

How Much Should Government Compensate Firms for Suspension of Their Businesses in Order to Fight off the New Coronavirus?

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How to cite this paper: Fujita, Y. (2020). How Much Should Government Compensate Firms for Suspension of Their Businesses in Order to Fight off the New Coronavirus? *Theoretical Economics Letters*, 10, 600-606.

<https://doi.org/10.4236/tel.2020.103038>

Received: April 18, 2020

Accepted: June 19, 2020

Published: June 22, 2020

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Abstract

Although it is considered to be vital to stop the businesses and reduce the overlaps of people in order to fight off the new coronavirus, firms are not willing to shut their businesses unless they receive the compensation for the losses in their businesses. In the present paper, we attempt to develop measures to induce firms to shut their businesses as a result of their maximizing problem. More precisely, we examine how much the government should compensate the firms for their losses during the business closure periods. We reveal that if the uncertainty or the government's budget increases, the government should increase the compensation per period for the losses in business and shorten the length of the business closure period.

Keywords

Coronavirus, Compensation for the Losses in Business, Government's Budget, Optimal Stopping Theory

1. Introduction

Although it is considered to be vital to stop the businesses and reduce the overlaps of people in order to fight off the new coronavirus, firms are not willing to shut their businesses unless they receive the compensation for the losses. In Japan, for example, although the government repeatedly calls for the business closures to firms, the firms are reluctant to obey, claiming this should go together with the compensation, which ends up with not so much reduction of the overlaps of people. Since the government is not strong enough to force the firms to close their businesses, it is necessary to develop some measures to induce firms to shut their businesses voluntarily as a result of their optimizing problem.

In the present paper, we examine how much the government should compensate the firms for their losses during the business closure period, by building a stochastic model based on the optimal stopping theory. The optimal stopping theory is a theory which deals with the problem of determining the optimal time to take a particular action in a stochastic environment, where the optimal time refers to the time to maximize an expected profit or minimize an expected cost. One of the most distinguished studies is McDonald and Siegel (1986), which demonstrated the value of waiting, followed by Dixit (1989), Farzin, Huisman, and Kort (1988), Fujita (2007, 2016) and so on. The present paper, by pushing forward these works, shows the usefulness of the optimal stopping theory in tackling the current coronavirus problem.

The most related work is Fujita (2020), which examined when the government should start the lockdown and how long it should be, by constructing a stochastic model where the government makes a decision as a central planner. The present paper is different from Fujita (2020) especially in that this paper pays attention to the strategic interaction between the firm and the government and formulates such an interaction as the second stage game. That is, in Fujita (2020), the government decides both the start time of business closure and its length; in the present paper, the firm decides the start time of business closure, while the government, taking into account the firm's decision, determines the length of the business closure period and how much money to pay the firm per period while it is closed. In the present paper, we reveal that if the uncertainty or the government's budget increases, the government should increase the compensation per period for the losses in business and shorten the length of the business closure period.

The structure of this paper is as follows. After constructing a basic stochastic model in section 2, section 3 formulates the firm's objective function and determines when the firm should shut its business. Based on these analyses, in section 4 we consider how long the business closure period should be and how much the government should pay to the firm during the business closure period. Concluding remarks are made in section 5.

2. Basic Model

Let us consider an inter-temporal economy that consists of households, representative firm and government, where the new coronavirus continues to spread over the economy. We assume that time passes continuously and the time horizon is infinite. The firm determines when to close its business given the sequential decline of its revenues; the government, taking into account this firm's decision, determines the length of the business closure period and how much to compensate the firm for its losses during the business closure periods

We let N , $n(t)$ and α denote the total number of households, the number of households who are infected with the coronavirus in period t and the rate of newly infected households to the total number of households, respectively. If we

assume the initial value of $n(t)$ is 0, motion of $n(t)$ is expressed as

$$\frac{dn}{dt} = \alpha(t)(N - n(t)), \quad (1)$$

and by solving this differential equation, we have the number of infected households in period t , $n(t)$, as

$$n(t) = N(1 - e^{-\alpha t}). \quad (2)$$

As in Fujita (2020), throughout this paper, we assume that $\alpha(t)$ follows the geometric Brownian motion of Equation (3).

$$d\alpha = \mu\alpha dt + \sigma\alpha dz, \quad (3)$$

with initial value of α being α_0 . μ and σ are parameters of drift and volatility, with both μ and σ being positive constants. Larger μ means that $\alpha(t)$ increases more quickly; larger σ means that the growth of $\alpha(t)$ is more uncertain. dz is Wiener process that expresses random movement, which has several real-world applications such as stock market fluctuations, exchange rate fluctuations and so on.

We assume that the profit of the firm goes down as the number of infected households, $n(t)$, increases and specify the amount of profit in period t , $Y(t)$, as

$$Y(t) = Y - \theta n(t), \quad (4)$$

where Y is the profit with no infected households and θ is a positive constant. By substituting (2) into (4), we have the profit in period t as

$$Y(t) = Y - \theta N + \theta N e^{-\alpha(t)t}, \quad (5)$$

as a function of $\alpha(t)$.

3. Objective Function of the Firm and Its Optimal Decision

The firm in the present paper closes the business if $\alpha(t)$ reaches α^* by incurring the cost K . We assume, for the simplicity of the analysis, the profit drops to zero at the moment of the business closure and it continues to stay at zero during the business closure periods; the government pays m amount of money to the firm in compensation for the losses in business. This sequence of the movements of the government and the firm is formulated as the second stage game where the first mover is the government and the second mover is the firm. We also assume that longer period of business closure removes more coronavirus and brings about more profits after the business closure. In order to simplify the analysis, we specify the profits after the business closure is constant at $Y - \frac{c}{S}$, where c is a positive parameter and S is the length of the business closure period.

Following the standard procedure, let us solve the problem backward and consider first the firm's behavior. Letting r denote the discount rate, and assuming the firm to maximize sum of the expected present value of profit minus expected present value of the cost K , we can express the firm's objective function V

as follows.

$$V = \int_0^{\tau} e^{-rt} \left(Y - \theta N + \theta N e^{-\alpha(t)t} \right) dt + \int_{\tau}^{\tau+S} e^{-rt} m dt + \int_{\tau+S}^{\infty} e^{-rt} \left(Y - \frac{c}{S} \right) dt + e^{-r\tau} K. \quad (6)$$

We assume that the firm maximizes V with respect to α^* .

In order to simplify (6), let us calculate the expected present value of one unit of profit at the moment when $\alpha(t)$ whose initial value is α_0 reaches α^* . If we let $G(\alpha_0)$ denote this value, we can express the general solution to $G(\alpha_0)$ as

$$G(\alpha_0) = A(\alpha_0)^{\beta_1} + B(\alpha_0)^{\beta_2}, \quad (7)$$

where $\beta_1 < 0$ and $\beta_2 > 0$ are solutions to $\frac{1}{2}\sigma^2 x(x-1) - r = 0$. Since $G(\alpha_0)$ satisfies $G(0) = 0$ and $G(\alpha^*) = 1$, it follows that $A = 0$ and $B = \frac{1}{\alpha^{*\beta_2}}$.

Since substituting these Equations into (2) yields $G(\alpha_0) = \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta_2}$, we obtain

$$G(\alpha) = \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta_2}, \quad (8)$$

by letting β denote β_2 . Thus, we can rewrite (6) as

$$V = \frac{1}{r} (Y - \theta N) \left(1 - \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta} \right) + \frac{1}{\mu + r} \theta N \left(1 - \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta} \alpha^* \right) + \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta} \frac{m}{r} (1 - e^{-rS}) + \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta} e^{-rS} \frac{1}{r} \left(Y - \frac{c}{S} \right) - \left(\frac{\alpha_0}{\alpha^*} \right)^{\beta} e^{-r\tau} K, \quad (9)$$

by making use of (8).

Since the firm in the present paper maximizes V with respect to α^* , from the first order condition, $\frac{\partial V}{\partial \alpha^*}$, we have

$$-\frac{1}{r} (Y - \theta N) \frac{1}{\theta N} - \frac{1}{\mu + r} - \beta \alpha_0^{\beta} \alpha^{*\beta-1} \frac{m}{r} (1 - e^{-rS}) - \beta \alpha_0^{\beta} \alpha^{*\beta-1} \frac{1}{r} \left(Y - \frac{c}{S} \right) + \beta \alpha_0^{\beta} \alpha^{*\beta-1} K = 0,$$

which yields the optimal value of α^* as

$$\alpha^* = \left[\frac{\left\{ rK - m - e^{-rS} \left(Y - \frac{c}{S} - m \right) \right\} \beta \alpha_0^{\beta}}{(Y - \theta N) \frac{1}{\theta N} + \frac{r}{\mu + r}} \right]^{\frac{1}{\beta+1}}. \quad (10)$$

4. Optimal Amount of Money the Government Should Compensate

Based on the above analyses, in this section, we consider how long the business

closure period should be and how much the government should pay to the firm during the business closure period, by assuming that the government minimizes the expected costs for the business suspension subject to, firstly, the firm's decision on the start time of business closure, i.e. (10), and, secondly, the budget constraint $B = mS$, where B is a positive constant that denotes the budget of the government.

As is obvious from the budget constraint $B = mS$, m is determined if S is determined. Thus, in the following, in order to simplify the analysis, we solve the minimization problem of the government with respect to S .

Since the expected costs for the business suspension C are expressed as $C = \int_{\tau}^{\tau+S} e^{-rt} m dt$, which reduces to

$$C = \left(\frac{\alpha_0}{\alpha \left(\frac{B}{S} \right)^*} \right)^{\beta} \frac{B}{rS} (1 - e^{-rS}) \quad (11)$$

by making use of (8) and $m = \frac{B}{S}$ that is derived by the budget constraint.

Therefore, by calculating $\frac{dC}{dS} = 0$, we have

$$\begin{aligned} & \left\{ \left\{ \frac{\beta}{2\beta+1} (1+rS) - 1 \right\} e^{-rS} - \frac{\beta+1}{2\beta+1} \left(\frac{1}{B} - 1 \right) \right\} \frac{B}{S} \\ & + \left\{ rK - \theta N + \left(\frac{1}{B} - 1 + e^{-rS} \right) \frac{B}{S} \right\} \frac{e^{-rS}}{1 - e^{-rS}} = rK - \theta N, \end{aligned} \quad (12)$$

as the relationship to determine the optimal S .

If we let $f(S)$ and g denote left hand side and right hand side of (12) respectively, we can determine the optimal S as the intersection of $f(S)$, downward sloping curve, and g , horizontal line as in **Figure 1**.

Since β is a positive solution of $\frac{1}{2} \sigma^2 x(x-1) - r = 0$, increase in σ reduces β , which shifts $f(S)$ downward keeping g unchanged, to reduce S to S' as is shown in **Figure 2**. Since the budget B is a positive constant, we see that decrease in S is equivalent with increase in m from the budget constraint $B = mS$. Thus, we have the following proposition.

Proposition 1: If the uncertainty increases, the government should increase the compensation per period for the losses in business and shorten the length of the business closure period.

Similarly, increase in B shifts $f(S)$ downward keeping g unchanged, to reduce S to S' as is shown in **Figure 3**. Since decrease in S means increase in m from the budget constraint $B = mS$, we have the following proposition.

Proposition 2: If the government's budget increases, the government should increase the compensation per period for the losses in business and shorten the length of the business closure period.

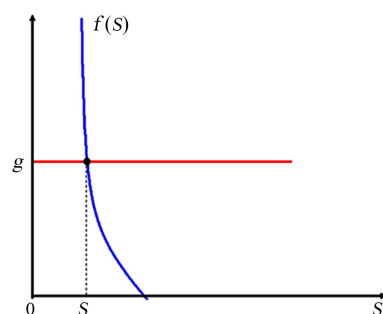


Figure 1. Determination of S .

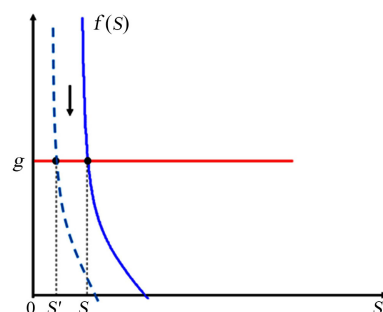


Figure 2. Effect of increase in σ on the optimal S .

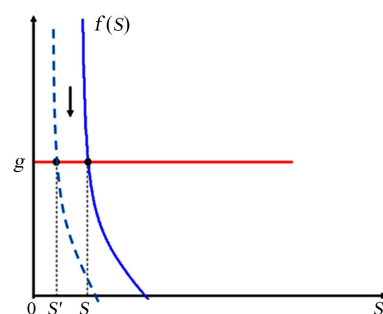


Figure 3. Effect of increase in B on the optimal S .

5. Conclusion

The present paper combined the game theory and the optimal stopping theory to formulate the strategic interaction between firm and government, and based on this formulation, examined how much the government should compensate firms for their losses during the business closure period so as to induce the firm to shut its businesses voluntarily as a result of its maximizing problem. The optimal length of the business closure period S is shown graphically as in **Figure 1**, which in turn determines the optimal amount of money the government pays to the firm in compensation for the losses in business per period as $\frac{B}{S}$, where B is the budget of the government. We also revealed that if the uncertainty or the government's budget increases, the government should increase the compensation per period for the losses in business and shorten the length of the business closure period.

In the present paper, we made simplifying assumptions such as that profit drops to zero at the moment of the business closure and it continues to stay at zero during the business closure periods, profits after the business closure is constant at $Y - \frac{c}{S}$, where Y is the profit with no infected households, c is a positive parameter and S is the length of the business closure period. Thus, it is necessary to improve the model by relaxing such simplifying assumptions and examine the robustness of our results in a more general framework. It is also of interest to make other assumptions on the government's objective than minimization of expected costs. We will undertake such analysis in future research.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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