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# Why the Spacetime Embedding Incompressible Cores of Pulsars Must Be Conformally Flat?

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## Abstract

The multi-messenger observations of the merger event in GW170817 did not rule out the possibility that the remnant might be a dynamically stable neutron star with $\mathcal{M}_{rm} \geq 2.69\mathcal{M}_{\odot}$. Based on this and other recent events, I argue that the universal maximum density hypothesis should be revived. Accordingly, the central densities in the cores of ultra-compact objects must be upper-limited by the critical density number $n_{cr}$, beyond which supranuclear dense matter becomes purely incompressible. Based on the spacetime-matter coupling in GR, it is shown that the topology of spacetime embedding incompressible quantum fluids with $n = n_{cr}$ must be Minkowski flat, which implies that spacetime at the background of ultra-compact objects should be bimetric.

## Keywords

Relativity: Numerical, General, Black Hole Physics, Pulsars, Neutron Stars, Pulsars, Superfluidity, Superconductivity, Incompressibility, Gluons, Quarks, Plasmas, QCD

Most theoretical investigations indicate that pulsars, neutron stars and magnetars, that comprise the family of ultra-compact objects (UCOs), whose masses $\mathcal{M}_{NS} \geq 1.4\mathcal{M}_{\odot}$, should have central densities much larger than the nuclear one [1] [2] [3] [4] [5]. Theoretically this posses an upper limit of their maximum mass $\mathcal{M}_{\max}^{NS} \leq 2.3M_{\odot}$ for almost all EOSs, though none of these objects have been ever observed with $\mathcal{M}_{\max}^{NS} \geq 2.1M_{\odot}$ (see [1] and the references therein).

Indeed, the multi-messenger observations of the merger of the two neutrons stars in GW170817 did not rule the formation of a massive NS with $\mathcal{M}^{NS} \approx 2.79M_{\odot}$ [5] [6] [7] [8]; hence the mechanisms that limit their theoretical mass range must be revisited.

Let us assume that there is a universal maximum energy density $\varepsilon_{\max}$, beyond which supranuclear dense fluids becomes incompressible. Under these circumstances the solution of the field equations:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G T^{\mu\nu} \tag{1}$$

reads:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \frac{1}{S^\kappa}\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{S} - r^2\mathrm{d}\Omega^2, \tag{2}$$

where $S = 1 - r_s/r$, $\kappa = \frac{1+3\alpha_0}{2}$, $\alpha_0 = P_b^L/\varepsilon_{\max}$ and $\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\varphi)$. $P_b^L$ here is the local baryonic pressure.

In order to find out whether this spacetime is Weyl-transformable into a conformally flat spacetime of the type:

$$\mathrm{d}s^2 = \hat{g}_{\mu\nu}\mathrm{d}\hat{x}^\mu \mathrm{d}\hat{x}^\nu = e^f\left(\mathrm{d}\tau^2 - \mathrm{d}\rho^2 - \rho^2\mathrm{d}\Sigma^2\right), \tag{3}$$

we need to analyse the relation between both coordinate systems, solve for the conformal factor $e^f = F(\tau, \rho, \vartheta, \psi)$ and discuss their physical consistences (see [9] [10] and the references therein). As the line element, d*s*, should be invariant under the transformation, both metrics are related to each other through:

$$\hat{g}_{\mu\nu} = \frac{\partial x^\mu}{\partial \hat{x}^\nu}\frac{\partial x^\mu}{\partial \hat{x}^\nu} g_{\mu\nu}. \tag{4}$$

Using algbraic manipulation and setting $\frac{\partial x^\mu}{\partial \hat{x}^\nu} = \delta_\nu^\mu$, the following equalities are obtained:

$$e^f = \left(\frac{1}{S}\right)^\kappa \left(\frac{\partial t}{\partial \tau}\right)^2 = \left(\frac{1}{S}\right)\left(\frac{\partial r}{\partial \varrho}\right)^2 = \left(\frac{r}{\rho}\right)^2\left(\frac{\partial \Omega}{\partial \Sigma}\right)^2. \tag{5}$$

A strictly flat spacetime would correspond to $e^f = 1$. In this case, the relation between both coordinates systems read:

$$\begin{aligned}
\frac{\mathrm{d}\tau}{\mathrm{d}t} &= \frac{1}{S^{\kappa/2}} \Leftrightarrow \mathrm{d}\tau = \frac{1}{\left(1 - a_0 r^2\right)^{\kappa/2}}\mathrm{d}t \\
\frac{\mathrm{d}\rho}{\mathrm{d}r} &= \frac{1}{\sqrt{S}} \Leftrightarrow \mathrm{d}\rho = \frac{1}{\sqrt{1 - a_0 r^2}}\mathrm{d}r \\
&\Rightarrow \rho = \frac{\arcsin\left(\sqrt{a_0}\, r\right)}{\sqrt{a_0}} r \\
\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} &= \frac{1}{S} \Leftrightarrow \mathrm{d}\Sigma = \frac{\sqrt{a_0}\, r}{\arcsin\left(\sqrt{a_0}\, r\right)}\mathrm{d}\Omega
\end{aligned} \tag{6}$$

Consequently, the spacetime:

$$\mathrm{d}s^2 = \left(\frac{1}{S}\right)^{\kappa/2}\mathrm{d}t^2 - \left(\frac{1}{S}\right)\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{7}$$

appears to be conformal to the flat spacetime:

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}\rho^2 - \rho^2\mathrm{d}\Sigma^2. \tag{8}$$

Mathematically, while $\kappa$ may accept other values, but $\kappa = 1$ is most reasonable as the volume-expansion rate of the incompressible core in both dual-spacetimes is equal and independent of the core's compactness, *i.e.*

$$\frac{\mathrm{d}\rho}{\mathrm{d}\tau} = \frac{\mathrm{d}r}{\mathrm{d}t}. \tag{9}$$

Note that since the matter inside the core is incompressible and stationary, then the spatial and temporal variations must vanish. This implies that the state of matter in both spacetimes are identical and that remote observers can measure the variations of the core's radius only. In most cases, modelings of the interiors of UCOs usually relay on using one single EOS throughout the entire object and obeying: $P_b^L \le 1/3\varepsilon$ This requires a strong spatial variation of $\varepsilon$ in the neighborhood of $r = 0$, where the regularity condition is imposed. However this egularity condition is practically equivalent to imposing zero-compressibility, *i.e.* $\nabla P = \nabla \varepsilon|_{r=0} = 0$. To overcome this inconsistency, the central density is manually increased to be much higher than the nuclear number density, $n_0$, therefore giving rise to causality violation. On the other hand, recently it was suggested that at $n_{cr} \approx 3 \times n_0$ and zero-temperature, the neutrons at the very central regions of massive pulsars ought to undergo a phase transition, through which they merge together to form embryonic super-baryons (SB, see **Figure 1** as well as [3] [4] and the references therein). The number density of the enclosed incompressible gluon-quark superfluid would attain the value $n_{cr}$, which could be easily reached by NSs with moderate masses. The state of the fluid on the verge of merger is said to be maximally compressible (MC) and the corresponding stress-energy tensor becomes traceless: $\boldsymbol{T} = \varepsilon_{cr} - 3P_b^L = 0$. Although the MC state is expected to be a short-living transient phase, $\boldsymbol{T} = 0$ insures that the matter-field correspondence, and specifically the incompressibility character of the quantum fluid, is invariant under Weyl and conformal transformations.

When the compressible matter surrounding the cores of NSs cools down on the cosmic time, the curved spacetime at the background would compress the baryons at the center together, thereby decreasing the separation distance between the baryons, $d_b$, down to values comparable to the average distance $d_g$, between quarks (see **Figure 2**), thereby giving rise a free darg energy, a free dark energy $\Delta\varepsilon_b^+$ of order $c\hbar/2d_b$. For $d_b = d_q$, $\Delta\varepsilon_b^+$ is comparable to the rest energy of an isolated baryon, *i.e.*

$$\frac{\Delta\varepsilon_b^+}{\Delta\varepsilon_q} \sim \frac{2d_q}{d_q + d_b} \to \begin{cases} 0; & d_b \gg d_q \\ 1; & d_b = d_q \end{cases} \tag{10}$$

where $\Delta\varepsilon_q = 0.939\ \mathrm{GeV}$. Hence $\Delta\varepsilon_b^+$ is capable of deconfining the quarks inside baryons, thereby rendering merger of the baryons possible. Moreover, the pressure $P_{UP}$, induced by the uncertainty principle, and $\Delta\varepsilon_b^+$ become duals that oppose compression/contraction of the core's matter, whilst enhancing the effective mass of the core. The state of matter in the post merger phase is governed by EOS: $P_{UP} = \Delta\varepsilon_b^+ = \varepsilon_{cr} = a_0 n_{cr}^2$. The mass of the field quanta goes to

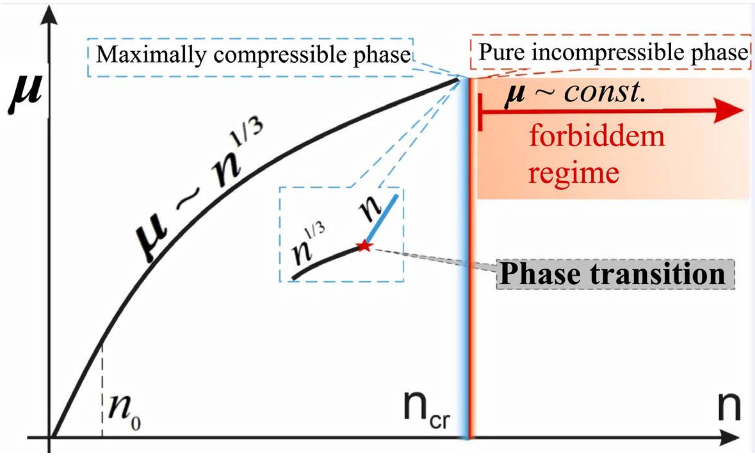


**Figure 1.** The development of the chemical potential $\mu$ of ultradense quantum fluids with the number density *n*. For $n_0 \le n < n_{cr}$, $\mu \sim n^{1/3}$. As $n \to n_{cr}$ the quantum fluid converges to the maximally compressible state and undergoes a phase transition into the incompressible state, where $P = \varepsilon_{cr} = c_0 n_{cr}^2$. The forbidden region correspond to number densities that are beyond the universal maximum allowable value $n_{cr}$.

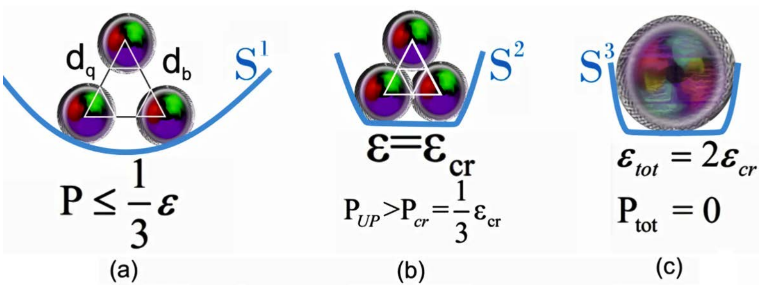


**Figure 2.** A schematic description of the spacetime curvature (denoted with $\mathcal{S}$) as opposed to the EOS and number density. $d_b$ and $d_q$ denote the mean distances between two arbitrary baryons and quarks, respectively. The case (a) corresponds to compressible quantum fluid embedded in a Schwarzschild-like spacetime ($\mathcal{S}^1$), (b) to the maximally compressible fluid phase, where the boundaries of baryons overlap, *i.e.* $d_b = d_g$ and the pressure $P_{UP}$ induced by the uncertainty principle becomes comparable to $\varepsilon_{cr}$. Here the spacetime start flattening to become nearly a Minkowski-type spacetime ($\mathcal{S}^2$). (c) corresponds to the final state in which baryons start merging to form a super-baryon, whose interior is made of incompressible gluon-quark superfluid embedded by a strictly flat spacetime ($\mathcal{S}^3$).

zero and therefore the quarks communicate with each other via the massless gluons at the speed of light [11] [12].

The dark energy contribution $\Delta\varepsilon^+$ must be stored locally; it goes specifically into enhancing the surface tension of the super-baryon and acts as a confining force for the enclosed ocean of quarks. At a certain point of the cosmic time, the core would decay and dark energy would be liberated, thereby provoking a hadronization process of the guon-quark superfluid.

For a given super-baryon resulting from the merger of *N*-baryons, the effective energy is expected to be:

$$\varepsilon_{SB}^{tot} \approx 2N \times \varepsilon_0. \tag{11}$$

This appears to be in line with the short-living pentaquark formation observed at the LHC-experiment (see [13] and the references therein), though the entropy and density regimes are totally different from those in the cores of UCOs.

Noting that the core must be 3D spherically symmetric with zero-entropy enclosed matter which behaves as a single quantum entity embedded in a flat spacetime, the governing physics is predicated to be mirrored onto its two-dimensional surface in accord with the holographic principle. However, it is not clear at all, how and what kind of information could be still storable on the surface under zero-entropy conditions?

Finally, when combining the result-presented here with the following arguments:

- The remnant of GW170817 didn't necessarily collapse into a BH.
- BHs with $\mathcal{M}_{BH} \leq 5M_{\odot}$ may be safely ruled out.
- The first generation of stars may have formed massive pulsars that should be dark by now.
- The glitch phenomena in pulsars are triggered by topological changes of the bimetric spacetime embedding pulsars (see [14] and the references therein); then the existence of a universal maximum energy density is an inevitable conclusion.

It should be noted however, that the existence of a universal maximum density, $n_{cr}$, does not necessary rule out BHs as astrophysical objects, whose existence is observationally well-verified; however, it argues against their classical formation scenario as well as against the existence of matter singularities at their centers.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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