

Capacity Calculation and Sub-Optimal Power Allocation Scheme for OFDM-Based Systems

Somnath Das, Abhishek Chakraborty, Sanjay Kumar

Birla Institute of Technology, Mesra, India

Email: infosomu1989@gmail.com, abhishek2003cemk@yahoo.co.in, skumar@bitmesra.in

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ABSTRACT

For emerging cellular wireless systems, the mitigation of inter-cell interference is the key to achieve a high capacity and good user experience. This paper is devoted to the performance analysis of interference mitigation techniques for the downlink in an orthogonal frequency division multiple access (OFDMA) network, with a focus on the Long Term Evolution-Advanced (LTE-A) standard. Here we have derived a general closed-form equation of system capacity taking multiple cells into consideration and then we have investigated a coordination technique for interference mitigation. For the given interference constraint, how power should be transmitted into each OFDM sub-carrier for prevailing channel condition such that the total transmission rate of the base station can be maximized.

Keywords: Interference Reduction; LTE-A; OFDMA; Power Loading; System Capacity

1. Introduction

Radio spectrum is one of the most scarce and valuable resources for wireless communications. Given this fact, new insights into the use of spectrum have challenged the traditional approaches to spectrum management. LTE-A system with OFDMA as downlink multiple access technique will have no intra-cell interference but will have a larger Inter-Cell Interference (ICI). Specifically, in [1] the authors have shown that OFDMA causes inter-cell interference among the users. The amount of interference introduced to the users by adjacent base station depends on the power allocated to the subcarrier as well as the spectral distance between particular subcarrier. The cell edge users are adversely affected due this ICI [2]. 3 GPP has proposed three different solutions to combat ICI *i.e.* randomization, cancellation and co-ordination, to counter this ICI problem [3]. An intelligent radio resource management is needed for the dynamic allocation of radio resource to its users so as to maintain the QoS and also to best utilize the available spectrum while considering maximization of system throughput and capacity. Transmitter power control is an efficient technique to mitigate the effect of interference specially co-channels interference. Thus, an effective power control algorithms can offer a significant improvement in the system throughput. As we know that OFDM sub-carriers have time-varying fading gains, various power loading schemes have been proposed in the literature [4]. These algorithms maximize the transmission capacity of a single cell scenario. But

the use of classical loading algorithms e.g., uniform power and water-filling algorithms, for multi-cellular scenario may result in higher interference since, there exists no coordination. In this paper a downlink transmission has been considered in a multi-cell scenario. Hence, the design problem is as follows. Under interference constraint, how much power should be transmitted into each OFDM sub-carrier for prevailing channels condition such that the total transmission rate of the base station can be maximized?

The organization of this paper is as follows. Section 2 gives a closed-form expression of system capacity for multi-cellular scenario. In Section 3, the system model is described. In Section 4, a suboptimal scheme has been proposed. In Section 5, numerical results are presented. Finally, Section 6 concludes the paper.

2. Closed-Form Expression of System Capacity

From the context of information theory, for M number of adjacent cells closely spaced and N number of nodes in the target cell—the normalized (with respect to bandwidth) system capacity or spectral efficiency can be expressed as

$$C = \sum_{n=1}^N \sum_{m=1}^M \log_2 \left(1 + \frac{p_n x_n^2}{N_0 + p_n h_m^2} \right) \quad (1)$$

where p_n is the delivered power to each sub-channel, N_0 is the noise spectral density, x_n is the sub-channel

gain of the cell under consideration, and h_m is the sub-channel gain associated with the adjacent sub-channels and thus the term $p_n h_m^2$ act as inter-cell interference to the system. To maximize the system capacity, we have to maximize the above expression. So, in order to calculate the maximized system capacity, along with the Karush-Kuhn-Tucker (KKT) conditions, the problem is described as follows

$$\text{maximize } \sum_{n=1}^N \sum_{m=1}^M \log_2 \left(1 + \frac{p_n x_n^2}{N_0 + p_n h_m^2} \right) \quad (2)$$

or,

$$\text{minimize } - \sum_{n=1}^N \sum_{m=1}^M \log_2 \left(1 + \frac{p_n x_n^2}{N_0 + p_n h_m^2} \right) \quad (3)$$

subjects to constraints,

$$\begin{aligned} p_n &\geq 0 \\ p_n \mu_n &\geq 0 \\ \delta \left(\sum p_n - P_x \right) &= 0 \end{aligned}$$

for

$$n = \{1, N\}, m = \{1, M\}$$

The above constraints are self explaining. Here δ is the dual variable and μ is the slack variable. KKT holds for strong duality, which can be solved for two different ways but having equivalent solutions. Here $\delta \geq 0$, $\mu_n \geq 0$ are called Lagrange's multipliers or dual variables. These KKT conditions are necessary and sufficient conditions for duality.

Now, taking the problem equation together with the KKT conditions, the partial Lagrangian equation is formulated as,

$$\begin{aligned} L(p_n, \delta, \mu_n) &= - \sum_{n=1}^N \sum_{m=1}^M \log_2 \left(1 + \frac{p_n x_n^2}{N_0 + p_n h_m^2} \right) \\ &+ \delta \left(\sum_{n=1}^N p_n - P_x \right) + \mu_n \left(- \sum_{n=1}^N p_n \right) \end{aligned} \quad (4)$$

To calculate the maximum capacity based on the above expression, partial derivatives of the Lagrangian function with respect to P_n has to be taken and by applying KKT condition, the resulting equation becomes

$$p_n X_{mn} + Y_{mn} = \frac{1}{\ln 2 \cdot \delta} \quad (5)$$

where,

$$X_{mn} = 1 + \frac{p_n h_m^2}{N_0}$$

$$Y_{mn} = \frac{(N_0 + p_n h_m^2)^2}{N_0 x_n^2}$$

From the above equation, the optimal power has to be calculated and thus the derived closed-form expression of optimized system capacity is given by

$$C = \sum_{n=1}^N \sum_{m=1}^M \log_2 \left[1 + \frac{\left(\frac{1}{X_{mn}} \cdot \frac{1}{\ln 2 \cdot \delta} - \frac{Y_{mn}}{X_{mn}} \right) \cdot x_n^2}{N_0 + \left(\frac{1}{X_{mn}} \cdot \frac{1}{\ln 2 \cdot \delta} - \frac{Y_{mn}}{X_{mn}} \right) \cdot h_m^2} \right] \quad (6)$$

This is a generalized closed-form expression of system capacity for multiple cells scenario.

3. System Model

Here we have considered a two-cell scenario. Each cell uses OFDMA and hence, uses frequency reuse factor 1. The available bandwidth is divided into k subcarriers, where k ranges from $1 \leq k \leq K$. It is assumed that the bandwidth for each sub-carrier is Hz and each user uses only one sub-carrier. The numbers of users under BS1 are j , where j ranges from $1 \leq j \leq J$ and users under BS2 are i , where i range from $1 \leq i \leq I$. $x_{bs1,j}$ and $x_{bs2,i}$ is the transmitted power of BS1 and BS2 respectively.

In downlink transmission scenario shown in **Figure 1**, there are four channel gains: i) between the BS2 and its i^{th} user for the sub-carrier denoted as h_i^{22} ii) between the BS2 and j^{th} BS1 user, denoted as h_i^{21} iii) between the BS1 and the i^{th} BS2 user, denoted as h_i^{11} iv) between the BS1 and its j^{th} user for sub-carrier denoted as h_j^{11} . The received signal at the receiver of j^{th} user of BS1 and i^{th} user of BS2 are $y_{bs1,j}$ and $y_{bs2,i}$ respectively

$$y_{bs1,j} = x_{bs1,j} h_j^{11} + \sum_{i=1}^I x_{bs2,i} h_i^{21} + n_{bs1,j} \quad (7)$$

$$y_{bs2,i} = x_{bs2,i} h_i^{22} + \sum_{j=1}^J x_{bs1,j} h_j^{12} + n_{bs2,i} \quad (8)$$

where $n_{bs1,j}$ and $n_{bs2,i}$ are the received noise at BS1

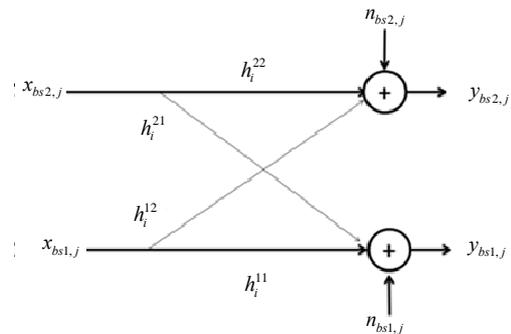


Figure 1. System model.

and BS2 respectively. For protection of BS1 users, we consider constraints on the interference introduced by BS2. The total interference introduced to BS1 can be written as

$$\sum_{i=1}^I p_{bs2,i} h_j^{21} \leq I_{th} \quad \forall j \in \{1, 2, \dots, J\} \quad (9)$$

4. Proposed Sub-Optimal Scheme

Considering this fact that most of the interference introduced to the BS1 users is induced by BS2 transmission over same sub-carriers. The problem can be formulated as follows

$$P_1 : \max_{p_i} \sum_{i=1}^{I_j} \log_2 \left(1 + \frac{p_i |h_i^{22}|}{\sigma_i^2} \right) \quad (10)$$

subject to

$$\sum_{i=1}^{I_j} p_i h_j^{21} \leq I_{th} \quad \forall j \in \{1, 2, \dots, J\} \in \{1, 2, \dots, K\} \quad (11)$$

$$\sum_{i=1}^{I_j} p_i \leq p_T \quad (12)$$

$$P_i \geq 0 \quad \forall i \in \{1, 2, \dots, I_j\} \in \{1, 2, \dots, K\} \quad (13)$$

where h_j^{22} is the channel gain from BS1 to its user and

$$\sigma_i^2 = \sigma_{AWGN}^2 + \sum_{i=1}^I p_{bs1,i} h_j^{12} \quad (14)$$

σ_{AWGN}^2 is the mean variance of the additive white Gaussian noise (AWGN). The interference is assumed to be the superposition of large number of independent components; hence, we can model the interference as Gaussian. Assuming that each sub-carrier band is narrow, sub-carriers can be approximated as channel having flat and constant gains during transmission. K denotes the total no of sub-carriers, I is the no users of BS2 users and J is the no users of BS1 users. I^{th} denotes the interference threshold prescribed by the BS1 users. Interference threshold is the maximum tolerable interference on the spectrum being utilized. It is highly variable depending on the allocation of channels to the users within the cell. However for simplicity we can assume a common threshold for all the channels. P_T is the fixed total power budget of the system. I_j denotes the set of the same sub-carriers belonging to the BS1. Using the same derivation in [5], we get

$$p_i^* = \left[\frac{1}{\sum_{j=1}^J \alpha'_j h_j^{21} + \beta'} - \frac{\sigma_i^2}{|h_i^{22}|^2} \right]^+ \quad (15)$$

where α and β are the non-negative dual variables corre-

sponding to the interference and power constraints respectively. The solution of the problem still has high computational complexity which encourages us to find a simpler, faster and efficient power allocation algorithm. The scheme proposed in this section is based on the fact that if the interference constraints are ignored in P_1 , the solution of the problem will follow the well-known Waterfilling interpretation [6].

$$p_i^{(P_T)} = \left[\lambda - \frac{\sigma_i^2}{|h_i^{22}|^2} \right]^+ \quad (16)$$

where λ is the waterfilling level and is given by

$$\lambda = P_T + \sum_{i=1}^I \frac{\sigma_i^2}{|h_i^{22}|^2} \quad (17)$$

On the other side, if the total power constraint is ignored, the Lagrangian of the problem can be written as [7]

$$G^{(Int)} = - \sum_{i=1}^I \log_2 \left(1 + \frac{p_i^{(Int)} |h_i^{22}|^2}{\sigma_i^2} \right) + \sum_{j=1}^J \alpha'_j \left(\sum_{i=1}^I p_i^{(Int)} h_j^{21} - I^{th} \right) \quad (18)$$

where is α'_j the Lagrange multiplier. Equating

$$\frac{dG^{(Int)}}{dp_i^{(Int)}}$$

to zero, we get

$$p_i^{(Int)} = \left[\frac{1}{\alpha'_j h_j^{21}} - \frac{\sigma_i^2}{|h_i^{22}|^2} \right]^+ \quad (19)$$

where value of $\alpha'^{(Int)}$ can be calculated by substituting Equation (19) into

$$\sum_{i=1}^{I_j} p_i^{(Int)} h_i^{21} = I_{th}$$

to get

$$\alpha'^{(Int)} = \frac{I_j}{I_{th} + \sum_{i \in I_j} \frac{h_j^{21} \sigma_i^2}{|h_i^{22}|^2}} \quad (20)$$

In order to solve the optimization problem P_1 , we can start by assuming that the maximum power that can be allocated for a given subcarrier p_i^n is determined according to the interference constraints only by using Equations (11) and (12) for every set of sub-carriers. By such an assumption, we can guarantee that the interference introduced to BS1 users will be under the pre-

specified threshold. Once the maximum power p_i^n is determined the total power constrain is tested. If the total power constrain is satisfied, then the solution has been found and is equal to maximum power that can be allocated to each subcarrier, i.e. $p_i' = p_i^n$. Otherwise, the available power budget should be distributed among the subcarriers giving that the power allocated to each subcarrier is lower than or equal to the maximum power that can be allocated to each subcarrier p_i^n , and hence the following problem should be solved:

$$P_2 : \max_{p_i} \sum_{i=1}^{I_j} \log_2 \left(1 + \frac{p_i^{WF} |h_i^{22}|}{\sigma_i^2} \right) \quad (21)$$

subject to

$$\sum_{i=1}^{I_j} p_i^{WF} \leq P_T \quad (22)$$

$$0 \leq p_i^{WF} \leq p_i^{\max} \quad (23)$$

The problem can be solved efficiently using the concept of the conventional waterfilling. Given the initial waterfilling solution, the channels that violate the maximum power p_i^{\max} are determined and upper bounded with p_i^{\max} . The total power budget is reduced by subtracting the power assigned so far. At the next step, the algorithm proceeds to successive water-filling over the sub-carriers that did not violate the maximum power p_i^{\max} in the last step.

This procedure is repeated until the allocated power p_i^{WF} does not violate the maximum power p_i^{\max} in any of the sub-carriers in the new iteration. The solution p_i^{WF} of the problem P_2 is satisfying the total power constraint of the problem P_2 with equality and also satisfies interference constraints. The suboptimal power allocation implementation algorithm is described in **Figure 2**. Since it is assumed that $p_i^{WF} \leq p_i^{\max}$ some of the powers allocated to sub-carriers will not reach the maximum allowable values. This will make the interference introduced to

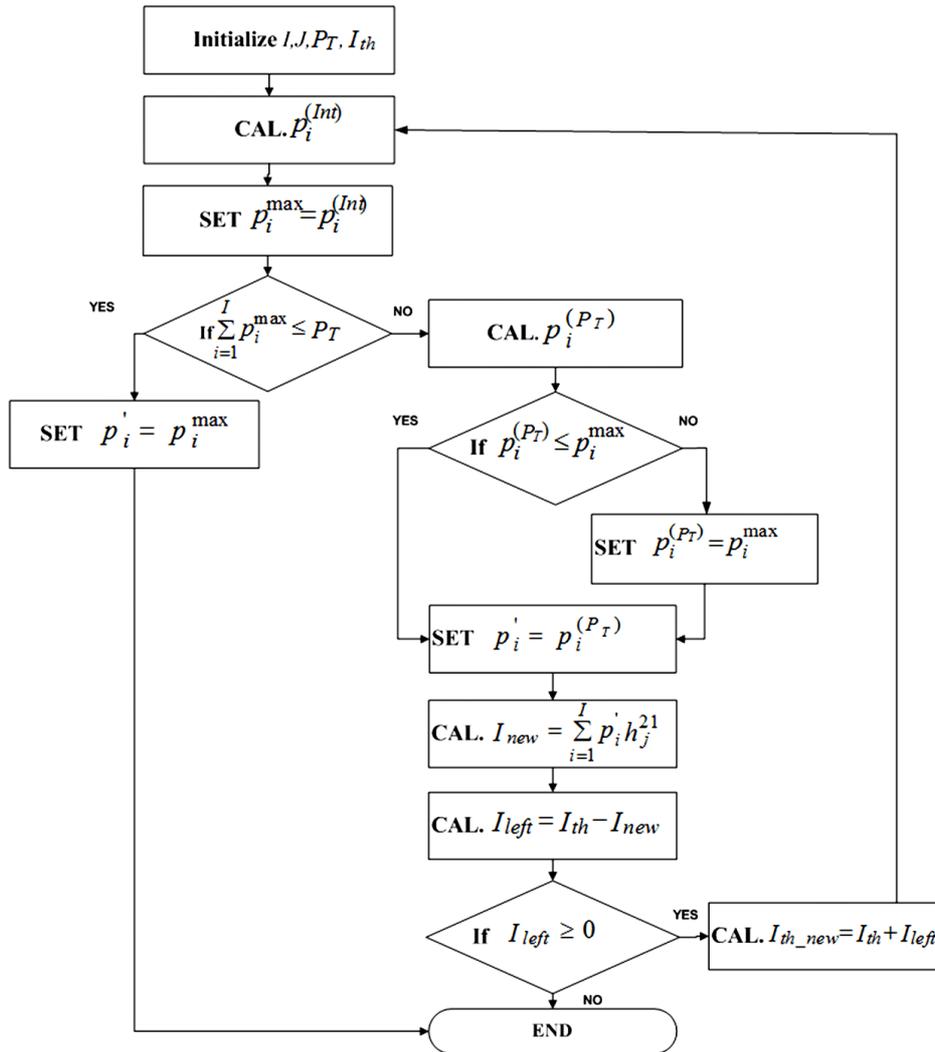


Figure 2. Algorithm flow.

the BS1 user below the threshold I^{th} . It is important to mention that the power allocation policy is indeed a waterfilling policy. However, the cut-off value for the channel gain or the threshold for this waterfilling policy is weighted by the inverse of the interference term I^{th} . Specifically, the policy suggests that more power should be allocated to the sub-carrier which has relatively better channel quality.

5. Results

In the results presented in this section, we assume the values of I and J to be 20 and 4, respectively. We assume the value of total available bandwidth as 1MHz and no of subcarriers to be 33. The value of σ_i is assumed to be 10^{-11} watts. The total power is assumed to be 10^{-3} watts. In **Figure 3**, we plot the achievable transmission rate of the BS2 versus interference threshold prescribed by BS1. The scheme tries to maximize the total throughput of the system under the constraint that each base station cannot

transmit more than a specific value P_T and the introduced interference to the neighbouring cell is within the interference threshold I^{th} .

In **Figure 4**, we have plotted the interference introduced to BS1 versus the transmit power of BS2. From this figure, we observe that for the same introduced interference level the proposed suboptimal scheme allows transmission of more power than the classical method like uniform power loading. This is possible because the suboptimal scheme considers the interference introduced to BS1 as one of its constraint. So there exists a harmony in power allocation over the sub-carriers which minimize the intercellular interference.

In **Figure 5**, we have plotted the transmit power and $1/SINR$ for individual sub-carriers of BS2. $1/SINR$ basically, represents the amount of interference present in the channel. The suboptimal scheme allows transmission of higher power over sub-carriers where the channel conditions are good and restricts power over sub-carriers where the channel conditions are bad.

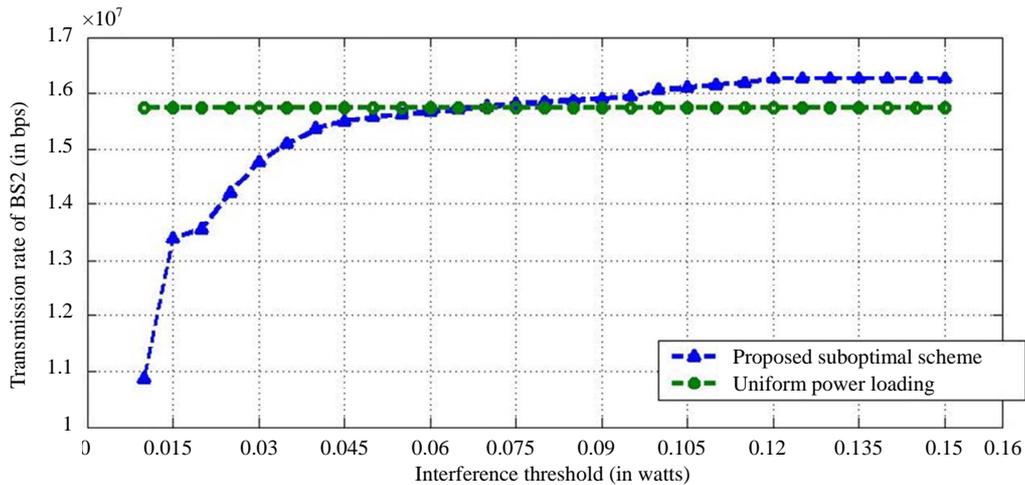


Figure 3. Interference introduced to BS1 vs. transmit power of BS2.

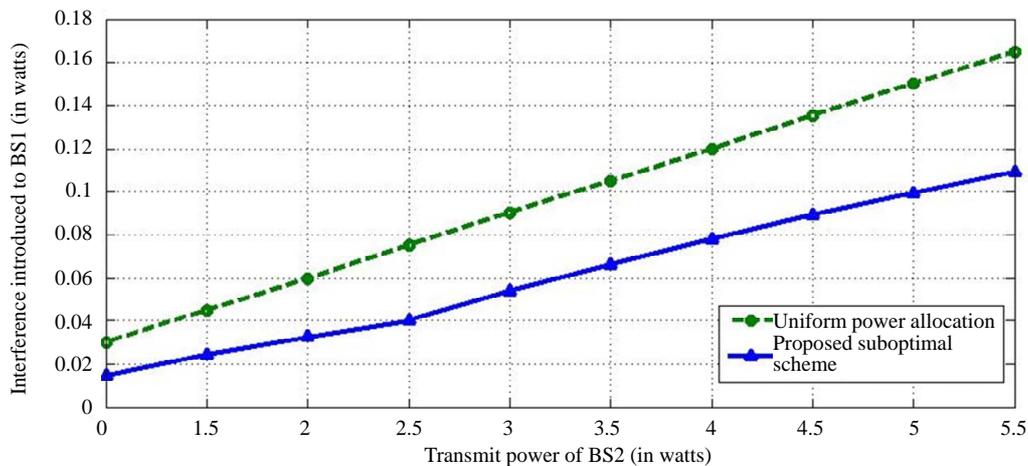


Figure 4. Transmission rate of BS2 vs. interference introduced to BS1.

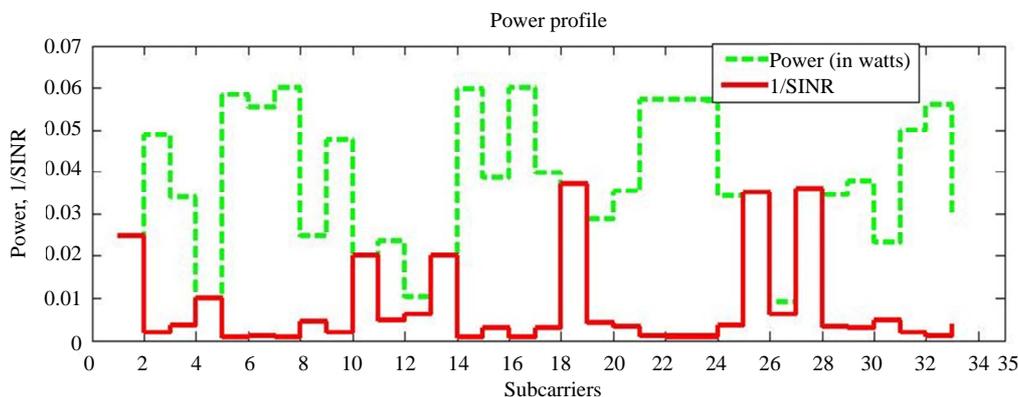


Figure 5. Power profile.

6. Conclusion

In this paper, we have developed a suboptimal power loading algorithm that maximizes the downlink transmission data rate of the BS2 while the interference introduced to the BS1 user remains within a given limit. The proposed algorithm is simpler and more efficient in terms of throughput performance.

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