

A Localization of Solow Growth Model with Labor Growth Pattern in China

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ABSTRACT

This paper investigates the Solow Growth Model on a country-specific level by applying the demographic growth pattern in China to it. To localize the neoclassic model, China population growth estimation function based on the Verhulst Population Model is introduced to transform the population growth rate from a constant to a function, altering the original model assumption. By inserting the population growth function into Solow's work, an economy growth phase diagram for China is obtained. MATLAB programming is used to depict the diagram in a three-dimensional space and to show that the set of optimal capital-labor ratio values lies in the intersecting line of two planes rather than in the intersecting point of two curves in the original model setting. An neoclassical aggregate feasible growth path for China's economy can be depicted based on a chosen optimal value. The dynamic equilibrium in this case should not be unique; instead, capital-labor ratio together with population growth situation at a certain time point should be jointly taken into consideration to solve the optimization problem in the country's long term economy development.

Keywords: Solow Growth Model; China Population Growth; Optimal Economy Development

1. Introduction

Researchers have been investigating on generalization of the neoclassical Solow Growth Model [1] in different directions by applying certain patterns or constraints to different model factors. The standard neoclassic Solow Growth Model has an essential assumption that the labor(population) growth in the economy is constant. Accinelli and Brida [2], Bucci and Guerrini [3], Ferrara and Guerrini [4] implemented several population (labor) models, for example, Ramsey population model and Uzawa-Lucase model [5] to Solow's work. However, researches have not explored enough in terms of revising the model to a regional level. China is a country worth attention in terms of its demographical situation, with the world's largest quantity and relatively high growing rate. Solow's assumption about a constant population growth does not suit in the case of China. By inserting a function simulating China's population growth to the original model set, a new phase diagram describing the country's economy growth process can be depicted. The paper helps study the dynamic equilibrium, namely the country's optimal growth pattern, in a more accurate stage, with both the values of population growth and capital-labor ratio taken in to consideration.

2. The neoclassical Solow Growth Model

Consider the most general case: at time t , an economy produces product called national product Y_t with an aggregate production function $F(L_t, K_t)$ having two factors, namely, labor (population) L_t and K_t capital. F is twice differentiable and indicates the diminishing return of each factor. By assuming population growths in a constant continuous rate r we can get the neoclassical aggregate feasible growth path $k_t = f(k_t) = (r + s\mu)k_t - c_t$, where μ is the depreciation rate of capital, s is the propensity to save. Also, the Solow's path (k_{solow}, c_{solow}) , which is called the optimal balanced growth path, could be attained through the formula:

$$k_t = sf(k_t) - (r + s\mu)k_t. \quad (1)$$

3. The localization to China Population Growth Pattern

According to Luo's [6] modeling work of China's population since year 2003, a function P_t , which simulates annual demographical situation in China, is derived by revising the Verhulst Population Model [8] with adjusting to country-specific indexes and using Binary Binomial regression. The function is formulated as:

$$P_t = \frac{-0.00185906e^{0.0014(t-2003)}}{1 - 1.3279e^{-0.0014(t-2003)}}, \quad t \geq 2003. \quad (2)$$

The graph of Equation (2) is shown in **Figure 1**. And we can get the growth rate of China's population r'_c by:

$$r'_c = \frac{P_t}{P_t} = -\frac{0.00185906e^{-0.0014(t-2003)}}{1 - 1.3279e^{-0.0014(t-2003)}}, \quad t \geq 2003. \quad (3)$$

The graph of Equation (3) is shown in **Figure 2**. We can definitely see that the population growth rate in China is not constant and we can also get that over time $\lim_{t \rightarrow \infty} r'_c = 0$, which is expected in the neoclassical model. In order to fit the discrete population growth rate to the continuous case in Solow's model, we transform the r'_c to $r_c = \ln(1 + r'_c)$, therefore satisfying:

$$\frac{P}{P} = \frac{P_{2003} e^{r_c t} \cdot r_c}{P_{2003} e^{r_c t}} = r_c. \quad (4)$$

With a certain population growth pattern localized to China, we should get a revised growth model for China. The revised optimal path should be described as:

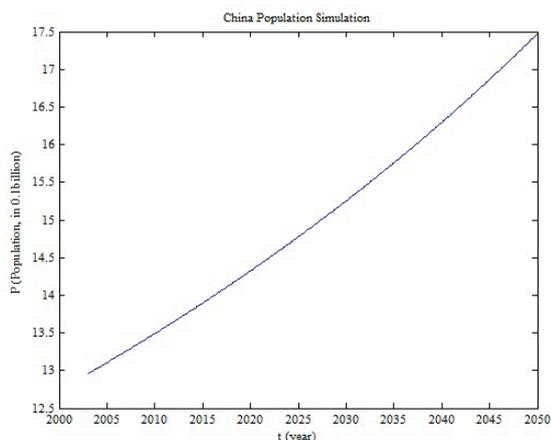


Figure 1. China Population Simulation.

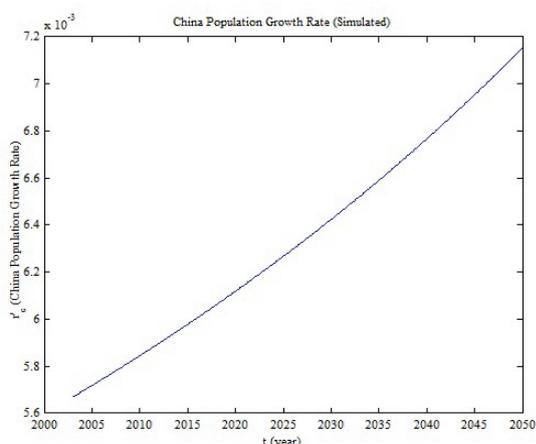


Figure 2. China Population Growth Rate (Simulated).

$$sy = sf(k'_{solow}) = (r_c + s\mu)k'_{solow}. \quad (5)$$

We present the determination process of optimal capital-labor ratio, namely k_{solow} by depicting a phase diagram (**Figure 3**) in three-dimensional space. By implementing China's population growth model, we finally generalize the neoclassical Solow Growth model to a country-specific level.

The three curve in the original model phase diagram turn to three planes. The intersecting line reflecting the situation described in formula (1) is shown in the diagram, pointing out a possible set of optimal capital-labor ratio values of China's economy. The dynamic equilibrium of the economy should be decided given consideration to both population growth and capital-labor ratio situation at a certain time point. To be specific, given a certain year t_m , corresponding population growth rate r_{cm} and capital-labor ratio k_m at the point can be obtained, and a short term optimal economy growth path towards the long term dynamic equilibrium point for the country can be depicted, with . Since the population growth rate varies continuously, the optimal path will turn slightly, changing its slope possibly from k_m to k_{m+1} . Over a long span of time, it should be expected that under Solow's model settings with China population growth pattern inserted, the country's economy develops to its long term optimum following a path not straight, but with certain radians. For other countries with diverse population growth patterns, the same methodology is appropriate to be adopted to describe economy development paths in a country-specific level.

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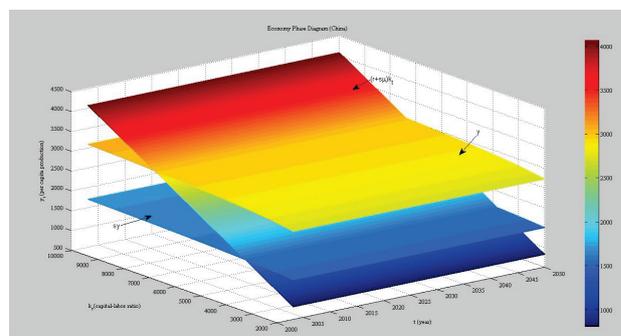


Figure 3. Economy Phase Diagram (China)

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