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LRS Bianchi Type-I Cosmological Model with Anisotropic Dark Energy and Special Form of Deceleration Parameter

Kishor Shankarrao Adhav, Rajesh Purushottam Wankhade, Abhijit Shankarrao Bansod
Department of Mathematics, Sant Gadge Baba Amravati University, Amravati, India
Email: ati_ksadhav@yahoo.co.in

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ABSTRACT

We have studied Locally Rotationally Symmetric (LRS) Bianchi type-I cosmological model filled with anisotropic fluid in general theory of relativity. The solutions of the field equations are obtained by using special form of deceleration parameter which gives early deceleration and late time accelerating cosmological model. The geometrical and physical aspect of the model is also studied.

Keywords: LRS Bianchi Type-I Space-Time; Special Form of Deceleration Parameter; Anisotropic Fluid; Dark Energy; Isotropization

1. Introduction

Our universe is undergoing a late-time accelerating expansion which has been evidenced by Riess *et al.* [1], Bahcall *et al.* [2], Bennett *et al.* [3], Spergel *et al.* [4], Cunha [5]. We live in a spatially flat universe composed of (approximately) 4% baryonic matter, 22% dark matter and 74% dark energy. Recently, Li *et al.* [6] studied the present acceleration of the universe by analyzing the sample of baryonic acoustic oscillation (BAO) with cosmic microwave background (CMB) radiation and concluded that such sample of BAO with CMB increases the present cosmic acceleration which has been further explained by plotting graphs for change of deceleration parameter q with redshift $z<2$.

Many authors suggested a number of ideas to explain the current accelerating universe, such as scalar field model, exotic equation of state (EoS), modified gravity, and the inhomogeneous cosmology model. The dark energy EoS parameter is $w=p/\rho$, where p is the dark-energy pressure and ρ is its energy density. The value $w<-1/3$ is necessary for comic acceleration. The simplest candidate for dark energy is the cosmological constant ($\wedge$) for which $w=-1$. The matter with $w<-1$ gives rise to Big Rip singularity (Caldwell [7]). Elizalde *et al.* [8] and Nojiri *et al.* [9] proposed several ideas to prevent the Big Rip singularity by introducing quantum effect terms in the action. Recently, Astashenok *et al.* [10] studied phantom cosmology without Big Rip singularity.

In the present paper, we have considered a LRS spatially homogeneous and anisotropic Bianchi type-I cosmological model with special form of deceleration parameter in general relativity. The physical and geometrical aspects of the model are also discussed. To have a general description of an anisotropic dark energy component, we consider a phenomenological parameterization of dark energy in terms of its equation of state (ω) and skewness parameter (δ). Some features of the evolution of the metric and the dynamics of the anisotropic dark energy fluid have been also examined. This paper is organized as follows. In Section 2, we have given the line element, energy momentum tensor and its parametrization and the field equations. In Section 3, isotropization and solutions are given. The physical and geometrical parameters such as the anisotropy parameter of expansion (Δ), the energy density (ρ), the deviation-free EoS parameter (ω) and the deviation parameter (δ) etc are also studied with proper interpretation.

2. Metric and Field Equations

The LRS Bianchi type-I line element is given by

$$\mathrm{d}s^2=\mathrm{d}t^2-A(t)^2\,\mathrm{d}x^2-B(t)^2\left(\mathrm{d}y^2+\mathrm{d}z^2\right), \quad (2.1)$$

where $A(t)$ and $B(t)$ are the scale factors (metric potential) and functions of the cosmic time t only (non-static case).

Here we are dealing only with an anisotropic fluid whose energy-momentum tensor is in the following form

$$T_j^i = \mathrm{diag}\left[T_0^0, T_1^1, T_2^2, T_3^3\right].$$

We parametrize it as follows:

$$T_j^i = \mathrm{diag}\left[\rho, -\omega\rho, -(\omega+\delta)\rho, -(\omega+\delta)\rho\right] \quad (2.2)$$

where ρ is the energy density of the fluid, ω is the equation of state (EoS) parameters of the fluid and δ is the skewness parameter.

Here ω and δ are not necessarily constants and can be taken as functions of the cosmic time *t*.

The Einstein field equations, in natural limits ($8\pi G = 1$ and $c = 1$) are

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij}. \quad (2.3)$$

The Einstein's field Equations (2.3) for metric (2.1) with the help of Equations (2.2) give

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = \rho \quad (2.4)$$

$$\left(\frac{\dot{B}}{B}\right)^2 + 2\frac{\ddot{B}}{B} = -\omega\rho \quad (2.5)$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = -(\omega+\delta)\rho \quad (2.6)$$

where dot $(\cdot)$ indicates the derivative with respect to *t*.

3. Isotropization and the Solutions

We have three linearly independent Equations (2.4)-(2.6) with five unknowns A, B, ρ, ω and δ. In order to solve this system completely, we use a special form of deceleration parameter as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^\alpha} \quad (3.1)$$

where a is mean scale factor of the universe and α (>0) is constant.

This form has been proposed by Singha and Debnath[11] for FRW metric. Adhav *et al.* [12] used this law for studying Bianchi type models and Kantowski-Sachs cosmological model with dynamical equation of state (EoS) parameter.

From Equation (3.1) after integrating, we obtain the Hubble parameter as

$$H = \frac{\dot{a}}{a} = m\left(1 + a^{-\alpha}\right), \quad (3.2)$$

where *m* is an arbitrary constant of integration.

Integrating twice Equation (3.1), we get $H = \frac{\dot{a}}{a}$ and the average scale factor as

$$a = \left(\mathrm{e}^{m\alpha t} - 1\right)^{\frac{1}{\alpha}}. \quad (3.3)$$

The spatial volume is given by

$$V = a^3 = AB^2, \quad (3.4)$$

i.e.

$$V = \left(\mathrm{e}^{\alpha t} - 1\right)^{\frac{3}{\alpha}}, \quad (3.5)$$

here we consider $m = 1$.

The mean Hubble parameter H for LRS Bianchi type-I metric may be given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right). \quad (3.6)$$

The directional Hubble parameters in the direction x, y and z respectively can be defined as

$$H_x = \frac{\dot{A}}{A} \text{ and } H_y = H_z = \frac{\dot{B}}{B}. \quad (3.7)$$

Subtracting Equation (2.5) from Equation (2.6), we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = -\delta\rho. \quad (3.8)$$

Now, from Equations (3.4) and (3.8), we get

$$\frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)}{\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)} + \frac{\dot{V}}{V} = \frac{-\delta\rho}{\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)}. \quad (3.9)$$

Integrating, this gives

$$\frac{V}{\lambda}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \mathrm{e}^{\int \frac{\delta\rho}{\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right)} \mathrm{d}t}, \quad (3.10)$$

λ = constant of integration.

In order to solve above Equation (3.10), we use the condition

$$\delta = \frac{\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right)\alpha}{\rho}. \quad (3.11)$$

Using Equation (3.11) in Equation (3.10), we obtain

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{\lambda}{V}\mathrm{e}^{\alpha t}. \quad (3.12)$$

Now integrating Equation (3.12) and using Equation (3.5), we obtain the scale factors as

$$A(t) = \left(\mathrm{e}^{\alpha t} - 1\right)^{\frac{1}{\alpha}} \exp\left\{\frac{2\lambda}{3(\alpha-3)}\left(\mathrm{e}^{\alpha t} - 1\right)^{\frac{\alpha-3}{\alpha}}\right\}, \quad (3.13)$$

$$B(t)=\left(e^{\alpha t}-1\right)^{\frac{1}{\alpha}}\exp\left\{\frac{-\lambda}{3(\alpha-3)}\left(e^{\alpha t}-1\right)^{\frac{\alpha-3}{\alpha}}\right\}. \quad (3.14)$$

Using Equations (3.13) and (3.14) the directional Hubble parameters are found as

$$H_x=\frac{2\lambda}{3}e^{\alpha t}\left(e^{\alpha t}-1\right)^{\frac{-3}{\alpha}}+e^{\alpha t}\left(e^{\alpha t}-1\right)^{-1} \quad (3.15)$$

$$H_y=H_z=\frac{-\lambda}{3}e^{\alpha t}\left(e^{\alpha t}-1\right)^{\frac{-3}{\alpha}}+e^{\alpha t}\left(e^{\alpha t}-1\right)^{-1}. \quad (3.16)$$

The mean Hubble parameter H for LRS Bianchi type-I metric may be given by

$$H=\frac{1}{\left(1-e^{-\alpha t}\right)}. \quad (3.17)$$

The anisotropy parameter of the expansion is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2,$$

where $H_i\,(i=1,2,3)$ represent the directional Hubble parameters in the directions of *x*, *y* and *z* axis respectively and is found as

$$\Delta=\frac{2\lambda^2}{9}\left(e^{\alpha t}-1\right)^{\frac{2(\alpha-3)}{\alpha}}. \quad (3.18)$$

The shear scalar σ^2, defined by $\sigma^2=\frac{3}{2}\Delta H^2$ is found as

$$\sigma^2=\frac{\lambda^2}{3}e^{2\alpha t}\left(e^{\alpha t}-1\right)^{\frac{-6}{\alpha}}. \quad (3.19)$$

Using Equations (3.15), (3.16) and (2.4) we obtain the energy density as

$$\rho=e^{2\alpha t}\left\{3\left(e^{\alpha t}-1\right)^{-2}-\frac{\lambda^2}{3}\left(e^{\alpha t}-1\right)^{\frac{-6}{\alpha}}\right\}. \quad (3.20)$$

Using Equations (3.16) and (3.20) in Equation (2.5), we obtain the deviation-free parameter as

Now using Equation (3.11), we obtain the deviation parameter as

$$\delta=\frac{-\alpha\lambda}{3e^{\alpha t}\left[\left(e^{\alpha t}-1\right)^{\frac{3}{\alpha}-2}-\frac{\lambda^2}{9}\left(e^{\alpha t}-1\right)^{\frac{-3}{\alpha}}\right]}. \quad (3.22)$$

The expansion scalar θ is found to be

$$\theta=3H=3e^{\alpha t}\left(e^{\alpha t}-1\right)^{-1}. \quad (3.23)$$

4. Discussion

1) From **Figure 1**, one can verify that *q* decreases from $(+1)$ to (-1) during evolution of the universe.

2) The dynamics of energy density (ρ) for LRS Bianchi type-I metric is illustrated in **Figure 2**. Here one can observe that all models [for different $f(x)$] start with Big Bang having infinite density and as time increases (for finite time), the energy density tends to a finite value. Hence, after some finite time, the models approach to a steady state.

3) In **Figure 3**, we have plotted anisotropy parameter of expansion (Δ) against cosmic time *t*. For LRS Bianchi type-I model, it is observed that anisotropy parameter decreases to zero after some time. Hence, the model reaches to isotropy after some finite time which matches with the recent observations as the universe is isotropic at large scale.

4) The evolution of expansion scalar θ for $\alpha=1$ is shown in **Figure 4**. It is observed that the expansion is infinite at $t=0$. As cosmic time *t* increases, it decreases to a finite value $(\theta=3)$ after some finite value of *t*.

5. Conclusions

We have verified that the energy density of the fluid, the deviation-free equation of state parameter and the deviation parameter are all dynamical.

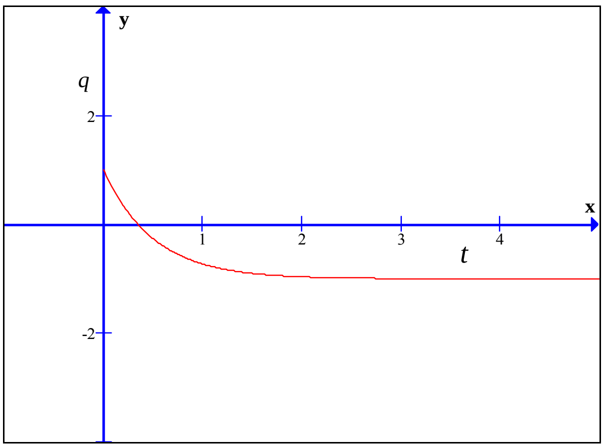

Figure 1. The variation of q vs t for $\alpha=2$.

$$\omega=-\left\{\frac{(3-2\alpha)\left(e^{\alpha t}-1\right)^{-2}+\frac{\lambda^2}{3}\left(e^{\alpha t}-1\right)^{\frac{-6}{\alpha}}-\frac{2\lambda\alpha}{3}e^{-\alpha t}\left(e^{\alpha t}-1\right)^{\frac{-3}{\alpha}}+2\alpha e^{-\alpha t}\left(e^{\alpha t}-1\right)^{-1}}{\left[3\left(e^{\alpha t}-1\right)^{-2}-\frac{\lambda^2}{3}\left(e^{\alpha t}-1\right)^{\frac{-6}{\alpha}}\right]}\right\}. \quad (3.21)$$

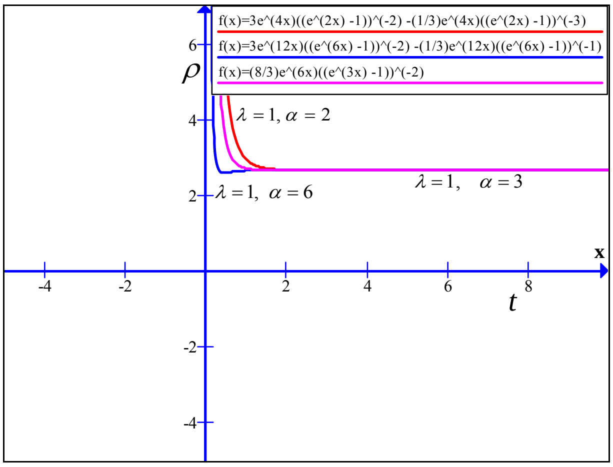

Figure 2. Energy density ρ vs cosmic time t.

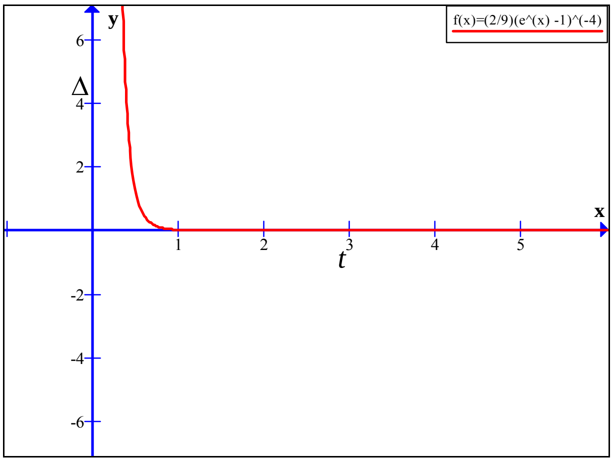

Figure 3. Anisotropy parameter Δ vs cosmic time t for $\lambda = 1 = \alpha$.

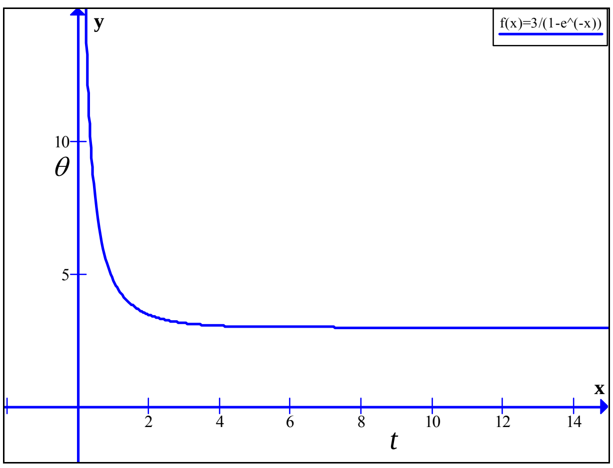

Figure 4. Expansion scalar θ vs cosmic time t.

It is observed that when $t \to \infty$, we get $\delta \to 0$, $\omega \to -1$ and $\rho \to 3 > 0$.

Which is mathematically equivalent to the cosmological constant (Λ CDM) model.

The SNe Ia data (Riess *et al.* [13], Astier *et al.* [14], Riess *et al.* [15]), the SDSS data (Eisenstein *et al.* [16]), and the three year WMAP data (Spergel *et al.* [17]) all indicate that the $\wedge$CDM model or the model that reduces to $\wedge$CDM model is described as a standard excellent model in cosmology to describe the cosmological evolution. Hence, one can conclude that LRS Bianchi type-I cosmological model is the best fitted model as it reduces to $\wedge$CDM model.

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REFERENCES

[1] A. G. Riess, *et al*., *The Astronomical Journal*, Vol. 116, 1998, p. 1009. doi:10.1086/300499

[2] N. Bahcall, *et al*., *Science*, Vol. 284, 1999, pp. 1481-1488. doi:10.1126/science.284.5419.1481

[3] C. L. Bennett, *et al*., *The Astronomical Journal*, Vol. 583, 2003. doi:10.1086/345346

[4] D. N. Spergel, *et al*., *The Astronomical Journal*, Vol. 148, 2003, p. 175.doi:10.1086/377226

[5] J. V. Cunha, *Physical Review D*, Vol. 79, 2009, Article ID: 047301. doi:10.1103/PhysRevD.79.047301

[6] Z. Li, *et al*., *Physics Letters B*, Vol. 695, 2011, p. 1-8. doi:10.1016/j.physletb.2010.10.044

[7] R. R. Caldwell, *et al*., *Physical Review Letters*, Vol. 91, 2003, Article ID: 071301. doi:10.1103/PhysRevLett.91.071301

[8] E. Elizalde, S. Nojiri and S. D. Odintsov, *Physical Review D*, Vol. 70, 2004, Article ID: 043539. doi:10.1103/PhysRevD.70.043539

[9] S. Nojiri and S. D. Odintsov, *Physics Letters B*, Vol. 595. 2004, p. 1. doi:10.1016/j.physletb.2004.06.060

[10] A. V. Astashenok, *et al*., *Physics Letters B*, Vol. 709, 2012, pp. 396-403.

[11] A. K. Singha and U. Debnath, *International Journal of Theoretical Physics*, Vol. 48, 2009, p. 351. doi:10.1007/s10773-008-9807-x

[12] K. S. Adhav, *et al*., *Central European Journal of Physics*, Vol. 9, 2011, p. 919.

[13] A. G. Riess, *et al*., *The Astronomical Journal*, Vol. 607, 2004, p. 665.doi:10.1086/383612

[14] P. Astier, *et al*., *Astronomy & Astrophysics*, Vol. 447, 2006, p. 31. doi:10.1051/0004-6361:20054185

[15] A. G. Riess, *et al*., *The Astronomical Journal*, Vol. 48, 2006, p. 886.

[16] D. J. Eisenstein, *et al*., *The Astronomical Journal*, Vol. 633, 2005, p. 560. doi:10.1086/466512

[17] D. N. Spergel, *et al*., *The Astronomical Journal*, Vol. 170, 2007.

