

Retraction Notice

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History

Expression of Concern:

yes, date: yyyy-mm-dd

no

Correction:

yes, date: yyyy-mm-dd

no

Comment:

This paper has inappropriately re-used content from a 2011 paper “Navigation in GPS-denied environments” published in IEEE.

This article has been retracted to straighten the academic record. In making this decision the Editorial Board follows [COPE's Retraction Guidelines](#). Aim is to promote the circulation of scientific research by offering an ideal research publication platform with due consideration of internationally accepted standards on publication ethics. The Editorial Board would like to extend its sincere apologies for any inconvenience this retraction may have caused.

Editor guiding this retraction: Prof. Theodore B. Trafalis (EiC of ICA)

Navigation Strategy with the Aid of the Theater Positioning System*

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ABSTRACT

The Theater Positioning System (TPS), which can perform in GPS-denied environments and can work with, or independently of, GPS systems, was presented in [3]. The principal difficulty in optimally combining this new system and GPS is caused by the somewhat unpredictable signal propagation of the TPS ground wave signal and thus results in less accurate performance when TPS works unaided in the environment while GPS is unavailable. A new navigation scheme that can provide an accurate position estimate for the GPS-denied user is developed. The essential of the scheme is to build a model to predict localization errors (transmission delays) caused by those propagation disturbances. In this work, we adopt three models: A state-space model which is represented by stochastic difference equations (SdEs), a linear combination of basis functions, and a special nonlinear model which is called the generalized linear model (GLM). We also propose a stochastic approximation method to solve the pseudo-range equations which does not require the prior knowledge of noise covariance matrices. A numerical example is provided and compared to illustrate the performance of the methods proposed in this paper.

Keywords: GPS; Theater Positioning System; State-Space Model; Basis Functions; GLM

1. Introduction

Global Positioning System (GPS) has been widely employed for both military and civil purposes, which makes it a necessity part in human lives. However, the nearly exclusive dependence on the GPS satellite constellation for accurate position information becomes a major operational concern for deploying US military and law-enforcement personnel. Such concern comes from the comparatively weakness of GPS, e.g., it may suffer from multipath and RF interference (intentional and unintentional), or even invasion from an adversary, which will result in inaccurate localization and further waste of time and capital. Differential GPS [4] is employed as an enhancement of GPS for improving the localization accuracy. A backup to GPS when GPS signals are out of reception is inertial navigation systems (INS) [5,6]. These units can be viewed as short-term backups to GPS but are in general costly, heavy, bulky, inaccurate, and/or power-hungry to be deployed except in a few specialized applications. Thus, a much more robust, inexpensive, and reliable GPS augmentation technique is badly needed for

dismounted personnel and most platforms.

A novel integrated radio-navigation system called Theater Positioning System (TPS) that is less expensive and offers far more consistent coverage than with GPS alone is developed in [3]. It operates at 90 - 110 kHz ground-wave radio-frequency (RF) and can be used with or without GPS. Like GPS, this enhanced low frequency (LF) component of the system uses spread-spectrum transmission for improved accuracy, exhibits a large processing gain for greater interference immunity, and thus has a significant advantage over conventional LORAN-C radio-navigation systems. It can be considered as a navigation system that uses terrestrial signal transmitters and a much lower RF compared with GPS. The low frequency property improves the capability of TPS signal penetration through obstacles such as buildings, canyons, and forests, where GPS may be disabled due to the signal blocking. For example, the first author's GPS did not work well or even provide a close navigation in Chicago downtown due to the skyscrapers and bridges. The independence of TPS allows it to provide localization information for users when GPS signal is denied.

*Parts of the paper has been published in conference versions [1,2].

The combination of GPS and TPS is explained as follows: In the usual operating mode, GPS serves as the principal positioning source. Continuity of their fixes are assured, since during the normal TPS tracking process, the TPS and GPS position data are continually compared. As long as the recent and current GPS signal quality is good, the displayed TPS fix will be automatically adjusted to overlay the GPS values; this is generally done to provide an ongoing in-situ calibration of the TPS signal propagation delay figures and thus “drag” the TPS fix in to match the GPS. If GPS suddenly fails to provide a clean or continuous fix, the TPS value will track the last good GPS coordinates. Once GPS signal integrity is restored for at least a few seconds and a new lock is satisfactorily obtained, the system will smoothly revert to the GPS fix and return to normal operation. In the event that GPS is jammed or otherwise unavailable for an extended period, TPS will be employed in a standalone mode to derive the unit’s fix, with a caution to the user that fix accuracies may be reduced. Another specific advantage of the TPS concept lies in the use of TPS as an anti-spoofing detector for GPS. For instance, if the TPS (presumed stable) and GPS planar fixes do not essentially coincide (*i.e.*, where the GPS solution is considerably off from the TPS fix), this could be an indicator of GPS receiver problems or of the presence of a spoofing signal.

As mentioned in the previous paragraph, the fix accuracies are reduced when TPS works alone. There are usually two reasons: 1) The lower radio frequency will limit the precision of the localization; 2) The transmission errors (η_k^i in Section 3.2) in the position estimation for TPS transmissions are caused by environmental factors such as the earth surface underlying the propagation path (e.g., water or land), as well as local variations in the surface types (e.g., terrain, soil types, vegetation). Unlike transmission errors in GPS such as ionospheric and tropospheric delays which have known models [7,8], the errors in TPS are harder to capture and difficult to be approximated by an exact model. The positioning accuracy thus degrades when TPS is used alone as lacking of an accurate delay model. In this work, we focus on the second point and build a dynamic model to capture the errors’ stochastic characteristics, by which they can be predicted, and thus the accuracies of TPS are improved. The proposed method can also be applied to the first case to improve the precision.

We also present an algorithm to solve the pseudorange equations in both GPS and TPS. Most of the techniques presented in the literature have applied Newton-Raphson [7,8], Kalman filter [9-11] or particle filter [12] methods to estimate the fixes. However, all of the algorithms either require the knowledge of noise statistics (Kalman filter and particle filter) or do not take the noise components into consideration (Newton-Raphson). An algo-

rithm based on stochastic approximation is proposed. The algorithm does not need any specific information on the noise variance but can still calculate the user’s position efficiently. Moreover, the algorithm uses less computation than those methods.

The contributions of this paper are summarized as follows:

- 1) A stochastic approximation (SA) approach is proposed to compute the position explicitly.
- 2) Three models: A dynamic state-space model, a linear combination of basis functions, and a GLM are introduced to model and predict transmission errors in TPS when GPS is absent.
- 3) A navigation scheme is put forward and demonstrated through the proposed error models and SA algorithm.

The paper is organized as follows. In Section 2, we briefly discuss the concept of TPS. In Section 3, navigation equations for GPS and TPS are presented. In Section 4, the detailed SA algorithm is proposed. In Sections 5, 6, and 7, methods of the state-space model, basis functions and GLM are presented, respectively. In Section 8, the navigation scheme is proposed. Finally, in Section 9, an example is given to illustrate the performance of the methods.

2. TPS Concept

The basic configuration of the TPS scheme is shown in **Figure 1** below. The spread-spectrum (SS) TPS signals originate from multiple widely-spaced, terrestrial transmitters. The radio-locating receiver acquires these transmitted signals and extracts the transmitter locations and times of transmission from data streams embedded in the respective SS signals, in a manner analogous to GPS units. The radio navigation solutions are then obtained by solving the usual systems of nonlinear pseudo-range equations by linearization techniques, Newton-Raphson estimation, Kalman filtering, particle filtering, multidimensional kernels or other means, but with downstream

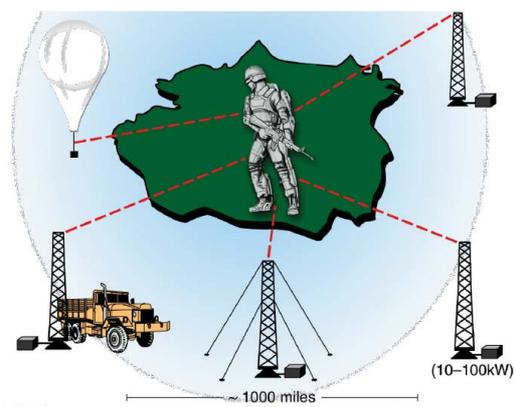


Figure 1. TPS deployment in large operational area [4].

corrections for the spherical-earth geometry and RF propagation factors governing the ground wave signals. However, there are several significant features of the TPS which differentiate it from GPS, including its operating frequency range (100 kHz), propagation modes, and signal security mechanisms [3].

TPS transmitters are typically, although not necessarily, deployed outside the main area of operations, in a reasonably regularly spaced array to provide favorable angles of reception from various transmitter locations (*i.e.*, “good system geometry” or “cuts”). The mathematical equations used to calculate the respective ranges from the TPS receiver to the transmitters in the area (which could easily exceed 1000 km in distance), called the pseudorange equations in GPS parlance, are similar to the GPS versions, except that the TPS transmissions are generally from stationary sources and, as such, do not need Doppler or relativistic corrections to the pseudorange values before computing the location solution in the receiver [3]. They do, however, require great-circle distance corrections for the ground-wave signal paths on the earth’s surface and adjustments to the propagation velocity values over the intervening terrain due to changes in the dielectric constant from varying soils, moisture content, etc. Like GPS, the TPS setup utilizes a precise common time base to provide highly accurate, stable time-of-day information for each transmitter. As in GPS, a stable clock in the TPS receiver permits faster initial signal acquisition and more accurate positioning via algorithms which incorporate strategic averaging among the various TPS signals [3].

3. Navigation Equations and Problem Formulation

In this paper, we consider the navigation of users on the surface of the earth that are subject to environmental conditions such as urban areas, very tough terrain, or in tropical or heavily forested regions. The calculation of the distance between the user and TPS transmitters should accommodate the ground-wave propagation and great-circle path distances. This is achieved by adjusting the equivalent speed of the wave for slower propagation along the earth’s surface; the curved-path distances may then be converted to the equivalent chord distances to utilize the normal rectilinear distance equations.

In the sequel, we first discuss the basic GPS pseudorange equation, then the corrected great-circle distance equation, after which an SA method is proposed to solve the pseudorange equations.

3.1. GPS Pseudorange Equation

The principle of GPS navigation can be represented as follows [7,8]: Each satellite is sending out signals with

the following content: I am satellite X, my position is Y and this information was sent at time Z. These orbital data (ephemeris and almanac data) are stored by the GPS receiver for later calculations. For the determination of its position, the GPS receiver compares the time when the signal was sent by the satellite with the time the signal was received. From this time difference the distance between receiver and satellite can be calculated. If data from other satellites are taken into account, the present position can be calculated by trilateration (the determination of a distance from three points). This means that at least three satellites are required to determine the position of the GPS receiver on the earth’s surface. The calculation of a position from 3 satellite signals is called a 2D-position fix (two-dimensional position determination); it is only two-dimensional because the receiver has to assume that it is located on the earth’s surface. By using four or more satellites, an absolute position in a three-dimensional space can be determined. A 3D-position fix also gives the height above the earth surface as a result.

The pseudorange of the i th transmitter at time k is given by the Equation [7]:

$$\rho_k^i = \rho_k^{T_i} + c(\zeta_k^i - \zeta_k^R) \quad (1)$$

where ρ_k^i is the pseudorange computed by the time difference between the receiver and the i th satellite and $\rho_k^{T_i}$ is the real range from the user to the i th GPS satellite at time k . The pseudorange contains two primary sources of error. One error is introduced by the receiver’s clock, which is denoted as ζ_k^R and called the receiver clock offset. This error remains the same in each pseudorange equation of each transmitter at time k . The other error is introduced in the transmission of GPS signal and denoted as ζ_k^i . This error can be modeled and approximated accurately [8], and thus is assumed known to the users. If we denote the i th satellite position by (X^i, Y^i, Z^i) relative to the center of the earth in Earth-Centered, Earth-Fixed (ECEF) coordinates, and the user’s position by (X_k, Y_k, Z_k) in the same coordinates, then the distance between the i th satellite and the user can be written as the non-linear expression:

$$\rho_k^{T_i} = \sqrt{(X_k - X^i)^2 + (Y_k - Y^i)^2 + (Z_k - Z^i)^2} \quad (2)$$

To solve the user positions and receiver clock offset, 4 satellites are needed to solve for $(X_k, Y_k, Z_k, \zeta_k^R)$ sufficiently.

3.2. TPS Great-Circle Distance

As mentioned above, the multilateration radiolocation algorithms for TPS are generally similar to those used in GPS except for the addition of great-circle corrections to accurately represent the lengths of the ground-wave propagation paths on the nearly spherical earth and (obvi-

ously) the deletion of the satellite almanac and ephemeris data. In most operational scenarios, the TPS transmitters will be locked to GPS time with very high-quality clocks. In addition, their locations will be pre-surveyed and known to fractions of a meter. The respective TPS data streams will thus provide all the information needed by the receiver (except for onboard-stored local propagation-correction tables) to accurately compute its position. Due to the finite conductivity of the earth's surface, and local variations due to surface types (*i.e.*, land or water), soil, moisture content, temperature, and (to a lesser extent) seasons, the average signal velocity must be reduced by very roughly 0.15%. In addition, the curved path on the earth's surface requires generic great-circle distance computations. As shown in **Figure 2**, the true range transmitted is along the spherical earth instead of the chord between A (the user) and T (the transmitter) and should be estimated by the great-circle distance.

The TPS ground wave follows the great-circle distance between two points on the earth's surface (assumed spherical), which can be computed by the following formula, where δ^i and φ^i are latitude and longitude, respectively and r is the radius of the earth (approximately 6371 km on average), then the great-circle distance d is approximately:

$$d(\delta^1, \varphi^1, \delta^2, \varphi^2) = r \cos^{-1} \left[\sin \delta^1 \sin \delta^2 + \cos \delta^1 \cos \delta^2 \cos(\varphi^1 - \varphi^2) \right] \quad (3)$$

The great-circle distance equation is employed to calculate the distance of a near-spherical earth path between the user and land-based TPS transmitters. In this paper, we consider only the navigation of the users near the surface, which means the height between the user and the earth surface is zero. For users at varying heights, the distances between the users and TPS transmitters do not quite follow the great-circle equations and should be calculated by taking the heights of the users into account.

Now assume there are M TPS transmitters. Then, the pseudorange equation at time k for TPS can be written similarly as that of GPS as follows:

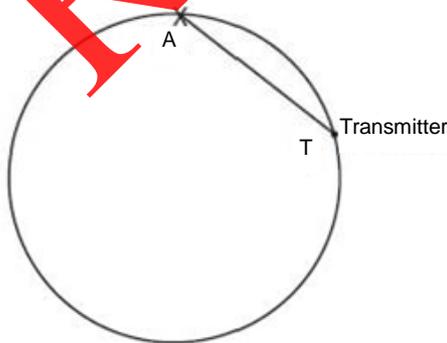


Figure 2. The users near the earth surface.

$$d_k^i = d_k^{T_i} + c_T (\eta_k^i - \eta_k^R) \quad (4)$$

where d_k^i is the pseudorange between the user and the i th TPS transmitter, and $d_k^{T_i}$ is the true range between the user and the i th ($1 \leq i \leq M$) TPS transmitter, which is approximated by the great-circle equation given above. η_k^i is the transmission error generated in the transmission of the TPS signal by the environment around the surface and is what we need to model. η_k^R is the receiver clock offset, equivalent to ζ_k^R in the GPS pseudorange equation.

3.3. Modeling and Identification of η_k^i

η_k^i is introduced through the transmission environment, which cause the change of the velocity of TPS transmission signal. Roughly, a 0.15% reduction of the velocity should be used, but this reduction seems more like an overall adjustment from experience. Thus, a more accurate model is needed to improve the estimation of the transmission errors. Unfortunately, it is difficult to model η_k^i accurately with a deterministic model (like GPS transmission errors) due to its characteristic irregularity.

In the sequel, we propose a navigation scheme for TPS that can improve the accuracy of modeling and identification based on statistics methods. In this work, we utilize past available GPS data to adjust TPS localization, *e.g.* model TPS transmission errors when GPS is available, then predict them when GPS is unavailable, and finally, obtain better users' positioning information. This procedure is illustrated in the **Figure 3**.

4. Stochastic Approximation Method

To obtain the position of the user exactly, we need to solve the GPS and TPS pseudorange equations explicitly. Numerous algorithms have already been proposed in the literature such as the Kalman Filter, Newton-Raphson method, particle filter, and the likes [7-12]. However, most of them require either the variance of the noise (Kalman Filter and particle filter) is known, or do not consider the effect of noise (*e.g.* Newton-Raphson). In this section, a stochastic approximation algorithm in [13], which is based on Kalman Filter is employed to compute the fixes explicitly. This method trains the Kalman gain

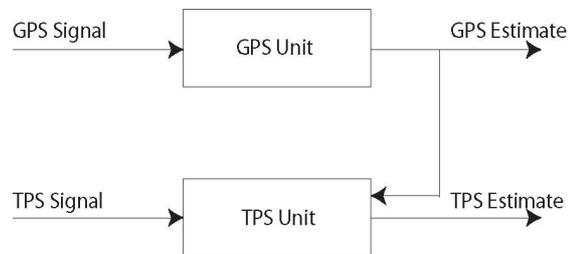


Figure 3. GPS and TPS localization.

matrix to its correct, steady-state form, when the plant noise and observation noise covariance matrices are unknown.

The dynamic model of the user's activity can be represented by the following discrete-time system [14]:

$$x_{k+1} = Ax_k + w_k \tag{5}$$

$$y_k = d(x_k) + v_k \tag{6}$$

where x_k is the state vector containing the longitude, latitude (or X, Y, Z fixes in ECEF coordinates) of the user at time k , velocities, and clock offsets; A is the corresponding system matrix; $d(x_k)$ is the pseudorange vector containing all the pseudoranges [as in (1) and (4)] sampled for each GPS/TPS transmitter and is a nonlinear function of x_k . For example, in TPS, x_k may be used to denote the longitude, latitude, and the clock offset of the user by $x_k = [\delta_k, \phi_k, \zeta_k^R]^T$, and $A = I_{4 \times 4}$ is an identity matrix. The state noise w_k represents the uncertainty in the movement of the user and v_k represents the sensor noise, both of which are assumed to be unmeasured noises with covariance W and V , respectively.

It is well known from extended Kalman filter theory that the posterior estimate of x_k is given by:

$$\hat{x}_{k+1} = A\hat{x}_k + K_{k+1}(y_{k+1} - d(A\hat{x}_k)) \tag{7}$$

where

$$K_{k+1} = P_{k+1|k} D_{k+1}^T (D_{k+1} P_{k+1|k} D_{k+1}^T + V)^{-1}$$

is the Kalman gain, where

$$P_{k+1|k} = A_{k+1}(I - K_k D_k) P_{k|k} A_{k+1}^T + W$$

and

$$D_k = \left. \frac{\partial d}{\partial x} \right|_{A\hat{x}_k}$$

Instead of the traditional Kalman gain, the stochastic approximation procedure provides a recursive gain adaptation algorithm in the form:

$$K_{k+1} = K_k + \mu_k \theta(K_k) \tag{8}$$

where μ_k is a decreasing sequence of real numbers and $\theta(K_k)$ is an unspecified stochastic vector that depends on K_k . One choice for $\theta(K_k)$ is $\theta(K_k) = A\hat{x}_{k+1}v_{k+1}^T$, and under certain conditions on μ_k in [13], K_{k+1} converges to the optimal Kalman gain.

The advantages of this SA algorithm over other algorithms are summarized as follows:

- 1) It does not assume knowledge of the noise covariance matrices;
- 2) The computation of its Kalman gain does not require the calculation of the estimation covariance, which can reduce the computation cost significantly over that of the Kalman filter;

- 3) Unlike Newton-Raphson, which needs N equations to solve for N unknowns, SA can estimate x_k accurately with a number of measurements smaller than the number of variables contained in the state x_k (partially observed).

5. State Space Model

In this section, we model the errors produced during the transmission of TPS via a dynamic state-space model which is based on SdEs. SdEs have been widely used previously to model discrete-time control systems and communication channels. For example, a mobile-to-mobile communication channels can be modeled as [15]:

$$\begin{aligned} x_{k+1}^c &= F_k^c x_k^c + G_k^c w_k^c \\ y_k^c &= H_k^c x_k^c + N_k^c v_k^c \end{aligned} \tag{9}$$

where x_k^c is the channel state; y_k^c is the measurement sampled at the output of the channel; w_k^c and v_k^c are noises; and F_k^c , G_k^c , H_k^c and N_k^c are parameters of the channel model.

The time-varying property of the parameters in (9) adapts dynamically to the variety of states. The noises w_k^c and v_k^c can also capture the range uncertainties introduced during the transmission. Due to these special characteristics, we propose to use the state-space model to track, estimate and predict the stochastic behavior of the transmission errors (η_k^i) (we suppress the superscript i below) in TPS transmissions. Consider the following time-invariant state-space predictor:

$$\begin{aligned} x_{k+1}^t &= F^t x_k^t + G^t e_k^t \\ \eta_k &= H^t x_k^t + e_k^t \end{aligned} \tag{10}$$

where η_k is the transmission error computed for each transmitter, and $e_k^t = \eta_k - H^t x_k^t$ is the error between the measured η_k and its prediction by the model. To estimate the parameters F^t , G^t , H^t , and x_k^t , the prediction error minimization (PEM) method [16] is employed. PEM estimates the parameters by minimizing a least-square cost function:

$$V_N = \sum_{k=1}^N (e_k^t)^T e_k \tag{11}$$

where N is the number of measurements stored for model identification. The details of the algorithm can be found in [16].

At each discrete-time instant k , the PEM algorithm estimates the parameters F^t, G^t and H^t from measurements available at k . With the estimated parameters and states of the model, the one step-ahead prediction of the transmission error can be computed as:

$$\hat{\eta}_{k+1} = \hat{H}_k^t \left(\hat{F}_k^t \hat{x}_k^t + \hat{G}_k^t \left(\eta_k - \hat{H}_k^t \hat{x}_k^t \right) \right) \tag{12}$$

where $\hat{\eta}_{k+1}$ denotes the predicted transmission error at $k+1$; $\hat{F}_k^t, \hat{G}_k^t, \hat{H}_k^t$ and \hat{x}_k^t are the parameters and state estimated by PEM at time k ; $\eta_k - \hat{H}_k^t \hat{x}_k^t$ is the prediction error at k ; $\hat{F}_k^t \hat{x}_k^t + \hat{G}_k^t (\eta_k - \hat{H}_k^t \hat{x}_k^t)$ is from the state evolution Equation in (10). The p -steps-ahead ($p > 1$) prediction of the transmission error can be computed as:

$$\hat{\eta}_{k+p} = \hat{H}_k^t \times (\hat{F}_k^t)^{p-1} (\hat{F}_k^t \hat{x}_k^t + \hat{G}_k^t (\eta_k - \hat{H}_k^t \hat{x}_k^t)) \quad (13)$$

where

$$(\hat{F}_k^t)^{p-1} (\hat{F}_k^t \hat{x}_k^t + \hat{G}_k^t (\eta_k - \hat{H}_k^t \hat{x}_k^t))$$

computes the predicted state after p steps from k .

The state-space model introduced in this section employs the PEM algorithm which numerically estimates the model parameters, however, such procedure may increase the computation cost. An autoregressive (AR) process can be viewed as a special case of the state-space model, where the parameters in the AR model can be estimated explicitly without involving numerical calculations. The details of the AR model can be found in the literature, e.g., [16]. The performance analysis of the AR model in our context is given in [2].

6. Basis Functions Method

In the state-space (or AR) model, the procedure of estimation is based on an implicit mathematical model of the process. In contrast to this, we employ the method of basis functions which is based on explicit model of parameter variation [17]. This method assumes that the measurements are governed by known functions, which are the so-called basis functions. It is commonly used in signal processing for signal decomposition. In this paper, we utilize this method as a comparison for the purpose of the modeling and prediction of transmission errors. In the sequel, we will introduce the procedure of this method.

Still assume there are N measurements that are available for model identification. Assume transmission errors η_k can be decomposed into an equation of basis functions below:

$$\eta_k = \sum_{j=1}^p A_j \phi_j(k), \quad k = 1, 2, \dots, N \quad (14)$$

where $\phi_j(k)$ is the basis function depending on time; A_j is the amplitude corresponding to each basis function. In this work, the basis functions are chosen as

$$\phi_j(k) = k^{(j/N)}, \quad \text{for } j = 1, \dots, p \quad (15)$$

With these basis functions, transmission errors vary with time, which represents the time characteristic of the errors. The amplitudes are estimated through the N measurements by the minimizing the following cost

function:

$$\hat{J}(\mathbf{A}) = \sum_{k=1}^N \left| \eta_k - \sum_{j=1}^p A_j \phi_j(k) \right|^2 \quad (16)$$

where $\mathbf{A} = [A_1, \dots, A_p]^T$ is the amplitude vector to be computed. Let $\boldsymbol{\eta} = [\eta_1, \dots, \eta_N]^T$, and define a $N \times p$ matrix

$$\begin{bmatrix} \phi_1(1) & \phi_2(1) & \dots & \phi_p(1) \\ \phi_1(2) & \phi_2(2) & \dots & \phi_p(2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(N) & \phi_2(N) & \dots & \phi_p(N) \end{bmatrix} \quad (17)$$

Thus, the amplitude vector \mathbf{A} can be computed by [17]

$$\mathbf{A} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T \boldsymbol{\eta} \quad (18)$$

To predict the future transmission errors in the absence of GPS data, we have

$$\hat{\eta}_k = \sum_{j=1}^p \hat{A}_j \phi_j(k), \quad \text{for } k > N \quad (19)$$

which will then be used in TPS localization.

7. Generalized Linear Model (GLM)

In the previous section, we introduce basis functions method which can be viewed as linear regression. In this section, we introduce a nonlinear model approach called generalized linear model for modeling purposes in TPS transmission. As its name stands, it generalizes linear regression by allowing the linear model to be related to the response variable via a link function and by allowing the magnitude of the variance of each measurement to be a function of its predicted value [18].

In a GLM, the responses Y are assumed to be generated from a particular distribution in the exponential family, including normal, binomial and poisson distributions, etc. The mean, μ , of the distribution, depends on a function of the linear combination of the independent variables X through the following expression [18]:

$$EY = \mu = g^{-1}(X\beta) \quad (20)$$

where $X\beta$ is called the linear predictor, a linear combination of unknown parameters β ; g is the link function. The unknown parameters, β , are typically estimated with maximum likelihood, maximum quasi-likelihood, or Bayesian techniques.

The GLM consists of three elements, including a probability distribution to describe the responses Y , a linear combination of X , $z = X\beta$, and a link function g such that $z = g(\mu)$. In this work, we consider the transmission errors η_k^i in TPS transmission as re-

sponses Y , and stipulate X to be independent variables, *i.e.*, the transmission errors from 1 to k are denoted by Y , while X is chosen as the standard Brownian motion $\{B_k\}_{k \geq 0}$, which is one of the most popular and fundamental stochastic processes. The relationship can then be set based on X, Y and β can be estimated. The estimated β is further used for the prediction of transmission errors when TPS works alone. More details of GLM can be found in [18] and will not be discussed here.

8. TPS Navigation Scheme Algorithm (NSA)

In this section, a navigation scheme to improve accuracy is introduced.

Assume GPS and TPS data are available from time 1 to time n ; also, from time n on, only TPS data is accessible. The procedure to navigate with TPS only at time $n+1$ is given as follows:

1) Compute the user's position (X_k, Y_k, Z_k) and ζ_k^R for each $k=1, \dots, n$ using (1) and (2) through the SA algorithm. Then convert (X_k, Y_k, Z_k) to latitude and longitude. As the user is assumed near the earth's surface, the height of the user H is estimated as 0.

2) Plug the latitude and longitude back into (3) to obtain $d_k^{T_i}$ terms for each $k=1, \dots, n$ and each transmitter.

3) As the receiver clock offset is constant in one single time slot, we can assume that $\eta_k^R = \zeta_k^R$. Then η_k^i can be computed in (4) for each k and each i (note $d_k^{T_i}$ are measured by TPS).

4) Build a statistical model for each $\{\eta_k^i\}_{k=1}^{k=n}$ and predict η_{n+1}^i for each transmitter using the models introduced in section 5, 6 and 7.

5) Plug η_{n+1}^i back into (4) and obtain the measurement equations. Together with the system Equation (5), the state x_{n+1} can now be estimated accurately by the SA algorithm.

After time $n+1$, continue to compute the user's positions with the previous algorithm when the GPS signal is lost. Once GPS becomes available again, update the model with the new η_k^i values computed from the latest GPS measurement.

Remark 1: The ground wave signal passes through a variety of different environments, which indicates that a fixed model can not capture the stochastic characteristics during the transmission. In different environments, *e.g.* canyon, forest, etc., the transmission errors introduced into the system are distinct, while the proposed models are well suited to such situations as they can be updated with new measurements. For example, once the signal transmission surroundings are changed, these models can be improved with the new incoming GPS measurements for the new surroundings.

Remark 2: For the users with varying heights ($H \neq 0$),

the distance equation to compute the true range should be updated. However, it is difficult to determine a unique equation in this case, as the user may be below the average earth altitude (*e.g.*, a canyon) or on a hill, where the equations are different (see **Figure 4**). In a practical situation, the range may be approximated by the great-circle equation. However, once the distance equation is altered for varying heights, the navigation scheme proposed in this paper is still applicable to the new distance equation.

9. Numerical Example

In this section, we present an example to illustrate the performance of the navigation algorithm proposed in this paper. The simulation result is based on MATLAB.

Assume $N = 3$ TPS transmitters are located with latitude and longitude pairs $(38.3127^\circ, 115.6443^\circ)$, $(39.2763^\circ, 116.0855^\circ)$, $(37.6414^\circ, 114.3173^\circ)$. The initial position of the user in ECEF coordinates is $(-2.172 \times 10^6, 4.390 \times 10^6, 4.074 \times 10^6)$.

The user is assumed to move along the earth's surface randomly. Thus, for convenience but without loss of generality, the distance equation can be written as (5) and (6), where $A = I_{4 \times 4}$, where $I_{4 \times 4}$ denotes the 4×4 identity matrix, the state vector $x_k^T = [\delta_k, \varphi_k, \eta_k^R]^T$ for TPS and $x_k^T = [X_k, Y_k, Z_k, \zeta_k^R]^T$ for GPS. The movement of the user w_k is simulated by a Gaussian random variable with mean 0.000001° and standard deviation 0.0000002° on both latitude and longitude.

From time 1 to 50, when both GPS and TPS data are available, $\{x_k^G\}_{k=1}^{50}$ are computed by the SA algorithm and then $\{\eta_k^i\}_{k=1}^{50}$ can be obtained by following the NSA described in section 8. From times 51 to 150, GPS is denied and only TPS is available. A scalar state-space model is employed to model $\{\eta_k^i\}_{k=1}^{k=50}$, and then $\{\eta_k^i\}_{k=51}^{k=150}$ are predicted by this model using the algorithm proposed in Section 5. Next, the positions of the user are estimated by the SA algorithm from time 51 to 150. The differences between real fixes and estimated

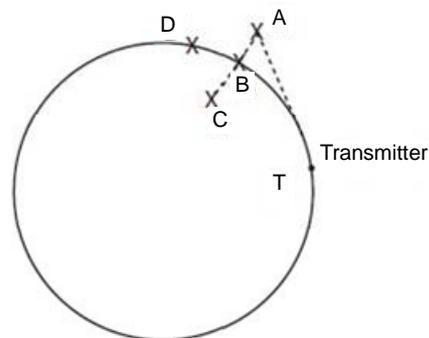


Figure 4. The users with varying heights.

fixes of all coordinates are presented in **Figure 5** (shown in ECEF coordinates for the sake of comparison). It is obvious that the positions estimated by the proposed navigation scheme are close to the real ones since the

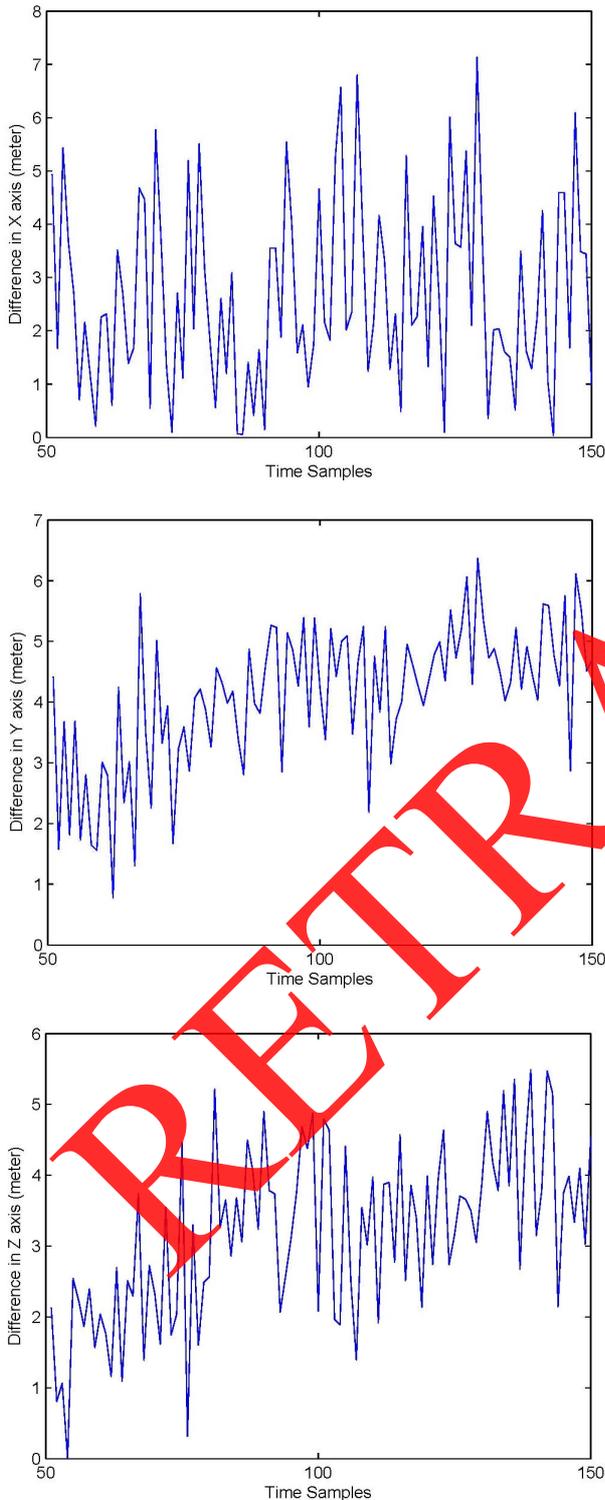


Figure 5. Position estimation errors in ECEF coordinates using state-space model.

differences between the estimated and the true fixes are small. The deviations from true positions are bounded by 8 m, 7 m and 6 m on each axis, respectively. The percentage of the error between actual $\{\eta_k^i\}$ and predicted ones $\{\hat{\eta}_k^i\}$ ($\{|\eta_k^i - \hat{\eta}_k^i|/\eta_k^i\}$) by the state-space model are plotted in **Figure 6**. These plots demonstrate that the proposed state-space model can predict the transmission errors with small errors.

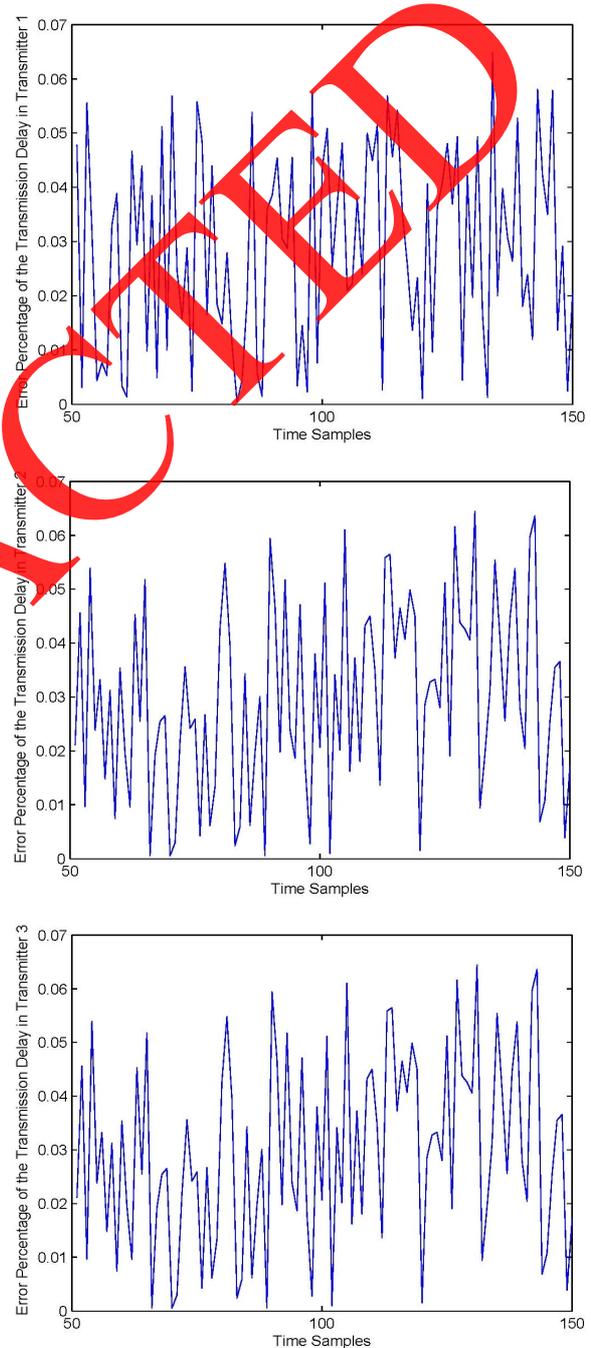


Figure 6. Error percentage of $\{\eta_i^k, i=1,2,3\}$ predicted by the state-space model.

Similarly, to illustrate the performance of the basis functions method, choose $p = 3$, bases in (15) to run the same simulation. The simulation results are shown in **Figures 7 and 8**. For GLM, the independent variables X corresponding to the measurement y_k are gene-

rated by normal distribution $N(0, k)$ and the simulation results are shown in **Figures 9 and 10**. Note although the navigation schemes based on the basis Functions and GLM both generate close estimation fixes,

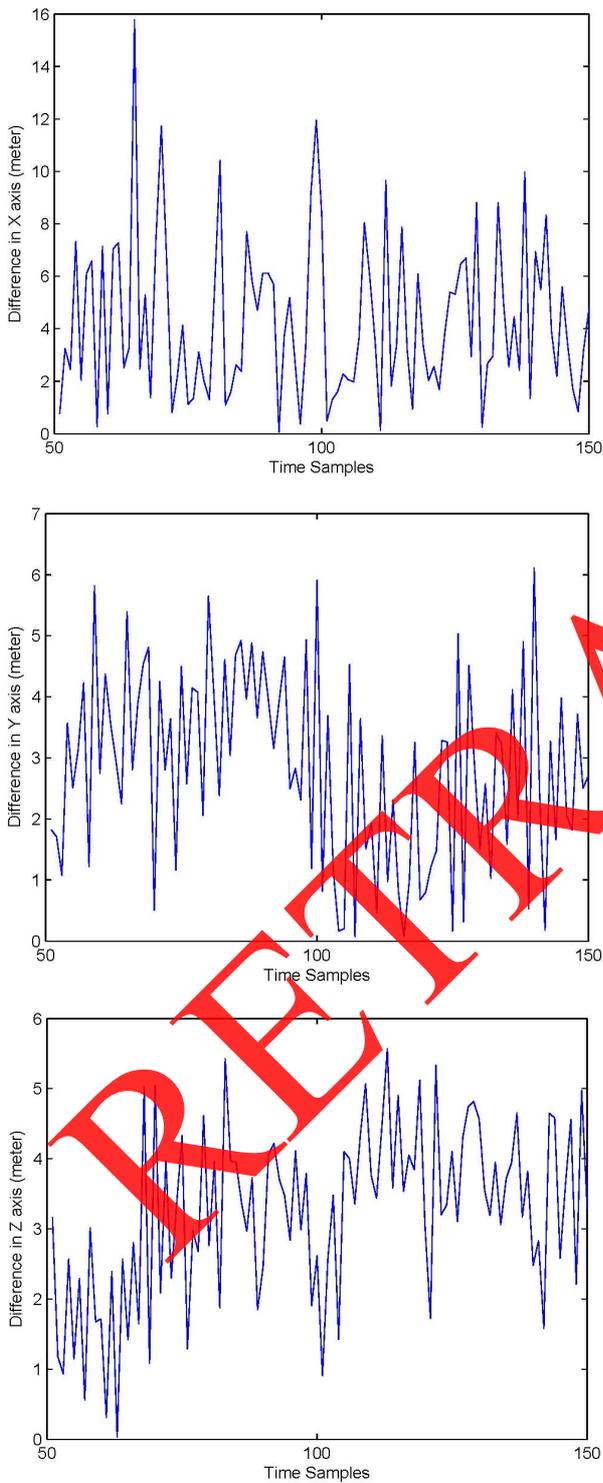


Figure 7. Position estimation errors in ECEF coordinates by basis functions.

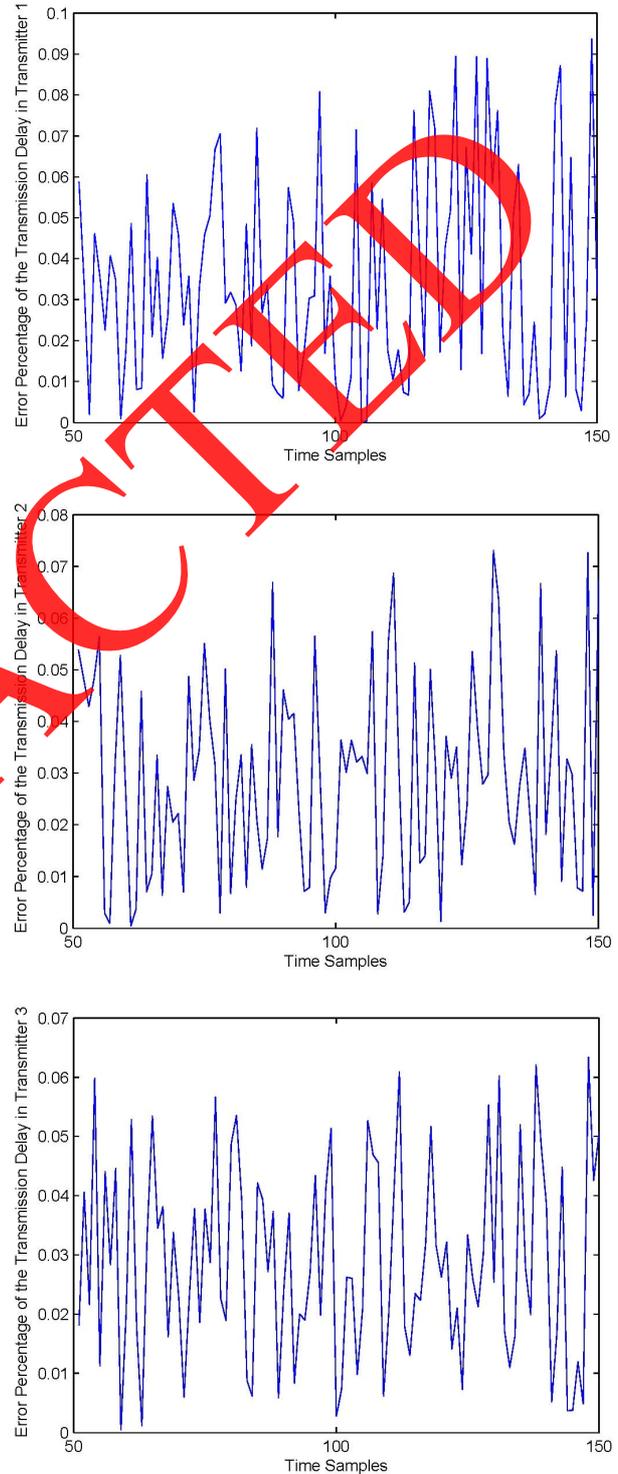


Figure 8. Error percentage of $\{\eta_i^k, i = 1, 2, 3\}$ predicted by basis functions.

the scheme based on the state-space model and GLM clearly offer better localization performances for this data set.

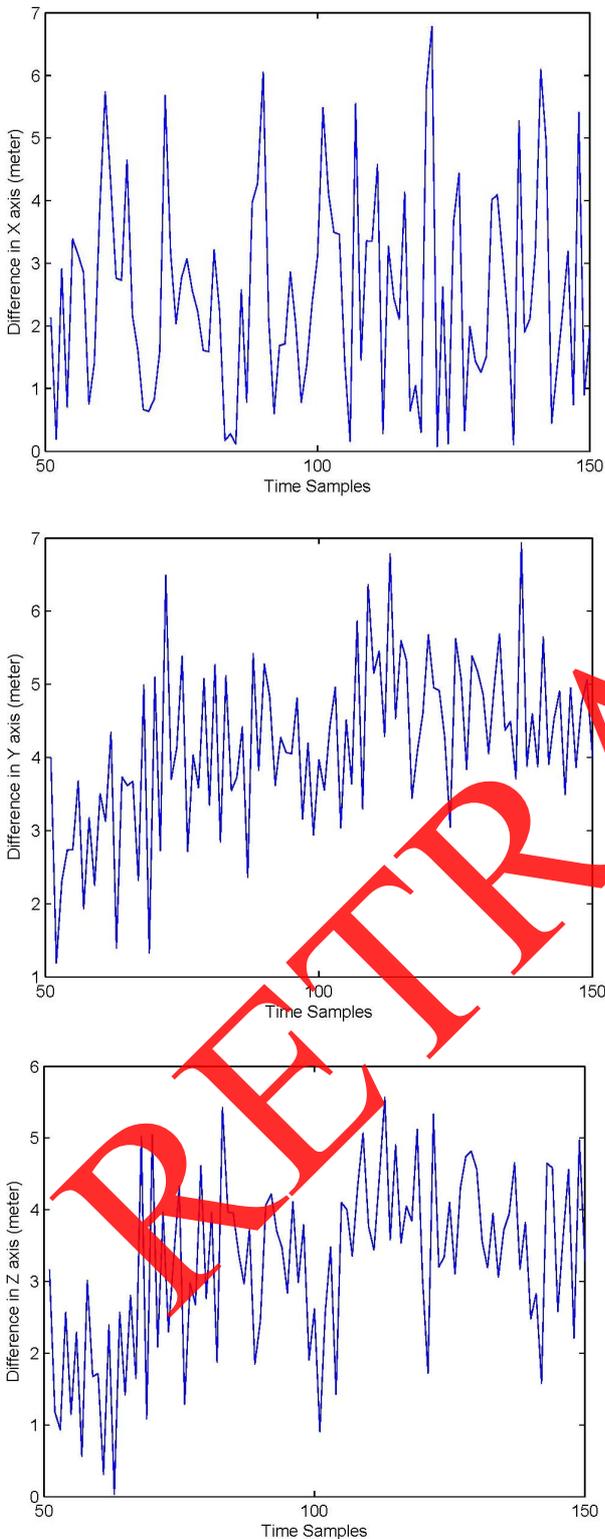


Figure 9. Position estimation errors in ECEF coordinates by the GLM.

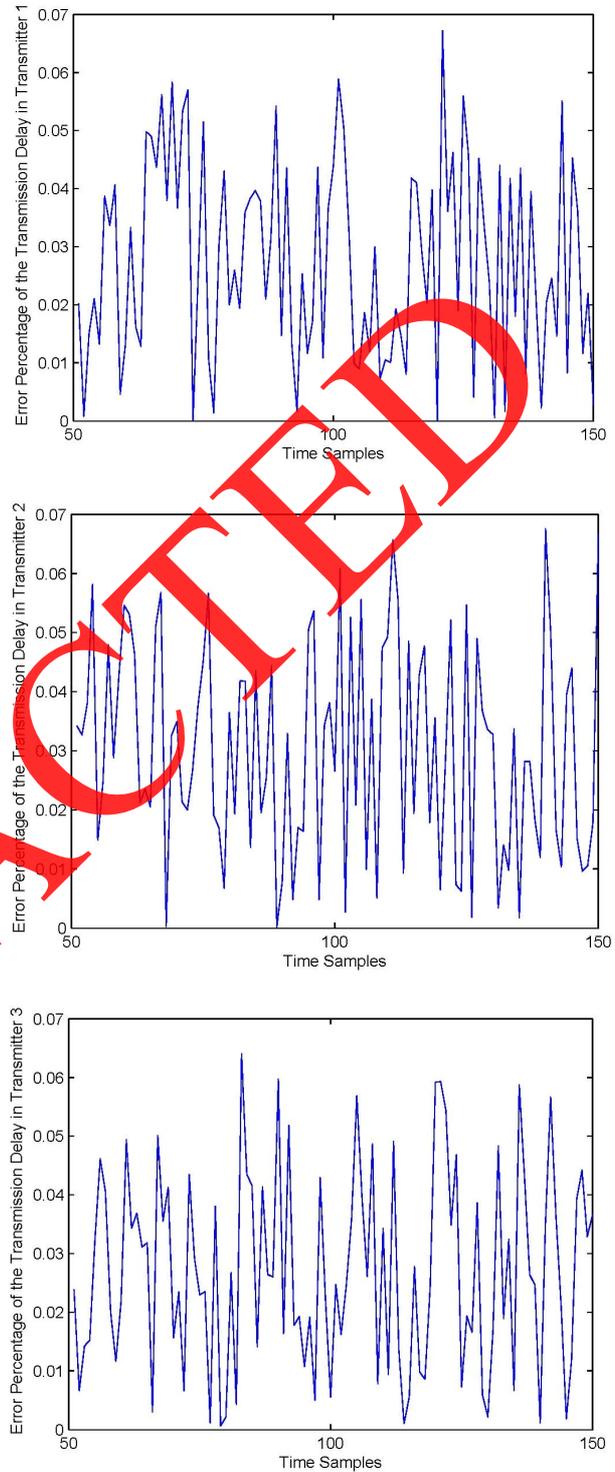


Figure 10. Error percentage of $\{\eta_i^k, i = 1, 2, 3\}$ predicted by the GLM.

10. Conclusion

In this paper, we have presented a new navigation system, the Theater Positioning System (TPS), which is largely intended to be used as a backup in GPS-denied environ-

ments. We have considered the user moving along the earth's surface and have employed three models: a state-space model, an AR process and a GLM model, to predict the error generated by environmental delays in the transmission, thus improving the estimation accuracy of TPS fixes. We have also proposed a stochastic approximation algorithm to solve the pseudorange equations. An example was provided to demonstrate that the estimation performances of all the models are quite satisfactory. The simulation results show that the state-space provides a better localization performance than the other two models. Future work will focus on more complex nonlinear models to further improve the accuracy.

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