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Delay-Dependent Robust Passive Control for Uncertain Discrete-Time Systems with Time Delays*

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ABSTRACT

This paper considers the problem of robust passive control for uncertain discrete systems with time-varying delays. We pay attention to designing a state feedback controller which guarantees the passivity of the closed-loop system for all admissible uncertainties. In terms of a linear matrix inequality, a sufficient condition for the solvability of this problem is presented and the explicit expression of the desired state feedback controller is given.

Keywords: Robust Passive Control; Discrete Systems; Time Delay

1. Introduction

In the past several years, much attention has been paid to the study of stability of systems with control input delay. Much of them is focused on the passivity analysis for classes of time-delay systems. Using classical definitions of passivity and positive realness, the conditions for a nonlinear system can be rendered passive via smooth state feedback, see [1,2]. The robust passive control problem for time-delay systems was dealt with in [3,4] via various approaches. The robust passivity synthesis problem for discrete-time-delay systems is investigated in [5,6], but all these time delays are constant. To the best knowledge of authors, the problem of robust passive control for discrete-time systems with time-varying delays has not been fully investigated, which is more complex.

In this paper, we deal with the problem of robust passive feedback control for discrete systems with parameter uncertainties and time-varying delays. The parameter uncertainties are assumed to be time-varying but norm-bounded. The purpose is to construct a state feedback controller such that the closed-loop system is strictly passive and obtain a delay-dependent condition for the solvability of the problem.

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2. Statement of the Problem

Consider the following uncertain discrete-time system with time-varying delays:

$$\begin{aligned} x(k+1) &= (A+\Delta A)x(k)+(A_d+\Delta A_d)x(k-\tau) \\ &\quad +(D_1+\Delta D_1)w(k) \end{aligned} \tag{2.1}$$

$$\begin{aligned} z(k) &= (E+\Delta E)x(k)+(E_d+\Delta E_d)x(k-\tau) \\ &\quad +(D_2+\Delta D_2)w(k) \end{aligned} \tag{2.2}$$

$$x(k)=\phi(k), k\in[-\bar{\tau},0] \tag{2.3}$$

where $x(k)\in R^n$ is the state, $z(k)\in R^q$ is the controlled output, $w(k)\in R^p$ the disturbance input which is assumed to belong to $l_2[0;\infty)$; τ is a positive integer representing the time-varying delay of the system, which satisfies the following assumption: $0\le\tau\le\bar{\tau}$. $\phi(k)$ is a real-valued initial function on $[-\bar{\tau},0]$; A, A_d, D_1, E, E_d and D_2 are known real constant matrices; $\Delta A, \Delta A_d, \Delta D_1, \Delta E, \Delta E_d$ and ΔD_2 are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta A & \Delta A_d & \Delta D_1 \\ \Delta E & \Delta E_d & \Delta D_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(k)\begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \tag{2.4}$$

where M_1, M_2, N_1, N_2 and N_3, are known real constant matrices and $F(k)$ satisfies:

$$F^{\mathrm{T}}(k)F(k)<I. \tag{2.5}$$

Our problem is to establish the passive control for systems (2.1)-(2.3) to determine the conditions. To this end, we introduce the following fact and related definition of passivity.

Lemma 2.1 Given constant symmetric matrices Σ_1, Σ_2, Σ_3, where $\Sigma_1 = \Sigma_1^{\mathrm{T}}$, and $0 < \Sigma_2 = \Sigma_2^{\mathrm{T}}$, then $\Sigma_1 + \Sigma_3^{\mathrm{T}}\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^{\mathrm{T}} \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^{\mathrm{T}} & \Sigma_1 \end{bmatrix} < 0 .$$

Lemma 2.2 Given constant matrices Σ_1, Σ_2, Σ_3 of appropriate dimensions with $\Sigma_1 = \Sigma_1^{\mathrm{T}}$. Then

$$\Sigma_1 + \Sigma_2 F(t)\Sigma_3 + \Sigma_3^{\mathrm{T}} F^{\mathrm{T}}(t)\Sigma_2^{\mathrm{T}} < 0.$$

where $F^{\mathrm{T}}(k)F(k) < I, \forall k$ if and only if for some scalar $\varepsilon > 0$

$$\Sigma_1 + \varepsilon^{-1}\Sigma_2\Sigma_2^{\mathrm{T}} + \varepsilon\Sigma_3^{\mathrm{T}}\Sigma_3 < 0.$$

Lemma 2.3 Let $y(K) = x(k) - x(k-1)$, then the following inequality holds for any matrices R, S_1, $S_2 \in R^{n\times n}$ and positive scalar $h > 0$:

$$-\sum_{k-\tau}^{k-1} y^{\mathrm{T}}(i)Ry(i) \le \xi^{\mathrm{T}}(k)\begin{bmatrix} S_1^{\mathrm{T}} + S_1 & -S_1^{\mathrm{T}} + S_2 \\ * & -S_2^{\mathrm{T}} - S_2 \end{bmatrix}\xi(k) + h\xi^{\mathrm{T}}(k)\begin{bmatrix} S_1^{\mathrm{T}} \\ S_2^{\mathrm{T}} \end{bmatrix} R^{-1}\begin{bmatrix} S_1^{\mathrm{T}} & S_2^{\mathrm{T}} \end{bmatrix}\xi(k).$$

Definition 2.1 The dynamical systems (2.1) - (2.3) is called passive if there exists a scalar $\beta \ge 0$ such that

$$\sum_{k=0}^{\infty} w^{\mathrm{T}}(k)z(k) \ge \beta,\ \forall \in L_2[0,\infty)$$

where β is some constant which depends on the initial condition of the system.

In addition, the systems (2.1)-(2.3) is said to be strictly passive if it is passive and $(D_2 + \Delta D_2) > 0$. In the sequel, we provide conditions under which a class of discrete-time linear dynamical systems with time-varying parameter uncertainties can be guaranteed to be strictly passive. First, we have the following result pertaining to the system (2.1)-(2.3).

3. Proof of Main Results

Theorem 3.1 The discrete-time systems with time delay (2.3) is strictly passive if there exist symmetric positive definite matrices P, R, Q and $M_1, M_2 \in R^{n\times m}$, such that the following LMI holds:

$$\Xi = \begin{bmatrix} \Xi_{11} & PA_d - S_1^{\mathrm{T}} + S_2 + \varepsilon N_1^{\mathrm{T}} N_2 & PD_1 - E^{\mathrm{T}} + \varepsilon N_1^{\mathrm{T}} N_3 & A^{\mathrm{T}}P & \tau A^{\mathrm{T}}R & \tau S_1^{\mathrm{T}} & PM_1 \\ * & -Q - S_2 - S_2^{\mathrm{T}} + \varepsilon N_2^{\mathrm{T}} N_2 & -E_d^{\mathrm{T}} + \varepsilon N_2^{\mathrm{T}} N_3 & A_d^{\mathrm{T}}P & \tau A_d^{\mathrm{T}}R & \tau S_2^{\mathrm{T}} & 0 \\ * & * & -D_2 - D_2^{\mathrm{T}} + \varepsilon N_3^{\mathrm{T}} N_3 & D_1^{\mathrm{T}}P & \tau D_1^{\mathrm{T}}R & 0 & -M_2 \\ * & * & * & -P & 0 & 0 & PM_1 \\ * & * & * & * & -\tau R & 0 & \tau RM_1 \\ * & * & * & * & * & -\tau R & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \qquad (3.1)$$

where $\Xi_{11} = PA + A^{\mathrm{T}}P + Q + S_1 + S_1^{\mathrm{T}} + \varepsilon N_1^{\mathrm{T}} N_1$.

Proof. Choose a Lyapunov function candidate for the system (2.1) - (2.3) as follows:

$$V(k) = x^{\mathrm{T}}(k)Px(k) + V_1(k) + V_2(k). \qquad (3.2)$$

where

$$V_1(k) = \sum_{\theta=-\tau+1}^{0}\sum_{i=k-1+\theta}^{k-1} y^{\mathrm{T}}(i)Ry(i),$$

$$V_2(k) = \sum_{i=k-\tau}^{k-1} x^{\mathrm{T}}(i)Qx(i),$$

$$y(k) = x(k+1) - x(k).$$

Now, by some calculations, we can get that

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= 2x^{\mathrm{T}}(t)Py(k) + y^{\mathrm{T}}(k)(P+\tau R)y(k) \\ &\quad - \sum_{i=k-\tau}^{k-1} y^{\mathrm{T}}(i)Ry(i) + x^{\mathrm{T}}(t)Qx(t) \\ &\quad - x^{\mathrm{T}}(t-\tau)Qx(t-\tau) \end{aligned} \qquad (3.3)$$

We define that $\eta(k) = col\{x(k), x(k-\tau), w(k)\}$, then have

$$y(k) = \Gamma_1\eta(k). \qquad (3.4)$$

From the Lemma 2.3, for $\forall S_1, S_2 \in R^{n\times n}$, we can have that

$$\begin{aligned} -\sum_{k-\tau}^{k-1} y^{\mathrm{T}}(i)Ry(i) \le& \begin{bmatrix} x^{\mathrm{T}}(k) \\ x(k-\tau) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} S_1^{\mathrm{T}} + S_1 & -S_1^{\mathrm{T}} + S_2 \\ * & -S_2^{\mathrm{T}} - S_2 \end{bmatrix} \begin{bmatrix} x^{\mathrm{T}}(k) \\ x(k-\tau) \end{bmatrix} \\ &+ \tau \begin{bmatrix} x^{\mathrm{T}}(k) \\ x(k-\tau) \end{bmatrix} \begin{bmatrix} S_1^{\mathrm{T}} \\ S_2^{\mathrm{T}} \end{bmatrix} R^{-1} \begin{bmatrix} S_1^{\mathrm{T}} & S_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} x^{\mathrm{T}}(k) \\ x(k-\tau) \end{bmatrix}. \end{aligned} \qquad (3.5)$$

We have (3.3) and (3.4) into (3.2), after some manipulation, then obtain the following inequality:

$$\begin{aligned}&\Delta V(k)-2z^{\mathrm{T}}(k)w(k)\\&=V(k+1)-V(k)-2z^{\mathrm{T}}(k)w(k)\\&=\eta^{\mathrm{T}}(t)\left(\Theta+\Gamma_1^{\mathrm{T}}P\Gamma_1+\tau\Gamma_1^{\mathrm{T}}R\Gamma_1+\tau\Gamma_2^{\mathrm{T}}R^{-1}\Gamma_2\right)\eta(t)\\&=\eta^{\mathrm{T}}(t)\tilde{\Theta}\eta(t)\end{aligned} \tag{3.6}$$

where

$$\Theta=\begin{bmatrix}PA+A^{\mathrm{T}}P+Q+S_1+S_1^{\mathrm{T}} & PA_d-S_1^{\mathrm{T}}+S_2 & PD_1-E^{\mathrm{T}}\\ * & -Q-S_2-S_2^{\mathrm{T}} & -E_d^{\mathrm{T}}\\ * & * & -D_2-D_2^{\mathrm{T}}\end{bmatrix}$$

$\Gamma_1=\begin{bmatrix}A & A_d & D_1\end{bmatrix}$, $\Gamma_2=\begin{bmatrix}S_1 & S_2 & 0\end{bmatrix}$.

If $\tilde{\Theta}<0$, then $\Delta V(k)-2z^{\mathrm{T}}(k)w(k)<0$, and from which it follows that

$$\begin{aligned}\sum_{j=k_0}^{k_f}w^{\mathrm{T}}(j)z(j)&>\frac{1}{2}\sum_{j=k_0}^{k_f}\Delta V(k)\\&=\frac{1}{2}\left[V(k_0)-V(k_f)\right]\end{aligned}. \tag{3.7}$$

Since $V(k)>0$ for $x\neq 0$ and $V(k)=0$ for $x = 0$, it follow as $k_f\to\infty$, that systems (2.1) - (2.3) is strictly passive. In view of Definition 2.1, the strictly passive condition is guaranteed if $\tilde{\Theta}<0$ and it can be expressed conveniently as

$$\begin{aligned}&\begin{bmatrix}PA+A^{\mathrm{T}}P+Q+S_1+S_1^{\mathrm{T}} & PA_d-S_1^{\mathrm{T}}+S_2 & PD_1-E^{\mathrm{T}}\\ * & -Q-S_2-S_2^{\mathrm{T}} & -E_d^{\mathrm{T}}\\ * & * & -D_2-D_2^{\mathrm{T}}\end{bmatrix}\\&+\Gamma_1^{\mathrm{T}}P\Gamma_1+\tau\Gamma_1^{\mathrm{T}}R\Gamma_1+\tau\Gamma_2^{\mathrm{T}}R^{-1}\Gamma_2<0.\end{aligned} \tag{3.8}$$

where $\Gamma_1=\begin{bmatrix}A & A_d & D_1\end{bmatrix}$, $\Gamma_2=\begin{bmatrix}S_1 & S_2 & 0\end{bmatrix}$.

Application of Lemma 2.1 to the above inequality, it puts into the following form:

$$\Pi=\begin{bmatrix}\Xi_{11} & PA_d-S_1^{\mathrm{T}}+S_2 & PD_1-E^{\mathrm{T}} & A^{\mathrm{T}}P & \tau A^{\mathrm{T}}R & \tau S_1^{\mathrm{T}}\\ * & -Q-S_2-S_2^{\mathrm{T}} & -E_d^{\mathrm{T}} & A_d^{\mathrm{T}}P & \tau A_d^{\mathrm{T}}R & \tau S_2^{\mathrm{T}}\\ * & * & -D_2-D_2^{\mathrm{T}} & D_1^{\mathrm{T}}P & \tau D_1^{\mathrm{T}}R & 0\\ * & * & * & -P & 0 & 0\\ * & * & * & * & -\tau R & 0\\ * & * & * & * & * & -\tau R\end{bmatrix}<0 \tag{3.9}$$

Substituting the uncertainty structure (2.5) into (3.9) and rearranging, we get the following inequality

$$\Pi+\begin{bmatrix}PM_1\\0\\-M_2\\PM_1\\\tau RM_1\\0\end{bmatrix}F(k)\begin{bmatrix}N_1^{\mathrm{T}}\\N_2^{\mathrm{T}}\\N_3^{\mathrm{T}}\\0\\0\\0\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}N_1^{\mathrm{T}}\\N_2^{\mathrm{T}}\\N_3^{\mathrm{T}}\\0\\0\\0\end{bmatrix}F^{\mathrm{T}}(k)\begin{bmatrix}M_1^{\mathrm{T}}P\\0\\-M_2^{\mathrm{T}}\\M_1^{\mathrm{T}}P\\\tau M_1^{\mathrm{T}}R\\0\end{bmatrix}^{\mathrm{T}}<0 \tag{3.10}$$

Then by Lemma 2.2, the inequality (3.10) holds if and only if for some $\varepsilon>0$

$$\Pi+\varepsilon^{-1}\begin{bmatrix}PM_1\\0\\-M_2\\PM_1\\\tau RM_1\\0\end{bmatrix}\begin{bmatrix}M_1^{\mathrm{T}}P\\0\\-M_2^{\mathrm{T}}\\M_1^{\mathrm{T}}P\\\tau M_1^{\mathrm{T}}R\\0\end{bmatrix}+\varepsilon\begin{bmatrix}N_1^{\mathrm{T}}\\N_2^{\mathrm{T}}\\N_3^{\mathrm{T}}\\0\\0\\0\end{bmatrix}\begin{bmatrix}N_1^{\mathrm{T}}\\N_2^{\mathrm{T}}\\N_3^{\mathrm{T}}\\0\\0\\0\end{bmatrix}^{\mathrm{T}}<0 \tag{3.11}$$

for all admissible uncertainties satisfying (2.4). On using Lemma 2.1 in (3.11), it becomes that $\Xi<0$ in (3.1). This completes the proof.

4. Robust Passive State Feedback Controller

We now build on the foregoing results by considering the passive control problem, that is, designing a state feedback controller to render the closed-loop time-delay system passive. Extending the system (2.1)-(2.3), we consider a class of time-delay systems of the form:

$$\begin{aligned}x(k+1)&=(A+\Delta A)x(k)+(A_d+\Delta A_d)x(k-\tau)\\&\quad+(D_1+\Delta D_1)w(k)+(B_1+\Delta B_1)u(k)\end{aligned} \tag{4.1}$$

$$\begin{aligned}z(k)&=(E+\Delta E)x(k)+(E_d+\Delta E_d)x(k-\tau)\\&\quad+(D_2+\Delta D_2)w(k)+(B_2+\Delta B_2)u(k)\end{aligned} \tag{4.2}$$

where $u(t)\in R^{n\times n}$ is the control input, B_1, B_2, are known real constant matrices; ΔB_1 and ΔB_2 are unknown matrices representing time-varying parametre uncertainties, and are assumed to be of the form:

$$\begin{bmatrix}\Delta B_1\\\Delta B_2\end{bmatrix}=\begin{bmatrix}M_1\\M_2\end{bmatrix}F(k)N_4. \tag{4.3}$$

Then the transformed system becomes

$$\begin{aligned}x(k+1)&=(A+\Delta A+B_1K+\Delta B_1K)x(k)\\&\quad+(A_d+\Delta A_d)x(k-\tau)+(D_1+\Delta D_1)w(k)\end{aligned} \tag{4.4}$$

$$\begin{aligned}z(k)&=(E+\Delta E+B_2K+\Delta B_2K)x(k)\\&\quad+(E_d+\Delta E_d)x(k-\tau)+(D_2+\Delta D_2)w(k)\end{aligned} \tag{4.5}$$

then we observe that

$$\begin{bmatrix} \Delta A+\Delta B_1K & \Delta A_d & \Delta D_1 \\ \Delta E+\Delta B_2K & \Delta E_d & \Delta D_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(k)\begin{bmatrix} N_1+N_4K & N_2 & N_3 \end{bmatrix}. \tag{4.6}$$

The following theorem establishes the main result.

Theorem 4.1 Consider the uncertain discrete-time delay system (4.4), (4.5). If there exists a positive scalar $\varepsilon > 0$, a real matrix Y, three symmetric positive definite matrices X, $\tilde{Q}$, R such that the following inequality holds:

$$\begin{bmatrix} \Phi_{11} & A_dX-W_1^{\mathrm{T}}+W_2 & \Phi_{13} & \Phi_{14} & \Phi_{15} & \tau W_1^{\mathrm{T}} & XN_1^{\mathrm{T}}+Y^{\mathrm{T}}N_4^{\mathrm{T}} \\ * & -\tilde{Q}-W_2-W_2^{\mathrm{T}} & -XE_d^{\mathrm{T}} & XA_d^{\mathrm{T}} & \tau XA_d^{\mathrm{T}} & \tau W_2^{\mathrm{T}} & XN_2^{\mathrm{T}} \\ * & * & \Phi_{33} & D_1^{\mathrm{T}}-\varepsilon M_2M_1^{\mathrm{T}} & \Phi_{35} & 0 & XN_3^{\mathrm{T}} \\ * & * & * & -X+\varepsilon M_1M_1^{\mathrm{T}} & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 \\ * & * & * & * & * & -\tau XRX & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{4.7}$$

where

$$\begin{aligned}
&\Phi_{11} = (AX+B_1Y)+(AX+B_1Y)^{\mathrm{T}}+\tilde{Q}+W_1+W_1^{\mathrm{T}}+\varepsilon M_1M_1^{\mathrm{T}}, \\
&\Phi_{13} = D_1-(EX+B_2Y)^{\mathrm{T}}-\varepsilon M_1M_2^{\mathrm{T}}, \\
&\Phi_{14} = (AX+B_1Y)^{\mathrm{T}}+\varepsilon M_1M_1^{\mathrm{T}}, \Phi_{15} = \tau(AX+B_1Y)^{\mathrm{T}}+\tau\varepsilon M_1M_1^{\mathrm{T}}, \\
&\Phi_{33} = -D_2-D_2^{\mathrm{T}}+\varepsilon M_2M_2^{\mathrm{T}}, \Phi_{35} = \tau D_1^{\mathrm{T}}-\tau\varepsilon M_2M_1^{\mathrm{T}}, \Phi_{55} = -\tau R^{-1}+\tau^2\varepsilon M_1M_1^{\mathrm{T}},
\end{aligned}$$

then the systems (4.4), (4.5) are strictly passive, and the state-feedback gain matrix is given by $K = YX^{-1}$.

Proof. Similar to Theorem 3.1.

Remark 4.1 It is noted that the matrix inequalities conditions in Theorem 4.1 are not LMIs. In order to solve the matrix inequalities conditions in Theorem 4.1, we can follow a similar line as in Lee *et al.* (2004) and Moon *et al.* (2001) to provide a nonlinear minimization problem subject to LMIs.

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