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An Interval-parameter Fuzzy Robust Nonlinear Programming Model for Water Quality Management*

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ABSTRACT

Planning for water quality management is important for facilitating sustainable socio-economic development; however, the planning is also complicated by a variety of uncertainties and nonlinearities. In this study, an interval-parameter fuzzy robust nonlinear programming (IFRNP) model was developed for water quality management to deal with such difficulties. The developed model incorporated interval nonlinear programming (INP) and fuzzy robust programming (FRP) methods within a general optimization framework. The developed IFRNP model not only could explicitly deal with uncertainties represented as discrete interval numbers and fuzzy membership functions, but also was able to deal with nonlinearities in the objective function.

Keywords: Water Quality Management; Interval Programming; Fuzzy Robust Programming; Nonlinear Programming; Uncertainty

1. Introduction

Water is one of the most important natural resources, and can provide support to human survival, sustainable socio-economic development, and ecosystem preservation. However, in recent years, due to speedy population growth and rapid socio-economic development throughout the world, many challenging water resources problems, such as water quality deterioration and water shortage, have caused widespread concerns. Water quality deterioration can lead to various adverse impacts on human health, environment, and industrial and agricultural production. Therefore, effective planning for water quality management is desired. However, water quality management is inevitably complicated since it involves in a variety of uncertainties and nonlinearities. For example, dynamic interactions, associated with a variety of uncertainties, exist between pollutant loadings and receiving waters [1,2]; uncertainties exist in many system components, and can affect the related decision processes [3]; moreover, representation of system costs for water quality management involves a number of nonlinear functions for projecting the interrelationships between environment and economy [4]. Such complexities lead to difficulties in formulating and solving the resulting optimization problems.

As a result, various research efforts were made for dealing with the above difficulties in water quality management through interval nonlinear programming (INP) [5], stochastic nonlinear programming (SNP) [6], and fuzzy nonlinear programming (FNP) [7]. The uncertainties were expressed as interval ranges, probability density functions, and fuzzy membership functions, respectively. In INP models, interval numbers are acceptable for the uncertain inputs under an implicit distribution assumption, and the distribution information for model parameters is not required. SNP models can deal with various probabilistic uncertainties; however, the increased data requirements for specifying the parameters' probability distributions may affect their practical applicability. FNP models can facilitate the analysis of systems with uncertainties being derived from vagueness or fuzziness, and be suitable for the situations that the uncertainties can not be expressed as interval numbers and probability density functions. Although each method can prove effectiveness in dealing with uncertainties expressed in a single format, it is confronted with difficulties when multiple uncertainties exist. In real-world problems, water quality management is associated with various uncertainties, and multiple formats of uncertainties need be investigated in

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the processes of problem definition as well as model formulation and solution. Therefore, an integrated inexact nonlinear programming model is desired for water quality management.

For water quality management problems, many studies for integrated inexact nonlinear programming models have been made. For instance, Li *et al.* [8] developed an interval-fuzzy two-stage quadratic programming method, which incorporated interval-parameter programming, two-stage stochastic programming, and fuzzy quadratic programming with a general optimization framework; Luo *et al.* [9] proposed an inexact two-stage stochastic nonlinear programming model, where uncertainties were expressed as intervals numbers and probability density functions; Qin *et al.* [4] developed an interval-parameter fuzzy nonlinear programming (IFNP) model, which was a hybrid of INP and FNP. IFNP model is effective in dealing with uncertainties expressed as discrete intervals and/or fuzzy membership functions; however, it is incapable of dealing with fuzziness in the left-hand-side coefficients of the constraints. In contrast, fuzzy robust programming (FRP) can effectively reflect the conditions when uncertainties in both left- and right-hand-side coefficients of the constraints are only expressed as fuzzy membership functions [10].

Therefore, as an extension of previous studies, an interval-parameter fuzzy robust programming (IFRNP) model is developed for water quality management, where techniques of INP and FRP are incorporated within a general optimization framework. The developed IFRNP model can address multiple uncertainties associated with nonlinear functions.

2. Modeling Formulation

Planning for water quality management is related to many socio-economic and environmental factors. The IFRNP model is formulated to minimize the system cost of wastewater treatment, and it is consist of a nonlinear objective function and a set of water quality constraints.

2.1. Water Quality Simulation Model

Previously, many water quality simulation models, such as the Streeter-Phelps, O'Conner, Dobbins and Thomas models, have been developed into useful tools for supporting environmental management. In this study, the Streeter-Phelps model is used to quantify water quality constraints related to biological oxygen demand (BOD) and dissolved oxygen (DO) discharges as well as their concentrations in stream waters.

According to conservation of mass, the basic equations of the Streeter-Phelps model can be written:

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -K_d L \tag{1a}$$

$$\frac{\mathrm{d}O}{\mathrm{d}t} = -K_d L + K_a\left(O_s - O\right) \tag{1b}$$

where L and O are the ultimate BOD and DO concentrations, respectively; O_s is the saturated DO concentration; K_d and K_a are the first-order deoxygenation and reaeration rate constants, respectively; t is the mean flow time. So the solutions of Equations (1a) and (1b) are:

$$L = L_0 \mathrm{e}^{-K_d t} \tag{2a}$$

$$O = O_s - \left(O_s - O_0\right)\mathrm{e}^{-K_a t} - \frac{K_d L_0}{K_a - K_d}\left(\mathrm{e}^{-K_d t} - \mathrm{e}^{-K_a t}\right) \tag{2b}$$

where L_0 and O_0 are BOD and DO concentrations at the starting point of the stream system, respectively.

Segmentation is necessary since a number of wastewater discharge outlets scatter along the stream, with temporal and spatial variations of their loadings. Water quality at each section is affected by various sources from the upper stream.

According to the previous study [4], a matrix expression for water quality prediction can be established:

$$\mathbf{L}_2 = \mathbf{UL} + \mathbf{m} \tag{3a}$$

$$\mathbf{O_2} = \mathbf{VL} + \mathbf{RO} + \mathbf{n} \tag{3b}$$

where **U**, **V** and **R** are $n \times n$ transformation matrixes; **m** and **n** are n-dimensional transformation vectors; **L** and **O** are n-dimensional transformation vectors associating with the observed BOD and DO loads in wastewater sources, respectively; $\mathbf{L_2}$ and $\mathbf{O_2}$ are n-dimensional transformation vectors associating with the predicted BOD and DO levels at different stream segments, respectively.

2.2. Optimization Model Formulation

The operating cost functions for handling different types of wastewater have different expressions. They are normally expressed in various empirical formulations [4]. In China, the most widely used function for reflecting the operational cost of a wastewater treatment unit can be expressed:

$$C = K_1 Q^{K_2} + K_3 Q^{K_2} \eta^{K_4} \tag{4}$$

where C is the wastewater treatment cost; Q is the wastewater discharge rate; η is the wastewater treatment efficiency, $0 \le \eta \le 1$; K_1 and K_3 are the cost-function coefficients, $K_1, K_3 > 0$; K_2 is the economy-of-scale index of treatment cost, $0 < K_2 < 1$; K_4 is the scale index of treatment efficiency, $K_4 > 1$.

For a water quality management system containing n industries (*i.e.*, wastewater discharger sources), a deterministic optimization model can be formulated as follows:

$$\mathrm{Min}\, f = \sum_{i=1}^{n}\left[K_{1i}\left(Q_i\right)^{K_{2i}} + K_{3i}\left(Q_i\right)^{K_{2i}}\left(\eta_i\right)^{K_{4i}}\right] \tag{5a}$$

Subject to:

$$Q_i L_i \left(1-\eta_i\right) \le T_{Bi},\ \forall i \quad (5b)$$

$$\sum_{i=1}^{n}\left[U_{ji} L_i \left(1-\eta_i\right)\right] + m_j \le L_{Cj},\ \forall j \quad (5c)$$

$$\sum_{i=1}^{n}\left[V_{ji} L_i \left(1-\eta_i\right)\right] + R_{ji} O_i + n_j \ge L_{Dj},\ \forall j \quad (5d)$$

$$0 \le \eta_i \le 1,\ \forall i \quad (5e)$$

where f is the wastewater treatment cost; i denotes the name of wastewater discharge sources; j denotes the index of stream segments. U_{ji}, V_{ji} and R_{ji} are the elements of transformation matrices; m_j and n_j are the elements of transformation vectors; L_i and O_i are the elements of raw BOD and DO concentration vectors, respectively; T_{Bi} are the elements of BOD discharge allowances vectors; L_{Cj} and L_{Dj} are the elements of water quality standards vectors for BOD and DO, respectively.

Model (5a)-(5e) describes the relationships between economical and environmental factors involved in water quality management. However, a number of parameters in the optimization model may not be deterministic due to the existence of uncertainties [11]. Some parameters, such as wastewater discharge rate and wastewater treatment efficiency, may be represented as discrete intervals; raw BOD and DO concentrations in wastewater and environmental requirements for BOD and DO discharge may be expressed as fuzzy values, where the L-R fuzzy membership functions are taken for the fuzzy boundaries. So an IFRNP model is formulated:

$$\mathrm{Min}\, f^{\pm} = \sum_{i=1}^{n}\left[K_{1i}\left(Q_i^{\pm}\right)^{K_{2i}} + K_{3i}\left(Q_i^{\pm}\right)^{K_{2i}}\left(\eta_i^{\pm}\right)^{K_{4i}}\right] \quad (6a)$$

Subject to:

$$Q_i^{\pm} \tilde{L}_i \left(1-\eta_i^{\pm}\right) \le \tilde{T}_{Bi},\ \forall i \quad (6b)$$

$$\sum_{i=1}^{n}\left[U_{ji}^{\pm} \tilde{L}_i \left(1-\eta_i^{\pm}\right)\right] + m_j^{\pm} \le \tilde{L}_{Cj},\ \forall j \quad (6c)$$

$$\sum_{i=1}^{n}\left[V_{ji}^{\pm} \tilde{L}_i \left(1-\eta_i^{\pm}\right)\right] + R_{ji}^{\pm} \tilde{O}_i + n_j^{\pm} \ge \tilde{L}_{Dj},\ \forall j \quad (6d)$$

$$0 \le \eta_i^{\pm} \le 1,\ \forall i \quad (6e)$$

where $f^{\pm}$, $Q_i^{\pm}$ and $\eta_i^{\pm}$ denote a set of interval numbers with deterministic lower and upper bounds; $U_{ji}^{\pm}$, $V_{ji}^{\pm}$ and $R_{ji}^{\pm}$ denote the elements of intervals matrixes; $m_j^{\pm}$ and $n_j^{\pm}$ denote the elements of intervals vectors; $\tilde{L}_i$, $\tilde{O}_i$, $\tilde{T}_{Bi}$, $\tilde{L}_{Cj}$ and $\tilde{L}_{Dj}$ denote the elements of a set of fuzzy vectors.

3. Solution Methods

The operating cost function (6a) can be generalized into:

$$f^{\pm} = \sum_{i=1}^{n}\left[a_i^{\pm} + b_i^{\pm}\left(\eta_i^{\pm}\right)^{K_{4i}}\right] \quad (7)$$

According to the previous study [12], by taking the method of the piecewise linearization, Equation (7) can be converted into:

$$f^{\pm} = \sum_{i=1}^{n}\left[a_i^{\pm} + \sum_{t=1}^{t_0} s_{it}^{\pm} y_{it}^{\pm}\right] \quad (8a)$$

$$y_{it}^{\pm} = \begin{cases} \eta_{it}^{\pm} - \eta_{i,\,t-1}^{\pm} & \left(\eta_i^{\pm} > \eta_{it}^{\pm}\right) \\ \eta_i^{\pm} - \eta_{i,\,t-1}^{\pm} & \left(\eta_{i,\,t-1}^{\pm} < \eta_i^{\pm} < \eta_{it}^{\pm}\right), \\ 0, & \left(\eta_i^{\pm} < \eta_{i,\,t-1}^{\pm}\right) \end{cases} \quad \sum_{t=1}^{t_0} y_{it}^{\pm} = \eta_i^{\pm} \quad (8b)$$

where $s_{it}^{\pm}$ and $y_{it}^{\pm}$ are a set of interval numbers associating with the relevant slopes of straight line and the wastewater efficiency, respectively; t is the number of segments. So models (6a)-(6e) can be converted into:

$$\mathrm{Min}\, f^{\pm} = \sum_{i=1}^{n}\left[K_{1i}\left(Q_i^{\pm}\right)^{K_{2i}} + \sum_{t=1}^{t_0} p_{it}^{\pm} z_{it}^{\pm}\right] \quad (9a)$$

Subject to:

$$Q_i^{\pm} \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^{\pm}\right) \le \tilde{T}_{Bi},\ \forall i \quad (9b)$$

$$\sum_{i=1}^{n}\left[U_{ji}^{\pm} \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^{\pm}\right)\right] + m_j^{\pm} \le \tilde{L}_{Cj},\ \forall j \quad (9c)$$

$$\sum_{i=1}^{n}\left[V_{ji}^{\pm} \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^{\pm}\right)\right] + R_{ji}^{\pm} \tilde{O}_i + n_j^{\pm} \ge \tilde{L}_{Dj},\ \forall j \quad (9d)$$

$$z_{it}^{\pm} \ge 0,\ \forall i, t \quad (9e)$$

where $p_{it}^{\pm}$ and $z_{it}^{\pm}$ are a set of interval numbers associating with the relevant slopes of straight line and the wastewater efficiency, respectively.

This IFRNP problem is converted into an interval-parameter fuzzy robust linear programming (IFRLP) problem through the piecewise linearization approach. Then the IFRLP problem can be solved within an ILP framework by utilizing FRLP optimization techniques.

According to the algorithms proposed by Huang *et al.* [13,14], the solution for models (9a)-(9e) can be obtained through a two-step method, where a submodel corresponding to f^- is first formulated and solved, and then the relevant submodel corresponding to f^+ can be formulated based on the solution of the first submodel. In detail, the first submodel can be formulated as follows:

$$\mathrm{Min}\, f^{-} = \sum_{i=1}^{n}\left[K_{1i}\left(Q_i^{-}\right)^{K_{2i}} + \sum_{t=1}^{t_0} p_{it}^{-} z_{it}^{-}\right] \quad (10a)$$

Subject to:

$$Q_i^- \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^-\right) \le \tilde{T}_{Bi},\ \forall i \tag{10b}$$

$$\sum_{i=1}^{n}\left[U_{ji}^- \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + m_j^- \le \tilde{L}_{Cj},\ \forall j \tag{10c}$$

$$\sum_{i=1}^{n}\left[V_{ji}^- \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + R_{ji}^+ \tilde{O}_i + n_j^+ \ge \tilde{L}_{Dj},\ \forall j \tag{10d}$$

$$z_{it}^- \ge 0,\ \forall i,t \tag{10e}$$

Thus, solutions of $z_{it\ opt}^-$ can be obtained, and then the submodel corresponding to f^+ can be formulated as follows:

$$\mathrm{Min}\, f^+ = \sum_{i=1}^{n}\left[K_{1i}\left(Q_i^+\right)^{K_{2i}} + \sum_{t=1}^{t_0} p_{it}^+ z_{it}^+\right] \tag{11a}$$

Subject to:

$$Q_i^+ \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^+\right) \le \tilde{T}_{Bi},\ \forall i \tag{11b}$$

$$\sum_{i=1}^{n}\left[U_{ji}^+ \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + m_j^+ \le \tilde{L}_{Cj},\ \forall j \tag{11c}$$

$$\sum_{i=1}^{n}\left[V_{ji}^+ \tilde{L}_i \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + R_{ji}^- \tilde{O}_i + n_j^- \ge \tilde{L}_{Dj},\ \forall j \tag{11d}$$

$$z_{it}^+ \ge 0,\ \forall i,t \tag{11e}$$

$$\sum_{t=1}^{t_0} z_{it}^+ \ge \sum_{t=1}^{t_0} z_{it\ opt}^-,\ \forall i \tag{11f}$$

Uncertainties still exist in these two submodels due to the fuzzy constraints (10b)-(10d) and (11b)-(11d). FRLP techniques are desired in the solution process to tackle fuzziness in the left- and right-hand sides of model's constraints. According to the concept of level set and representation theorem [10], (10b)-(10d) and (11b)-(11d) can be respectively concerted into:

$$Q_i^- \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right) \le \overline{T_{Bi}}^s,\ \forall i \tag{10b'}$$

$$Q_i^- \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right) \ge \underline{T_{Bi}}^s,\ \forall i \tag{10b''}$$

$$\sum_{i=1}^{n}\left[U_{ji}^- \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + m_j^- \le \overline{L_{Cj}}^s,\ \forall j \tag{10c'}$$

$$\sum_{i=1}^{n}\left[U_{ji}^- \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + m_j^- \ge \underline{L_{Cj}}^s,\ \forall j \tag{10c''}$$

$$\sum_{i=1}^{n}\left[V_{ji}^- \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + R_{ji}^+ \overline{O_i}^s + n_j^+ \le \overline{L_{Dj}}^s,\ \forall j \tag{10d'}$$

$$\sum_{i=1}^{n}\left[V_{ji}^- \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^-\right)\right] + R_{ji}^+ \underline{O_i}^s + n_j^+ \ge \underline{L_{Dj}}^s,\ \forall j \tag{10d''}$$

and

$$Q_i^+ \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right) \le \overline{T_{Bi}}^s,\ \forall i \tag{11b'}$$

$$Q_i^+ \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right) \ge \underline{T_{Bi}}^s,\ \forall i \tag{11b''}$$

$$\sum_{i=1}^{n}\left[U_{ji}^+ \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + m_j^+ \le \overline{L_{Cj}}^s,\ \forall j \tag{11c'}$$

$$\sum_{i=1}^{n}\left[U_{ji}^+ \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + m_j^+ \ge \underline{L_{Cj}}^s,\ \forall j \tag{11c''}$$

$$\sum_{i=1}^{n}\left[V_{ji}^+ \overline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + R_{ji}^- \overline{O_i}^s + n_j^- \le \overline{L_{Dj}}^s,\ \forall j \tag{11d'}$$

$$\sum_{i=1}^{n}\left[V_{ji}^+ \underline{L_i}^s \left(1-\sum_{t=1}^{t_0} z_{it}^+\right)\right] + R_{ji}^- \underline{O_i}^s + n_j^- \ge \underline{L_{Dj}}^s,\ \forall j \tag{11d''}$$

where the marks "$\overline{\ }s$" and "$\underline{\ }s$" denote the superior and inferior limits among the set ***s***.

Following these decomposition and replacements, the final solutions for model (9a-9e) as $f_{opt}^{\pm} = [f_{opt}^-, f_{opt}^+]$ and $\eta_{i\ opt}^{\pm} = [\eta_{i\ opt}^-,\ \eta_{i\ opt}^+]$ can be obtained.

4. Conclusions

In this study, an interval-parameter fuzzy robust nonlinear programming (IFRNP) model was developed for water quality management under uncertainty. As an integration of interval nonlinear programming (INP) and fuzzy robust programming (FRP), the developed IFRNP model could explicitly address the complexities of various system uncertainties, where parameters were represented as interval numbers and/or fuzzy membership functions; moreover, it could deal with the nonlinearities in the objective function.

The method of piecewise linearization was developed to deal with the nonlinear cost function in the IFRNP model. Such a linearization approach was suitable for nonlinear functions; however, the approach could be further improved by 0 - 1 piecewise linearization approach. Moreover, other inexact optimization methods, such as stochastic programming and multi-objective programming, could be incorporated into IFRNP model to handle various types of uncertainties and enhance the model's application.

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