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On the Behavior of the Positive Solutions of the System of Two Higher-Order Rational Difference Equations

Qi Wang[1], Gengrong Zhang[2*], Linlin Fu[2]
[1]College of Science, Guangxi University of Science and Technology, Liuzhou, China
[2]College of Mathematics and Information Science, Guangxi University, Nanning, China
Email: *zgrzaw@gxu.edu.cn

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ABSTRACT

We study the convergence of the positive solutions of the system of the following two difference equations:

$$x_{n+1}=\frac{x_{n-2k+1}}{Ay_{n-k+1}x_{n-2k+1}+\alpha},y_{n+1}=\frac{y_{n-2k+1}}{Bx_{n-k+1}y_{n-2k+1}+\beta},n\geq 0,$$

where k is a positive integer, the parameters A,B,α,β and the initial conditions are positive real numbers. Our results generalize well known results in [1,2].

Keywords: System of Difference Equations; Convergence; Positive Solution; Behavior

1. Introduction

Difference equations and the system of difference equations play an important role in the analysis of mathematical models of biology, physics and engineering. The study of dynamical properties of nonlinear difference equations and the system of difference equations have been an area of intense interest in recent years (for example, see [1-11]).

In [1], Kurbanlı, Çinar and Yalçinkaya studied the behavior of the positive solutions of the following system of difference equations

$$x_{n+1}=\frac{x_{n-1}}{y_nx_{n-1}+1},y_{n+1}=\frac{y_{n-1}}{x_ny_{n-1}+1},n\geq 0.$$

In [2], Stevo Stević investigated the system of the following difference equations

$$x_{n+1}=\frac{ax_{n-1}}{by_nx_{n-1}+c},y_{n+1}=\frac{\alpha y_{n-1}}{\beta x_ny_{n-1}+\gamma},n\geq 0.$$

Motivated by the above studies, in this note, we consider the system of the following difference equations

$$x_{n+1}=\frac{x_{n-2k+1}}{Ay_{n-k+1}x_{n-2k+1}+\alpha},y_{n+1}=\frac{y_{n-2k+1}}{Bx_{n-k+1}y_{n-2k+1}+\beta},n\geq 0, \tag{1}$$

where k is a positive integer, the parameters A, B,α,β and the initial conditions are positive real numbers.

System (1) is a particular case of the system of the following difference equations

$$\begin{aligned}x_{n+1}&=\frac{a_1x_{n-2k+1}}{b_1y_{n-k+1}x_{n-2k+1}+c_1},\\ y_{n+1}&=\frac{a_2y_{n-2k+1}}{b_2x_{n-k+1}y_{n-2k+1}+c_2},n\geq 0.\end{aligned} \tag{2}$$

If $a_1a_2=0$, then system (2) is trivial. If $a_1a_2\neq 0$, system is reduced to system (1) with $A=\frac{b_1}{a_1},\alpha=\frac{c_1}{a_1},B=\frac{b_2}{a_2}$ and $\beta=\frac{c_2}{a_2}$. Hence from now on, we will consider system (1).

On the other hand, system (1) is a natural generalizetion of the equation

$$x_{n+1}=\frac{x_{n-2k+1}}{Ax_{n-k+1}y_{n-2k+1}+\alpha},n\geq 0, \tag{3}$$

where k is a positive integer, the parameters A,α are positive real numbers. Hence the results which we obtained can also apply to (3).

The essential problem we consider in this paper is the behavior of the positive solutions of system (1). We es-

*Corresponding author.

tablish the convergence of the positive solutions of system (1). To some extent, this generalizes the results obtained in [1,2]. It is interesting that a modification of our method enables us to investigate the form of the positive solutions of system (1).

2. Main Results

For convenience, set $r=\alpha\beta$,

$$l_n=\begin{cases} n, & \text{when } r=1,\\ \dfrac{1-r^n}{1-r}, & \text{when } r\neq 1.\end{cases} \quad \text{for } n\geq 0$$

Let $\{(x_n,y_n)\}_{n=-2k+1}^{\infty}$ be a positive solution of (1). Set

$$x_n=\frac{1}{s_n},\ y_n=\frac{1}{t_n} \quad \text{for } n\geq -2k+1, \tag{4}$$

then (1) translates into

$$s_{n+1}=\alpha s_{n-2k+1}+\frac{A}{t_{n-k+1}},\ t_{n+1}=\beta t_{n-2k+1}+\frac{B}{s_{n-k+1}},\ n\geq 0. \tag{5}$$

Set

$$p_{1,j,n}=\alpha+\frac{A}{r^n t_{-k+j}s_{-2k+j}+l_n(\beta A+B)},$$

$$q_{1,j,n}=\beta+\frac{B}{r^n s_{-k+j}t_{-2k+j}+l_n(\alpha B+A)},$$

$$p_{2,j,n}=\alpha+\frac{A}{r^n(\beta s_{-k+j}t_{-2k+j}+B)+l_n(\beta A+B)},$$

$$q_{2,j,n}=\beta+\frac{B}{r^n(\alpha t_{-k+j}s_{-2k+j}+A)+l_n(\alpha B+A)},$$

for $n\geq 0,\ j=1,2,3,\cdots,k$.

Lemma 2.1 For (5), we have

$$\begin{aligned} &s_{2kn+j}=s_{-2k+j}\prod_{i=0}^{n}p_{1,j,i},\ t_{2kn+j}=t_{-2k+j}\prod_{i=0}^{n}q_{1,j,i},\\ &s_{2kn+k+j}=s_{-2k+j}\prod_{i=0}^{n}p_{2,j,i},\ t_{2kn+k+j}=t_{-k+j}\prod_{i=0}^{n}q_{2,j,i}. \quad \text{for } n\geq 0,\ j=1,2,3,\cdots,k.\end{aligned} \tag{6}$$

Proof. Since

$$s_j=\alpha s_{-2k+j}+\frac{A}{t_{-k+j}}=s_{-2k+j}p_{1,j,0},t_j=\beta t_{-2k+j}+\frac{B}{s_{-k+j}}=t_{-2k+j}q_{1,j,0},$$

$$s_{k+j}=\alpha s_{-k+j}+\frac{A}{t_j}=\alpha s_{-k+j}+\frac{As_{-k+j}}{\beta s_{-k+j}t_{-2k+j}+B}=s_{-k+j}p_{2,j,0},$$

$$t_{k+j}=\beta t_{-k+j}+\frac{B}{s_j}=\beta t_{-k+j}+\frac{Bt_{-k+j}}{\alpha t_{-k+j}s_{-2k+j}+A}=t_{-k+j}q_{2,j,0}.$$

Hence (6) holds for $n=0,\quad j=1,2,3,\cdots,k$.

For $n=m-1,\ j=1,2,3,\cdots,k$. assume that (6) is true. Note that, for $n\geq 0,\ j=1,2,3,\cdots,k, l_n+r^n=l_{n+1}$, $rl_n=l_{n+1}-1$,

$$\prod_{i=0}^{n}p_{1,j,i}\cdot\prod_{i=0}^{n}q_{2,j,i}=r^{n+1}+l_{n+1}\frac{\beta A+B}{t_{-k+j}s_{-2k+j}},\quad \prod_{i=0}^{n}q_{1,j,i}\cdot\prod_{i=0}^{n}p_{2,j,i}=r^{n+1}+l_{n+1}\frac{\alpha B+A}{s_{-k+j}t_{-2k+j}}.$$

Then for# $n=m,\ \ j=1,2,3,\cdots,k,$ we have

$$\begin{aligned} s_{2km+j}&=\alpha s_{2k(m-1)+j}+\frac{A}{t_{2k(m-1)+k+j}}=\alpha s_{-2k+j}\prod_{i=0}^{m-1}p_{1,j,i}+\frac{A}{t_{-k+j}\prod_{i=0}^{m-1}q_{2,j,i}}\\ &=\alpha s_{-2k+j}\prod_{i=0}^{m-1}p_{1,j,i}+\frac{A}{t_{-k+j}}\cdot\frac{t_{-k+j}s_{-2k+j}\prod_{i=0}^{m-1}p_{1,j,i}}{r^m t_{-k+j}s_{-2k+j}+l_m(\beta A+B)}\\ &=s_{-2k+j}\prod_{i=0}^{m-1}p_{1,j,i}\left[\alpha+\frac{A}{r^m t_{-k+j}s_{-2k+j}+l_m(\beta A+B)}\right]=s_{-2k+j}\prod_{i=0}^{m}p_{1,j,i}\end{aligned}$$

$$t_{2km+j}=\beta t_{2k(m-1)+j}+\frac{B}{s_{2k(m-1)+k+j}}=\beta t_{-2k+j}\prod_{i=0}^{m-1}q_{1,j,i}+\frac{B}{s_{-k+j}\prod_{i=0}^{m-1}p_{2,j,i}}$$
$$=\beta t_{-2k+j}\prod_{i=0}^{m-1}q_{1,j,i}+\frac{B}{s_{-k+j}}\cdot\frac{s_{-k+j}t_{-2k+j}\prod_{i=0}^{m-1}q_{1,j,i}}{r^m s_{-k+j}t_{-2k+j}+l_m(\alpha B+A)}$$
$$=t_{-2k+j}\prod_{i=0}^{m-1}q_{1,j,i}\left[\beta+\frac{B}{r^m s_{-k+j}t_{-2k+j}+l_m(\alpha B+A)}\right]=t_{-2k+j}\prod_{i=0}^{m}q_{1,j,i}$$

$$s_{2km+k+j}=\alpha s_{2k(m-1)+k+j}+\frac{A}{t_{2km+j}}=\alpha s_{-k+j}\prod_{i=0}^{m-1}p_{2,j,i}+\frac{A}{t_{-2k+j}\prod_{i=0}^{m}q_{1,j,i}}$$
$$=\alpha s_{-k+j}\prod_{i=0}^{m-1}p_{2,j,i}+\frac{A}{t_{-2k+j}}\cdot\frac{s_{-k+j}t_{-2k+j}\prod_{i=0}^{m}p_{2,j,i}}{r^{m+1}s_{-k+j}t_{-2k+j}+l_{m+1}(\alpha B+A)}$$
$$=s_{-k+j}\prod_{i=0}^{m-1}p_{2,j,i}\left[\alpha+\frac{Ap_{2,j,m}}{r^{m+1}s_{-k+j}t_{-2k+j}+l_{m+1}(\alpha B+A)}\right]=s_{-k+j}\prod_{i=0}^{m}p_{2,j,i}$$

$$t_{2km+k+j}=\beta t_{2k(m-1)+k+j}+\frac{B}{s_{2km+j}}=\beta t_{-k+j}\prod_{i=0}^{m-1}q_{2,j,i}+\frac{B}{s_{-2k+j}\prod_{i=0}^{m}p_{1,j,i}}$$
$$=\beta t_{-k+j}\prod_{i=0}^{m-1}q_{2,j,i}+\frac{B}{s_{-2k+j}}\cdot\frac{t_{-k+j}s_{-2k+j}\prod_{i=0}^{m}q_{2,j,i}}{r^{m+1}t_{-k+j}s_{-2k+j}+l_{m+1}(\beta A+B)}$$
$$=t_{-k+j}\prod_{i=0}^{m-1}q_{2,j,i}\left[\beta+\frac{Bq_{2,j,m}}{r^{m+1}t_{-k+j}s_{-2k+j}+l_{m+1}(\beta A+B)}\right]=t_{-k+j}\prod_{i=0}^{m}q_{2,j,i}$$

Hence (6) is true.

Theorem 2.2 Let $\{(x_n,y_n)\}_{n=-2k+1}^{\infty}$ be a positive solution of (1). For $0\le j\le k, n=0,1,2,\cdots$

1) When $r\ge 1$, we have

$$\lim_{n\to\infty}x_n=\begin{cases}0, & \text{when } \alpha>1,\\ +\infty, & \text{when } \alpha<1,\end{cases}$$

$$\lim_{n\to\infty}y_n=\begin{cases}0, & \text{when } \beta>1,\\ +\infty, & \text{when } \beta<1,\end{cases}$$

$$\lim_{n\to\infty}x_{2kn+j}=a_{-2k+j},$$

$$\lim_{n\to\infty}y_{2kn+j}=b_{-2k+j},$$

where

$$a_{-2k+j},b_{-2k+j}$$

satisfy:

$$a_{-2k+j}=\frac{a_{-2k+j}}{Ab_{-k+j}a_{-2k+j}+\alpha},b_{-2k+j}=\frac{b_{-2k+j}}{Ba_{-k+j}b_{-2k+j}+\beta}.$$

2) When $r<1$.

a) Suppose $\frac{\alpha B+A}{\beta A+B}>1$, then

$$\lim_{n\to\infty}x_n=0,\lim_{n\to\infty}y_n=+\infty.$$

b) Suppose $\frac{\alpha B+A}{\beta A+B}<1$, then

$$\lim_{n\to\infty}x_n=+\infty,\lim_{n\to\infty}y_n=0.$$

c) Suppose $\frac{\alpha B+A}{\beta A+B}=1$

i) If $Ay_{-k+j}x_{-2k+j}+\alpha=1$, then $x_{2kn+j}=x_{-2k+j}$, for $n=0,1,2,\cdots$.

ii) If $Bx_{-k+j}y_{-2k+j}+\beta=1$, then $y_{2kn+j}=y_{-2k+j}$, for $n=0,1,2,\cdots$.

iii) If $Ay_{-k+j}x_{-2k+j}+\alpha\ne 1$, then

$$\lim_{n\to\infty}x_{2kn+j}=a_{-2k+j},\lim_{n\to\infty}x_{2kn+k+j}=a_{-k+j}.$$

iv) If $Bx_{-k+j}y_{-2k+j}+\beta\ne 1$, then

$$\lim_{n\to\infty}y_{2kn+j}=b_{-2k+j},\lim_{n\to\infty}y_{2kn+k+j}=b_{-k+j}.$$

where

$$a_{-2k+j},b_{-2k+j}$$

satisfy

$$a_{-2k+j}=\frac{a_{-2k+j}}{Ab_{-k+j}a_{-2k+j}+\alpha},b_{-2k+j}=\frac{b_{-2k+j}}{Ba_{-k+j}b_{-2k+j}+\beta}.$$

Proof. Note that for $j=1,2,\cdots,k$,

$$\lim_{n\to\infty} p_{1,j,n} = \lim_{n\to\infty} p_{2,j,n} = \begin{cases} \alpha, & \text{when } r \ge 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} > 1, \\ \dfrac{\alpha B + A}{\beta A + B}, & \text{when } r < 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} < 1, \end{cases}$$

$$\lim_{n\to\infty} q_{1,j,n} = \lim_{n\to\infty} q_{2,j,n} = \begin{cases} \beta, & \text{when } r \ge 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} > 1, \\ \dfrac{\beta A + B}{\alpha B + A}, & \text{when } r < 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} < 1. \end{cases}$$

Hence

$$\lim_{n\to\infty} \ln\left(s_{2kn+j}\right) = \ln\left(s_{-2k+j}\right) + \sum_{n=0}^{\infty} \ln\left(p_{1,j,n}\right) = \begin{cases} +\infty, & \text{when } \alpha > 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} > 1, \\ -\infty, & \text{when } \alpha < 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} < 1. \end{cases} \tag{7}$$

$$\lim_{n\to\infty} \ln\left(s_{2kn+k+j}\right) = \ln\left(s_{-k+j}\right) + \sum_{n=0}^{\infty} \ln\left(p_{2,j,n}\right) = \begin{cases} +\infty, & \text{when } \alpha > 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} > 1, \\ -\infty, & \text{when } \alpha < 1 \text{ or } \dfrac{\alpha B + A}{\beta A + B} < 1. \end{cases} \tag{8}$$

$$\lim_{n\to\infty} \ln\left(t_{2kn+j}\right) = \ln\left(t_{-2k+j}\right) + \sum_{n=0}^{\infty} \ln\left(q_{1,j,n}\right) = \begin{cases} +\infty, & \text{when } \beta > 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} > 1, \\ -\infty, & \text{when } \beta < 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} < 1. \end{cases} \tag{9}$$

$$\lim_{n\to\infty} \ln\left(t_{2kn+k+j}\right) = \ln\left(t_{-k+j}\right) + \sum_{n=0}^{\infty} \ln\left(q_{2,j,n}\right) = \begin{cases} +\infty, & \text{when } \beta > 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} > 1, \\ -\infty, & \text{when } \beta < 1 \text{ or } \dfrac{\beta A + B}{\alpha B + A} < 1. \end{cases} \tag{10}$$

1) When $r \ge 1$. In view of (4), (6), (7) and (8), we drive

$$\lim_{n\to\infty} x_{2kn+j} = \lim_{n\to\infty} x_{2kn+k+j} = \begin{cases} 0, & \text{when } \alpha > 1, \\ +\infty, & \text{when } \alpha < 1, \end{cases}$$

Thus

$$\lim_{n\to\infty} x_n = \begin{cases} 0, & \text{when } \alpha > 1, \\ +\infty, & \text{when } \alpha < 1, \end{cases}$$

Similarly, in view of (4), (6), (9) and (10), we get

$$\lim_{n\to\infty} y_n = \begin{cases} 0, & \text{when } \beta > 1, \\ +\infty, & \text{when } \beta < 1, \end{cases}$$

When $\alpha = 1$ note that the positive series

$$\sum\nolimits_{n=0}^{\infty} \ln\left(p_{1,j,n}\right) = \sum\nolimits_{n=0}^{\infty} \ln\left(1 + \frac{A}{r^n t_{-k+j} s_{-2k+j} + l_n\left(\beta A + B\right)}\right)$$

is convergent, in view of (4) and (6), we drive $\lim_{n\to\infty} x_{2kn+j} = a_{-2k+j}$.

Similarly, we get

$$\lim_{n\to\infty} x_{2kn+k+j} = a_{-k+j}, \lim_{n\to\infty} y_{2kn+j} = b_{-2k+j}, \lim_{n\to\infty} y_{2kn+j} = b_{-k+j}.$$

2) When $r < 1$.

a) b) The proof is similar to the proof of 1, we get

$$\lim_{n\to\infty} x_n = \begin{cases} 0, & \text{when } \dfrac{\alpha B + A}{\beta A + B} > 1, \\ -\infty, & \text{when } \dfrac{\alpha B + A}{\beta A + B} < 1. \end{cases},$$

$$\lim_{n\to\infty} y_n = \begin{cases} 0, & \text{when } \dfrac{\beta A + B}{\alpha B + A} > 1, \\ -\infty, & \text{when } \dfrac{\beta A + B}{\alpha B + A} < 1. \end{cases}$$

c) Suppose $\dfrac{\alpha B + A}{\beta A + B} = 1$.

i) If $Ay_{-k+j} x_{-2k+j} + \alpha = 1$ In view of (1), by induction, we drive $x_{2kn+j} = x_{-2k+j}$, for $n = 0, 1, 2, \cdots$.

ii) If $Bx_{-k+j} y_{-2k+j} + \beta = 1$, Similarly, we drive

$y_{2kn+j} = y_{-2k+j}$ for $n = 0,1,2,\cdots$.

iii) Note that

$$p_{1,j,n} = 1 + \frac{r^n\left[(\alpha-1)t_{-k+j}s_{-2k+j} + A\right]}{r^n t_{-k+j}s_{-2k+j} + l_n(\beta A+B)} = 1 + \frac{r^n\left(Ay_{-k+j}x_{-2k+j} + \alpha - 1\right)}{r^n + y_{-k+j}x_{-2k+j}l_n(\beta A+B)}$$

$$q_{1,j,n} = 1 + \frac{r^n\left[(\beta-1)s_{-k+j}t_{-2k+j} + B\right]}{r^n s_{-k+j}t_{-2k+j} + l_n(\alpha B+A)} = 1 + \frac{r^n\left(Bx_{-k+j}y_{-2k+j} + \beta - 1\right)}{r^n + x_{-k+j}y_{-2k+j}l_n(\alpha B+A)}$$

$$p_{2,j,n} = 1 + \frac{\beta r^n\left[(\alpha-1)s_{-k+j}t_{-2k+j} + A\right]}{r^n\left(\beta s_{-k+j}t_{-2k+j} + B\right) + l_n(\beta A+B)} = 1 + \frac{\beta r^n\left(Ax_{-k+j}y_{-2k+j} + \alpha - 1\right)}{r^n\left(\beta + Bx_{-k+j}y_{-2k+j}\right) + x_{-k+j}y_{-2k+j}l_n(\beta A+B)}$$

$$q_{2,j,n} = 1 + \frac{\alpha r^n\left[(\beta-1)t_{-k+j}s_{-2k+j} + B\right]}{r^n\left(\alpha t_{-k+j}s_{-2k+j} + A\right) + l_n(\alpha B+A)} = 1 + \frac{\alpha r^n\left(By_{-k+j}x_{-2k+j} + \beta - 1\right)}{r^n\left(\alpha + Ay_{-k+j}x_{-2k+j}\right) + y_{-k+j}x_{-2k+j}l_n(\alpha B+A)}$$

Hence, if $Ay_{-k+j}x_{-2k+j} + \alpha > 1$ the positive series

$$\sum_{n=0}^{\infty}\ln\left(p_{1,j,n}\right) = \sum_{n=0}^{\infty}\ln\left(1 + \frac{r^n\left(Ay_{-k+j}x_{-2k+j} + \alpha - 1\right)}{r^n + y_{-k+j}x_{-2k+j}l_n(\beta A+B)}\right)$$

is convergent. If $Ay_{-k+j}x_{-2k+j} + \alpha < 1$, the negative series

$$\sum_{n=0}^{\infty}\ln\left(p_{1,j,n}\right) = \sum_{n=0}^{\infty}\ln\left(1 + \frac{r^n\left(Ay_{-k+j}x_{-2k+j} + \alpha - 1\right)}{r^n + y_{-k+j}x_{-2k+j}l_n(\beta A+B)}\right)$$

is convergent.

In view of (4) and (6), thus if $Ay_{-k+j}x_{-2k+j} + \alpha \neq 1$ then $\lim_{n\to\infty} x_{2kn+j} = a_{-2k+j}$. Similarly, we get $\lim_{n\to\infty} x_{2kn+k+j} = a_{-k+j}$.

iv) If $Bx_{-k+j}y_{-2k+j} + \beta \neq 1$ Similarly, we get $\lim_{n\to\infty} y_{2kn+j} = b_{-2k+j}, \lim_{n\to\infty} y_{2kn+j} = b_{-k+j}$.

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