_4 + x_3 + \overline{x_2} + \overline{x_1})(\overline{x_4} + x_1 + x_0)(x_2 + \overline{x_0}) \\ &\cdot (x_4 + + \overline{x_2} + \overline{x_1} + x_0) \end{aligned}$$

$$\begin{aligned} a &= (\overline{x_4} + x_1)(x_4 + \overline{x_2} + \overline{x_1} x_0) \\ b &= x_2 \quad (5) \\ a + b &= x_2 + \overline{x_4 x_2} + \overline{x_4 x_1} x_0 + x_4 x_1 + \overline{x_2} x_1 = x_2 + \overline{x_4} + x_1 \end{aligned}$$

The expression of no static hazard in relation with x_0 input:

$$\begin{aligned} y &= (x_4 + x_3 + \overline{x_2} + \overline{x_1})(\overline{x_4} + x_1 + x_0)(x_2 + \overline{x_0}) \\ &\cdot (x_4 + \overline{x_2} + + \overline{x_1} + x_0)(\overline{x_4} + x_2 + x_1) \end{aligned} \quad (6)$$

2. Method of Resolving of Boolean Equations [5]

In this paragraph we apply the consensus method [5] and the method of solving some specific Boolean equations.

If $y = (a + x_i)(b + \overline{x_i})c$, by resolving the next system equations it can be determined the vectors input values which presents static hazard.

$$\begin{aligned} a &= 0 \\ b &= 0 \quad (7) \\ c &= 1 \end{aligned}$$

If the (7) system has no solution, the function doesn't presents static hazard in relation with x_i.

Therefore, the expression of the function becomes:

$$\begin{aligned} a &= (\overline{x_4} + x_1)(x_4 + \overline{x_2} + \overline{x_1} + x_0) = 0 \\ b &= x_2 = 0 \quad (8) \\ c &= x_4 + x_3 + \overline{x_2} + \overline{x_1} = 1 \end{aligned}$$

Therefore, $x_2 = 0$ imposes the reduction of the system to: $\overline{x_4} + x_1 = 0$ or $x_4 \overline{x_1} = 1$

So, the solution is:

$$\begin{aligned} x_4 &= 1 \\ x_3 &= \Phi \\ x_2 &= 0 \quad (8.1) \\ x_1 &= 0 \end{aligned}$$

So, the function will have hazard at commutation

$$\begin{aligned} &x_4 x_3 x_2 x_1 x_0 \\ &1\,0\,0\,0\,0 \leftrightarrow 1\,0\,0\,0\,1 \quad (16-17) \\ &1\,1\,0\,0\,0 \leftrightarrow 1\,1\,0\,0\,1 \quad (24-25) \end{aligned}$$

So, in the POS relation will be added the multiplied prime implicant $\overline{x_4} + x_2 + x_1$.

The function will have the same expression like in (7).

3. Static Hazards in Two-Level Combinational Logic Circuits

We will consider two analytical methods to detect and eliminate this type of hazard:

(*A*) *Consensus method* [1]

We will initially define:

- Coupled variable; a variable input is complemented within a term of function and uncomplemented in another term of the same function.
- Coupled term; one of two terms containing only one coupled variable.
- Residue; the part of a coupled term that remains after removing the coupled variable.
- Hazard cover (or consensus term).

The RPI (Redundant Prime Implicant) required to eliminate the static hazards:

- AND the residues of coupled p-term to obtain the SOP hazard cover,
- OR the residues of coupled s-term to obtain the POS hazard cover.

Example 1. Lets consider the logic function $f(x_2 x_1 x_0) = R_1(2,3,5,7)$.

a) *SOP example*: will be determined the prime implicants using Veitch-Karnaugh or Quine-McCluskey methods, as:

$$\begin{aligned} A &= \overline{x_2} \cdot x_1 \quad (2,3) \\ B &= x_1 \cdot x_0 \quad (3,7) \quad (9) \\ C &= x_2 \cdot x_0 \quad (5,7) \end{aligned}$$

One of the minimal equations is:

$$y = A + C = \overline{x_2} \cdot x_1 + x_2 \cdot x_0 \qquad (10)$$

where we have:

- coupled variable: x_2
- coupled terms: $\overline{x_2} \cdot x_1, x_2 \cdot x_0$
- residues: x_1, x_0
- consensus term: $x_1 \cdot x_0$

Therefore, the logic expression that has no static hazard in relation to x_2 variable is:

$$y = \overline{x_2} \cdot x_1 + x_2 \cdot x_0 + x_1 \cdot x_0 \qquad (11)$$

b) *POS example*: will be determined the prime implicants using Veitch-Karnaugh or Quine-Mc Cluskey methods, as:

$$\begin{aligned} a &= x_2 + x_1 \quad (0,1) \\ b &= x_1 + x_0 \quad (0,4) \\ c &= \overline{x_2} + x_0 \quad (4,6) \end{aligned} \qquad (12)$$

One of the minimal equations is:

$$y = (x_2 + x_1) \cdot (\overline{x_2} + x_0) \cdot (x_1 + x_0) \qquad (13)$$

where we have:

- coupled variable: x_2
- coupled terms: $\overline{x_2} + x_0, x_2 + x_1$
- residues: x_1, x_0
- consensus term: $x_1 + x_0$

The equation (13) shows no static 0 hazard.

Example 2. Let's consider the function of four variables $y = f(x_3 x_2 x_1 x_0) = R_1(0,1,2,5,6,7,8,9,10,14)$.

SOP hazard: will be determined the prime implicants using Quine-McCluskey method, as:

$$\begin{aligned} A &= \overline{x_3} \cdot \overline{x_1} \cdot x_0 \quad (1,5) \\ B &= \overline{x_3} \cdot x_1 \cdot \overline{x_0} \quad (2,6) \\ C &= \overline{x_3} \cdot x_2 \cdot x_0 \quad (5,7) \\ D &= \overline{x_3} \cdot x_2 \cdot x_1 \quad (6,7) \\ E &= x_3 \cdot x_1 \cdot \overline{x_0} \quad (10,14) \\ F &= \overline{x_2} \cdot \overline{x_1} \quad (0,1,8,9) \\ G &= \overline{x_2} \cdot \overline{x_0} \quad (0,2,8,10) \\ H &= x_1 \cdot \overline{x_0} \quad (2,6,10,14) \end{aligned} \qquad (14)$$

Applying the Patrick method [6], going from prime implicants table will be determined all SOP solutions.

Let's consider the logical p_i variables attached to the prime implicants as follows: $p_0 \leftrightarrow A$, if $p_0 = 1$, the A prime implicant is present in the logical function expression, otherwise $p_0 = 0$ (A prime implicant is not present in the logical function expression), etc.

Therefore, considering the correspondence $p_1 \leftrightarrow B$, $p_2 \leftrightarrow C$, $p_3 \leftrightarrow D$, $p_4 \leftrightarrow E$, $p_5 \leftrightarrow F$, $p_6 \leftrightarrow G$, $p_7 \leftrightarrow H$, in the table illustrated in **Table 1** is shown the

Table 1. The SOP coverage table.

dec. equiv.	0	1	2	5	6	7	8	9	10	14
p_i										
p_0		1		1						
p_1			1		1					
p_2				1		1				
p_3					1	1				
p_4									1	1
p_5	1	1					1	1		
p_6	1		1				1		1	
p_7			1		1				1	1

Patrick coverage:

It writes the coverage equation:

$$\begin{aligned} &(p_5 + p_6) \cdot (p_0 + p_5) \cdot (p_1 + p_6 + p_7) \\ &\cdot (p_0 + p_2) \cdot (p_1 + p_3 + p_7) \cdot (p_2 + p_3) \\ &\cdot (p_5 + p_6) \cdot p_5 \cdot (p_4 + p_6 + p_7) \cdot (p_4 + p_7) \equiv 1 \end{aligned} \qquad (15)$$

Simplifications are made by using the laws of Boolean algebra: the redundance law, the identity law and the distributive law.

$$\begin{aligned} &p_5 \cdot (p_1 + p_6 + p_7) \cdot (p_1 + p_3 + p_7) \\ &\cdot (p_4 + p_7) \cdot (p_2 + p_0) \cdot (p_2 + p_3) = 1 \\ &\text{or } p_5 \cdot (p_7 + p_1 + p_3 \cdot p_6) \cdot (p_7 + p_4) \\ &\quad \cdot (p_2 + p_0 \cdot p_3) = 1 \\ &\text{or } p_5 \cdot (p_7 + p_1 \cdot p_4 + p_3 \cdot p_4 \cdot p_6) \\ &\quad \cdot (p_2 + p_0 \cdot p_3) = 1 \\ &\text{or } (p_5 \cdot p_7 + p_1 \cdot p_4 \cdot p_5 + p_3 \cdot p_4 \cdot p_5 \cdot p_6) \\ &\quad \cdot (p_2 + p_0 \cdot p_3) = 1 \end{aligned} \qquad (16)$$

A version of the optimal solution corresponds to $p_5 \cdot p_7 \cdot p_2$ triplet, *i.e.*

$$\begin{aligned} y &= f(x_3, x_2, x_1, x_0) = F + H + C \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \end{aligned} \qquad (17)$$

The cost of this function in SOP implementation is:

$$C(y) = C(\overline{x_2} \cdot \overline{x_1}) + C(x_1 \cdot \overline{x_0}) + C(\overline{x_3} \cdot x_2 \cdot x_0) + 3 = 10$$

(It was considered the variables $x_i, \overline{x_i}$, available at input).

It can verify that any other coverage has a higher cost. For example, the coverage $p_5 \cdot p_7 \cdot p_0 \cdot p_3$ which corresponds to

$$\begin{aligned} y &= F + H + A + D \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot \overline{x_1} \cdot x_0 + \overline{x_3} \cdot x_2 \cdot x_1 \end{aligned} \qquad (18)$$

has the cost $C_1(y) = 14$.

4. The Static Hazard Elimination

(*B*) *The consensus method*

We apply the same method as in [7], only that it has a strong computing nature. Any logic function can be written as:

$$y = e_0 \cdot \overline{x_i} + e_1 \cdot x_i \quad (11),$$

where

$$e_0 = y(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 0, x_{i-1}, \cdots, x_0),$$

$$e_1 = y(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 1, x_{i-1}, \cdots, x_0).$$

Obviously, if $y = a \cdot \overline{x_i} + b \cdot x_i + c$ (12), then $e_0 = a + c$, $e_1 = b + c$.

If we add the term $e_0 \cdot e_1$ to relation (11), the function presents no hazard towards x_i.

In terms of the consensus method, the term that covers the static 1 hazard is

$$e_0 \cdot e_1 = (a + c) \cdot (b + c) = c + a \cdot b \quad (19),$$

therefore for the form (11) will be $e_0 \cdot e_1$, and for the form (12), $a \cdot b$.

Considering the second example, we will have:
hazard in relation to the input x_0:

$$y = F + H + C = \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \quad (20),$$

where

$$\begin{aligned} e_0 &= x_1 + \overline{x_2} \cdot \overline{x_1} = x_1 + \overline{x_2}, \\ e_1 &= \overline{x_3} \cdot x_2 + \overline{x_2} \cdot \overline{x_1} = \overline{x_3} \cdot x_2 + \overline{x_2} \cdot \overline{x_1}, \\ e_0 \cdot e_1 &= (x_1 + \overline{x_2}) \cdot (\overline{x_3} \cdot x_2 + \overline{x_2} \cdot \overline{x_1}) \\ &= \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_2} \cdot \overline{x_1} = F + D \end{aligned} \quad (21)$$

By adding $F + D$ term to relation (14), it obtains:

$$\begin{aligned} y &= F + H + C + (F + D) \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 + \overline{x_3} \cdot x_2 \cdot x_1 \end{aligned} \quad (22)$$

hazard in relation to the input x_1:

$$\begin{aligned} e_0 &= \overline{x_2} + \overline{x_3} \cdot x_2 \cdot x_0 = \overline{x_2} + x_3 \cdot x_0 \\ e_1 &= \overline{x_0} + \overline{x_3} \cdot x_2 + \overline{x_3} \cdot x_2 \cdot x_0 = \overline{x_0} + \overline{x_3} \cdot x_2 \\ e_0 \cdot e_1 &= (\overline{x_2} + x_3 \cdot x_0) \cdot (\overline{x_0} + \overline{x_3} \cdot x_2) = \overline{x_2} \cdot \overline{x_0} = G \end{aligned} \quad (23)$$

Therefore, the expression of the function becomes:

$$\begin{aligned} y &= F + H + C + D + G \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \\ &\quad + \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_2} \cdot \overline{x_0} \end{aligned} \quad (24)$$

hazard in relation to the input x_2:

$$e_0 = \overline{x_1} + \overline{x_0} + x_1 \cdot \overline{x_0} = \overline{x_1} + \overline{x_0}$$

$$e_1 = \overline{x_3} \cdot x_0 + \overline{x_3} \cdot x_1 + x_1 \cdot \overline{x_0}$$

Therefore,

$$\begin{aligned} e_0 \cdot e_1 &= (\overline{x_1} + \overline{x_0}) \cdot (\overline{x_3} \cdot x_0 + \overline{x_3} \cdot x_1 + x_1 \cdot \overline{x_0}) \\ &= \overline{x_3} \cdot \overline{x_1} \cdot x_0 + \overline{x_3} \cdot x_1 \cdot \overline{x_0} + x_1 \cdot \overline{x_0} \\ &= x_1 \cdot \overline{x_0} + \overline{x_3} \cdot \overline{x_1} \cdot x_0 = H + A \end{aligned} \quad (25)$$

The expression of the function becomes:

$$\begin{aligned} y &= F + H + C + D + G + A \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \\ &\quad + \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_2} \cdot \overline{x_0} + \overline{x_3} \cdot \overline{x_1} \cdot x_0 \\ &= \overline{x_3} \cdot x_2 \cdot x_0 + \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_1} \cdot x_0 + t \end{aligned} \quad (26)$$

hazard in relation to the input x_3:

$$e_0 = x_2 \cdot x_0 + x_2 \cdot \overline{x_1} + \overline{x_1} \cdot x_0 + t,$$

where

$$t = \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_2} \cdot \overline{x_0} + \overline{x_3} \cdot \overline{x_1} \cdot x_0$$

$$e_1 = t$$

$$e_0 \cdot e_1 = (x_2 \cdot x_0 + x_2 \cdot \overline{x_1} + \overline{x_1} \cdot x_0 + t) \cdot t = t \quad (27)$$

so that remains the same expression (20), which has no hazards in relation to x_3.

From the relation (20), it sees that the expression of the function without SOP hazards contains all prime implicants without B and E.

(*C*) *The method of solving of some Boolean equations* [8]

A logic function can be written as:

$$y = a \cdot \overline{x_i} + b \cdot x_i + c \quad (28)$$

where

$$a + c = f(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 0, x_{i-1}, \cdots, x_0),$$

$$b + c = f(x_{n-1}, x_{n-2}, \cdots, x_{i+1}, 1, x_{i-1}, \cdots, x_0).$$

According to a theorem from [8], a logic function expressed as SOP, presents a static hazard in the situation $x_i + \overline{x_i} = 1$, a situation deducted by solving the following system of logical equations:

$$\begin{aligned} a &= 1 \\ b &= 1 \\ c &= 0 \end{aligned} \quad (29)$$

We return to the same function, (14):

$$y = F + H + C = \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0.$$

hazard in relation to the input x_0:

$$y = (x_1) \cdot \overline{x_0} + (\overline{x_3} \cdot x_2) \cdot x_0 + \overline{x_2} \cdot \overline{x_1} \quad (30)$$

The function will present SOP hazard, if

$$\begin{aligned} &x_1 = 1 \\ &\overline{x_3} \cdot x_2 = 1 \\ &\overline{x_2} \cdot \overline{x_1} = 0 \end{aligned} \quad (31)$$

Therefore, $x_3 = 0$, $x_2 = 1$, $x_1 = 1$, which imposes a hazard at commutation

$$\begin{aligned} &x_3 x_2 x_1 x_0 \\ &0\,1\,1\,0 \leftrightarrow 0\,1\,1\,1, \end{aligned}$$

which imposes the adding of the prime implicant $D = (6,7)$ to function.

The function becomes

$$\begin{aligned} y &= F + H + C + D \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 + \overline{x_3} \cdot x_2 \cdot x_1 \end{aligned} \quad (32)$$

hazard in relation to the input x_1:

$$\begin{aligned} &a = \overline{x_2} = 1 \\ &x_2 = 0 \\ &b = \overline{x_0} + \overline{x_3} \cdot x_2 = 1 \\ &\text{or } x_0 = 0 \\ &c = \overline{x_3} \cdot x_2 \cdot x_0 = 0 \\ &x_3 = \phi \end{aligned} \quad (33)$$

Therefore, we will have hazards in the following cases:

$$\begin{aligned} &x_3 x_2 x_1 x_0 \\ &0\,0\,0\,0 \leftrightarrow 0\,0\,1\,0 \quad (0,2) \\ &1\,0\,0\,0 \leftrightarrow 1\,0\,1\,0 \quad (8,10) \end{aligned}$$

The previous commutations are equivalent to the implicant $G = (0,2,8,10)$.

The function becomes

$$\begin{aligned} y &= F + H + C + D + G \\ &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \\ &\quad + \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_2} \cdot \overline{x_0} \end{aligned} \quad (34)$$

hazard in relation to the input x_2:

$$\begin{aligned} &a = \overline{x_1} + \overline{x_0} = 1 \\ &x_3 = 0 \\ &b = \overline{x_3} \cdot x_0 + \overline{x_3} \cdot x_1 = 1 \\ &\Rightarrow \text{and } c = x_1 \cdot \overline{x_0} = 0, x_0 + x_1 = 1 \end{aligned} \quad (35)$$

We will have the solution:

$$\begin{aligned} &x_3 = 0 \\ &x_1 = 0 \\ &x_0 = 1 \end{aligned} \quad (35.1)$$

The corresponding commutation is:

$$\begin{aligned} &x_3 x_2 x_1 x_0 \\ &0\,0\,0\,1 \leftrightarrow 0\,1\,0\,1 \quad (1,5). \end{aligned}$$

Therefore, the term $A = (1,5)$ is added to the function.

And therefore:

$$y = F + H + C + D + G + A \quad (36)$$

hazard in relation to the input x_3:

$$\begin{aligned} y &= \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_3} \cdot x_2 \cdot x_0 \\ &\quad + \overline{x_3} \cdot x_2 \cdot x_1 + \overline{x_2} \cdot \overline{x_0} + \overline{x_3} \cdot \overline{x_1} \cdot x_0 \end{aligned} \quad (37)$$

$$\begin{aligned} &a = x_2 \cdot x_0 + x_2 \cdot x_1 + \overline{x_1} \cdot x_0 = 1 \\ &b = 0 \\ &c = \overline{x_2} \cdot \overline{x_1} + x_1 \cdot \overline{x_0} + \overline{x_2} \cdot \overline{x_0} = 0 \end{aligned} \quad (38)$$

Because one of the terms (a,b) is zero, we have no hazards in relation to that variable.

5. Conclusions

The contribution of the authors consists in that by analysis of two methods of detection/elimination of the static hazard, insisting of the POS method for the logic function which wasn't analyzed in [1].

The boolean equation [2,3], presents some advantages instead the consensus methods, the most important to determine the transactions which causes static hazard.

It concludes that the classical method of the 70s, the method of solving some specific Boolean equations [4], presents some advantages compared to consensus method [5], which has a strong heuristic nature.

In the first method it used the consensus theorem to determine the cover term that is equal with the product of the two residual implicants [6], and in the second method it resolved a Boolean equation system [7]. The authors observed that in the second method the digital hazard can be earlier detected. If the Boolean equation system is incompatible (doesn't have solutions), the considered logical function doesn't have the static 1 hazard regarding the coupled variable. Using the logical computations, this method permits to determine the needed transitions to eliminate the digital hazard.

From the both methods, we can observe that static 1 hazard can be removed by adding the prime implicants step by step.

The same method with the same conclusions was applied to the static 0 hazard (POS), using the duality theorem [8,9].

The authors observed that in the second method the digital hazard can be earlier detected. If the Boolean equation system is incompatible (doesn't have solutions), the considered logical function doesn't have the static 1 hazard regarding the coupled variable. Using the logical computations, this method permits to determine the needed transitions to eliminate the digital hazard.

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