World Journal of Nuclear Science and Technology, 2023, 13, 73-90
https://www.scirp.org/journal/wjnst
ISSN Online: 2161-6809
ISSN Print: 2161-6795

Simulation of the Traweling Wave Burning Regime on Epithermal Neutrons

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How to cite this paper: Tarasov, V., Chernezhenko, S., Korduba, I. and Vashchenko, V. (2023) Simulation of the Traweling Wave Burning Regime on Epithermal Neutrons. *World Journal of Nuclear Science and Technology*, **13**, 73-90.
https://doi.org/10.4236/wjnst.2023.134006

Received: August 25, 2023
Accepted: October 28, 2023
Published: October 31, 2023

Abstract

New results of two computer experiments on modeling of superthermal neutron-nuclear combustion of natural uranium for two different flux densities of external neutron source and duration of half a year each are presented. The simulation results demonstrate the dependence of the autowave combustion modes on the parameters of the external source.

Keywords

Wave Reactor, Computer Modeling, Neutron Nuclear Combustion, Neutron Thermal Spectrum, Natural Uranium Combustion

1. Introduction

Today, in addition to the first two results of numerical modeling of epithermal burning of natural uranium on epithermal neutrons, presented in our previous work [1], we have the results of several more calculations confirming the wave burning of natural uranium in the epithermal region at that, one of them is a repetition of the second numerical calculation from [1] (recall that in [1] this calculation was performed for the flux density of an external neutron source equal to 10^{23} $cm^{-2}\cdot s^{-1}$, and the simulation time was 48 days.), but for a significantly longer simulation time of 150 days, and another calculation was carried out for an external source with a significantly lower flux density equal to 10^{15} $cm^{-2}\cdot s^{-1}$ and for a simulation time of 150 days.

2. Objective

To carry out computer experiments to simulate the neutron-nuclear combustion of natural uranium for two different flux densities of an external neutron source.

3. New Results of Modeling of Wave Neutron-Nuclear Burning of Natural Uranium At Suprathermal Neutrons

Below in Figures 1-5 are the results of modeling for the external source neutron flux density equal to 10^{23} n/cm^{2}s, and in Figures 5-10 for the neutron density of the external source equal to 10^{15} n/cm^{2}s. Below in Figures 1-5 show the simulation results for the neutron flux density of an external source equal to 10^{23} cm^{-2}·s^{-1}, and Figures 6-10 for the neutron density of an external source equal to 10^{15} cm^{-2}·s^{-1}.

The kinetic system of 20 equations, initial and boundary conditions remained the same as in [1], except for the neutron flux density of an external source. The

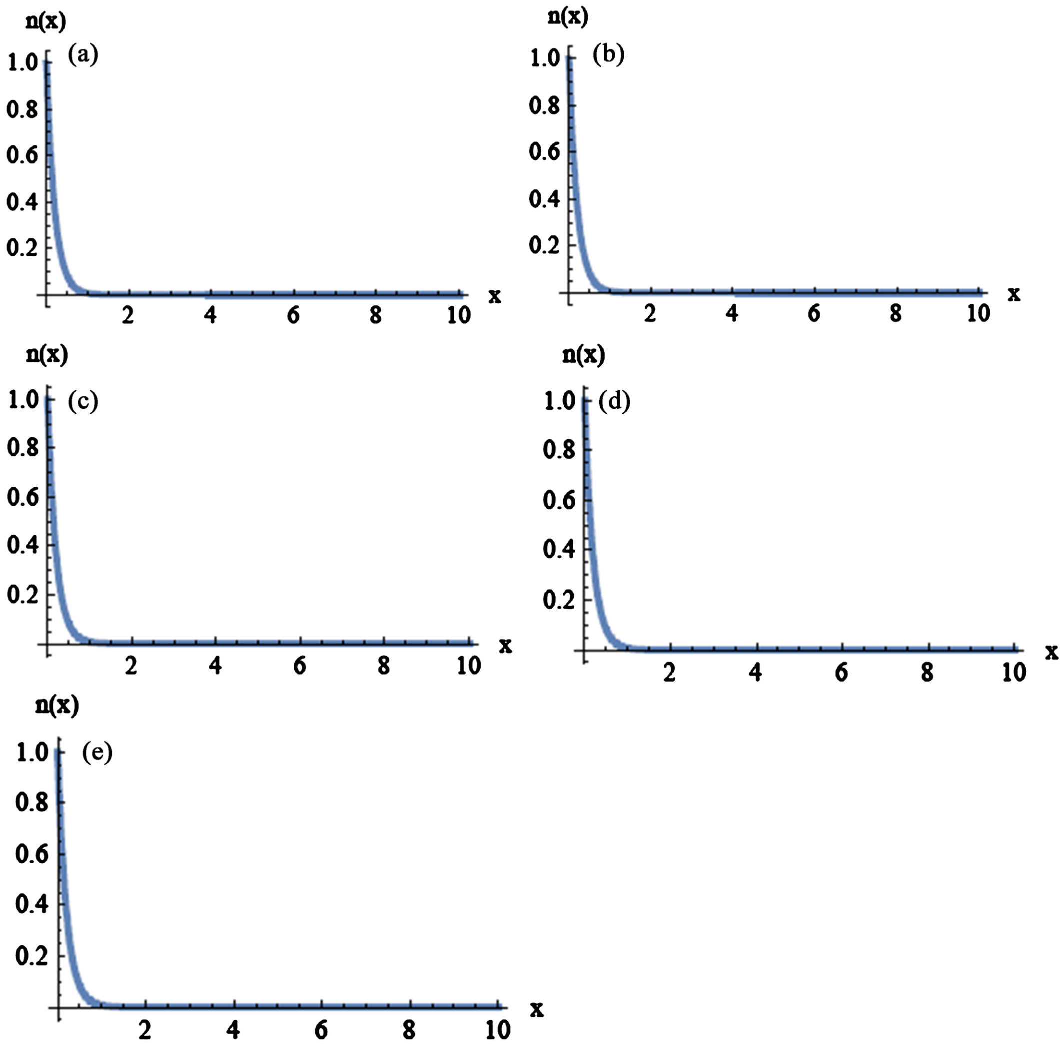

Figure 1. The kinetics of neutrons in the wave neutron-nuclear burning of natural uranium (a) the dependence of the dimensionless neutron density on the spatial coordinate $n^*(x)$ for the time point of calculation $t = 30$ days; (b) $n^*(x)$ for $t = 60$ days; (c) $n^*(x)$ for $t = 90$ days; (d) $n^*(x)$ for $t = 120$ days; (e) $n^*(x)$ for $t = 150$ days).

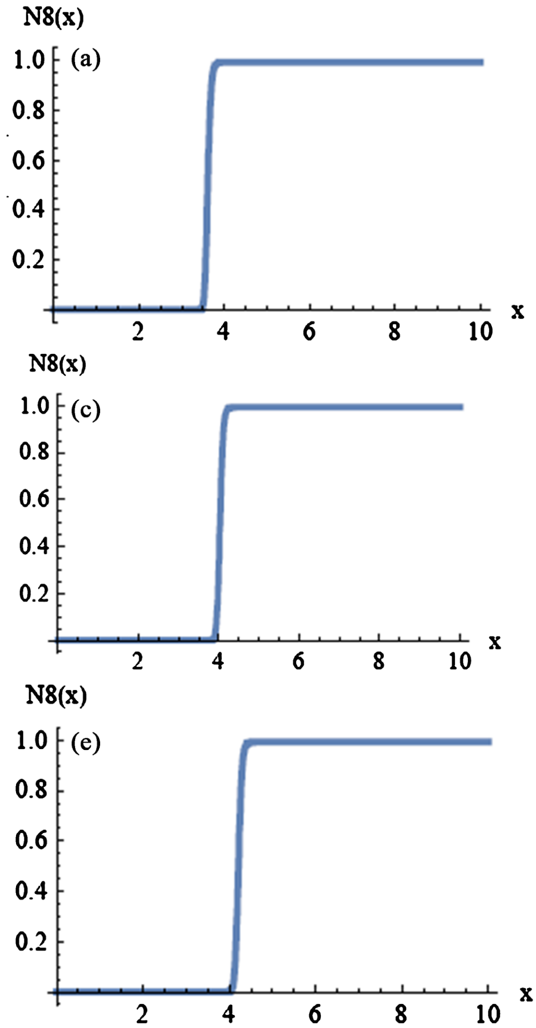

Figure 2. Kinetics of the density of uranium 238 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of uranium 238 nuclei on the spatial coordinate $N_8^*(x)$ for the time point of calculation t = 30 days; (b) $N_8^*(x)$ for t = 60 days; (c) $N_8^*(x)$ for t = 90 days; (d) $N_8^*(x)$ for $t = 120$ days; (e) $N_8^*(x)$ for $t = 150$ days).

kinetic system of 20 equations, the initial and boundary conditions remained the same as in [1], except for the neutron flux density of the external source.

In the present work, as in [1], the numerical solution of the system of kinetic equations was carried out using the Mathematica 8 software package and, to optimize the process of numerical solution of the system of equations, we passed to dimensionless quantities

$$n(x,t) \to n^*(x,t) \text{ и } N_i(x,t) \to N_i^*(x,t),$$

According to the following relationships:

$$n(x,t) = \frac{\Phi_0}{V_n} n^*(x,t), \quad N(x,t) = \frac{\rho_8 N_A}{\mu_8} N^*(x,t) \tag{1}$$

At the first calculation, the following numerical values of constant coefficients of differential equations were set:

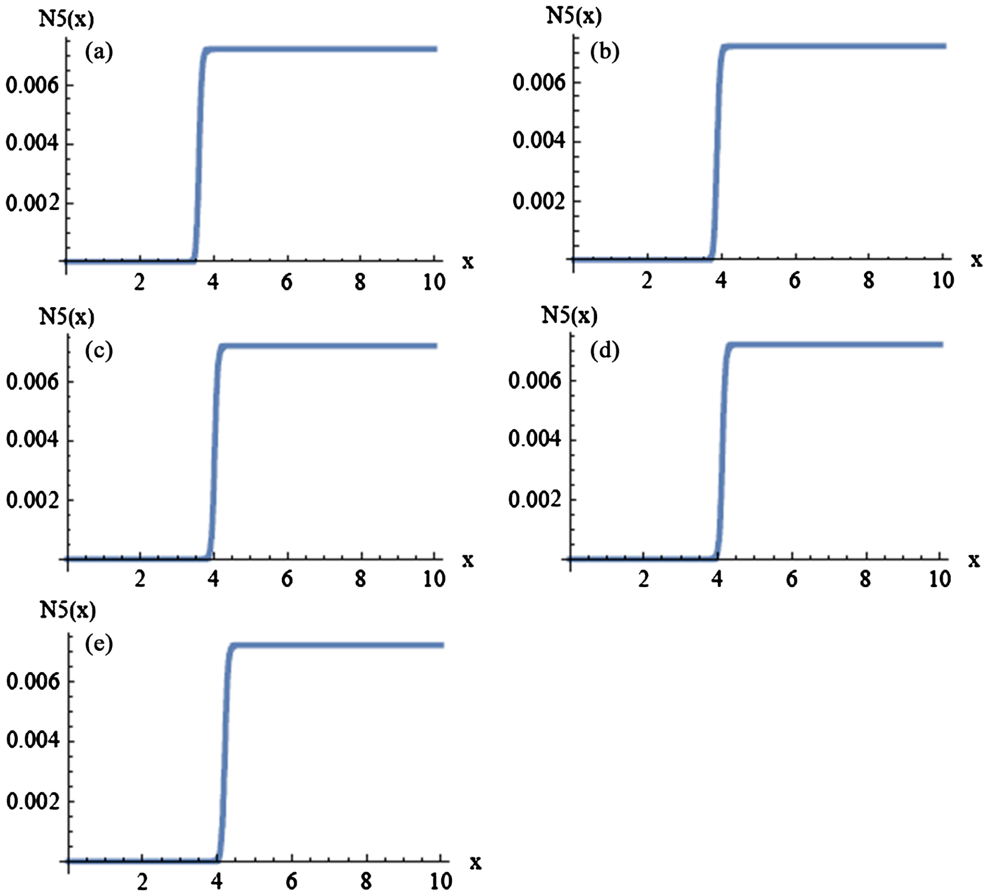

Figure 3. Kinetics of the density of uranium 235 nuclei during wave neutron-nuclear burning of natural uranium (a) the dependence of the dimensionless density of uranium 235 nuclei on the spatial coordinate $N_5^*(x)$ for the time point of calculation t = 30 days; (b) $N_5^*(x)$ for t = 60 days; (c) $N_5^*(x)$ for $t = 90$ days; (d) $N_5^*(x)$ for $t = 120$ days; (e) $N_5^*(x)$ for $t = 150$ days).

$$
\begin{aligned}
&D = 2.0\times10^{4}\ \text{cm}^2/\text{s};\ F_0 = 1.0\times10^{23}\ \text{cm}^{-2}\cdot\text{s};\ \tau_\beta \sim 3.3\ \text{days};\ \nu^{(Pu)} = 2.90;\\
&\nu^{(5)} = 2.41;\ \sigma_f^{Pu} = 477.04\times10^{-24}\ \text{cm}^2;\ \sigma_c^{Pu} = 286.15\times10^{-24}\ \text{cm}^2;\\
&\sigma_c^{8} = 252.50\times10^{-24}\ \text{cm}^2;\ \sigma_f^{5} = 136.43\times10^{-24}\ \text{cm}^2;\ \sigma_c^{5} = 57.61\times10^{-24}\ \text{cm}^2;\\
&\sigma_c^{9} = 4.80\times10^{-24}\ \text{cm}^2;\ \sigma_c^{eff(Pu)} = 10.10\times10^{-24}\ \text{cm}^2;\\
&\sigma_c^{i(Pu)} = 1.00\times10^{-24}\ \text{cm}^2, i = 1\text{ - }6;\ \sigma_c^{i(5)} = 1.00\times10^{-24}\ \text{cm}^2, i = 1\text{ - }6;\\
&\sigma_c^{eff(5)} = 10.10\times10^{-24}\ \text{cm}^2;\ T_1^{(Pu)} = 54.28\ \text{s};\ T_2^{(Pu)} = 23.04\ \text{s};\ T_3^{(Pu)} = 5.60\ \text{s};\\
&T_4^{(Pu)} = 2.13\ \text{s};\ T_5^{(Pu)} = 0.62\ s;\ T_6^{(Pu)} = 0.26\ \text{s};\ p_1^{(Pu)} = 0.072\times10^{-3};\\
&p_2^{(Pu)} = 0.626\times10^{-3};\ p_3^{(Pu)} = 0.444\times10^{-3};\ p_4^{(Pu)} = 0.685\times10^{-3};\\
&p_5^{(Pu)} = 0.180\times10^{-3};\ p_6^{(Pu)} = 0.093\times10^{-3};\ p^{(Pu)} = \sum_{i=1}^{6} p_i^{(Pu)} = 0.0021;\\
&\sigma_c^{eff} = 1.10\times10^{-24}\ \text{cm}^2;\ T_1^{(5)} = 55.72\ \text{s};\ T_2^{(5)} = 22.72\ \text{s};\ T_3^{(5)} = 6.22\ \text{s};\\
&T_4^{(5)} = 2.30\ \text{s};\ T_5^{(5)} = 0.61\ \text{s};\ T_6^{(5)} = 0.23\ \text{s};\ p_1^{(5)} = 0.210\times10^{-3};\\
&p_2^{(5)} = 1.400\times10^{-3};\ p_3^{(5)} = 1.260\times10^{-3};\ p_4^{(5)} = 2.520\times10^{-3};\\
&p_5^{(5)} = 0.740\times10^{-3};\ p_6^{(5)} = 0.27\times10^{-3};\ p^{(5)} = \sum_{i=1}^{6} p_i^{(5)} = 0.0064.
\end{aligned}
\tag{2}
$$

Figures 1-5 show the results of a calculation that coincides with the second numerical experiment in [1], but for a significantly longer burning time, that is, for external source neutron flux density of 10^{23} n/cm^{-2}·s and simulation time equal to 150 days.

The length of the fissile medium, in which the wave of neutron-nuclear burning propagates, is 1000 cm, the total simulation time is t = 150 days, the time step is $\Delta t = 10$ minutes, and the step along the spatial coordinate.

$$\Delta x = 1\ \text{cm}$$

From the presented results, for example, Figure 8 shows that after 90 days (Figure 5(c)), the amplitude of plutonium 239 concentration reaches a steady-state maximum and the plutonium maximum has shifted by 20 cm during the simulation time of 60 days, which allows us to make an estimate of the

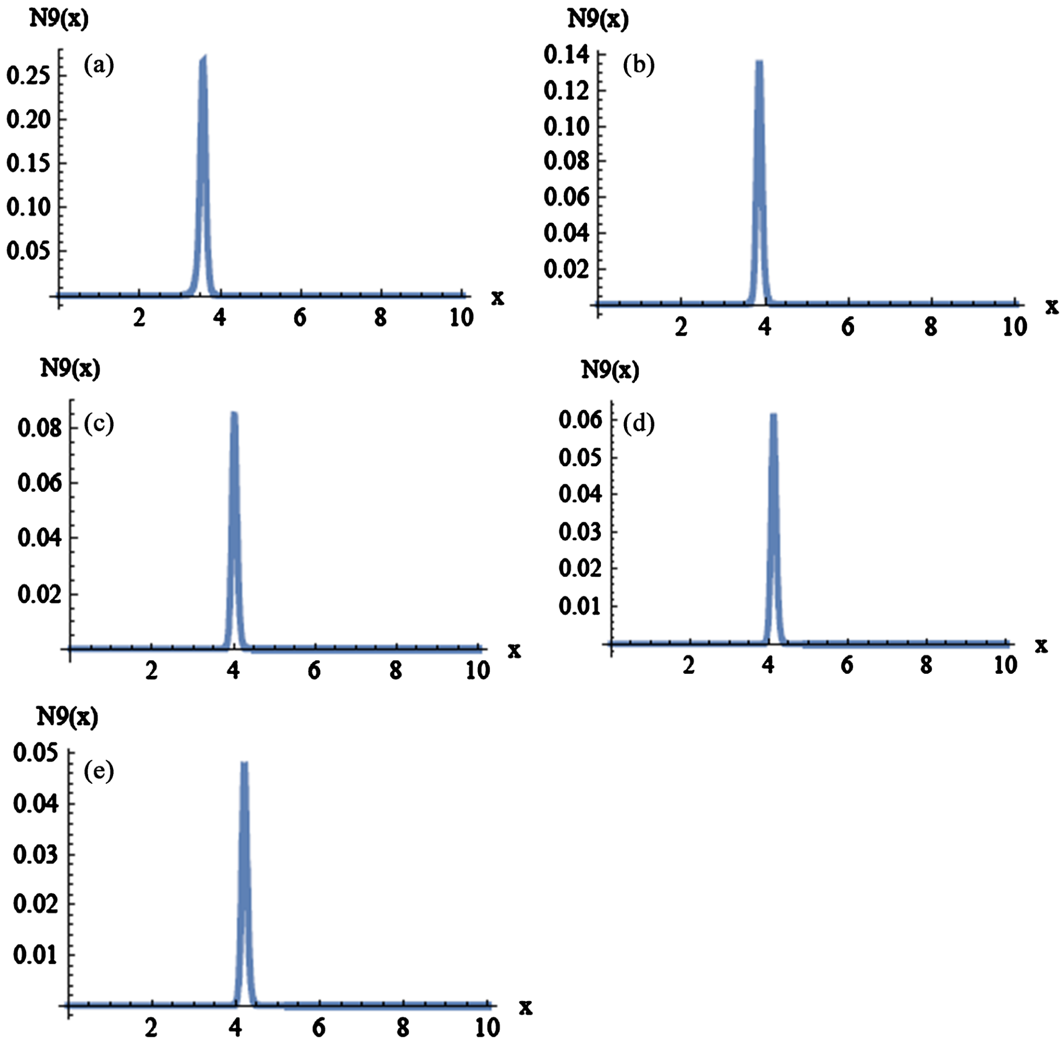

Figure 4. Kinetics of the density of uranium 239 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of uranium 239 nuclei on the spatial coordinate $N_9^*(x)$ for the time point of calculation t = 30 days; (b) $N_9^*(x)$ for t = 60 days; (c) $N_9^*(x)$ for t = 90 days; (d) $N_9^*(x)$ for $t = 120$ days; (e) $N_9^*(x)$ for $t = 150$ days).

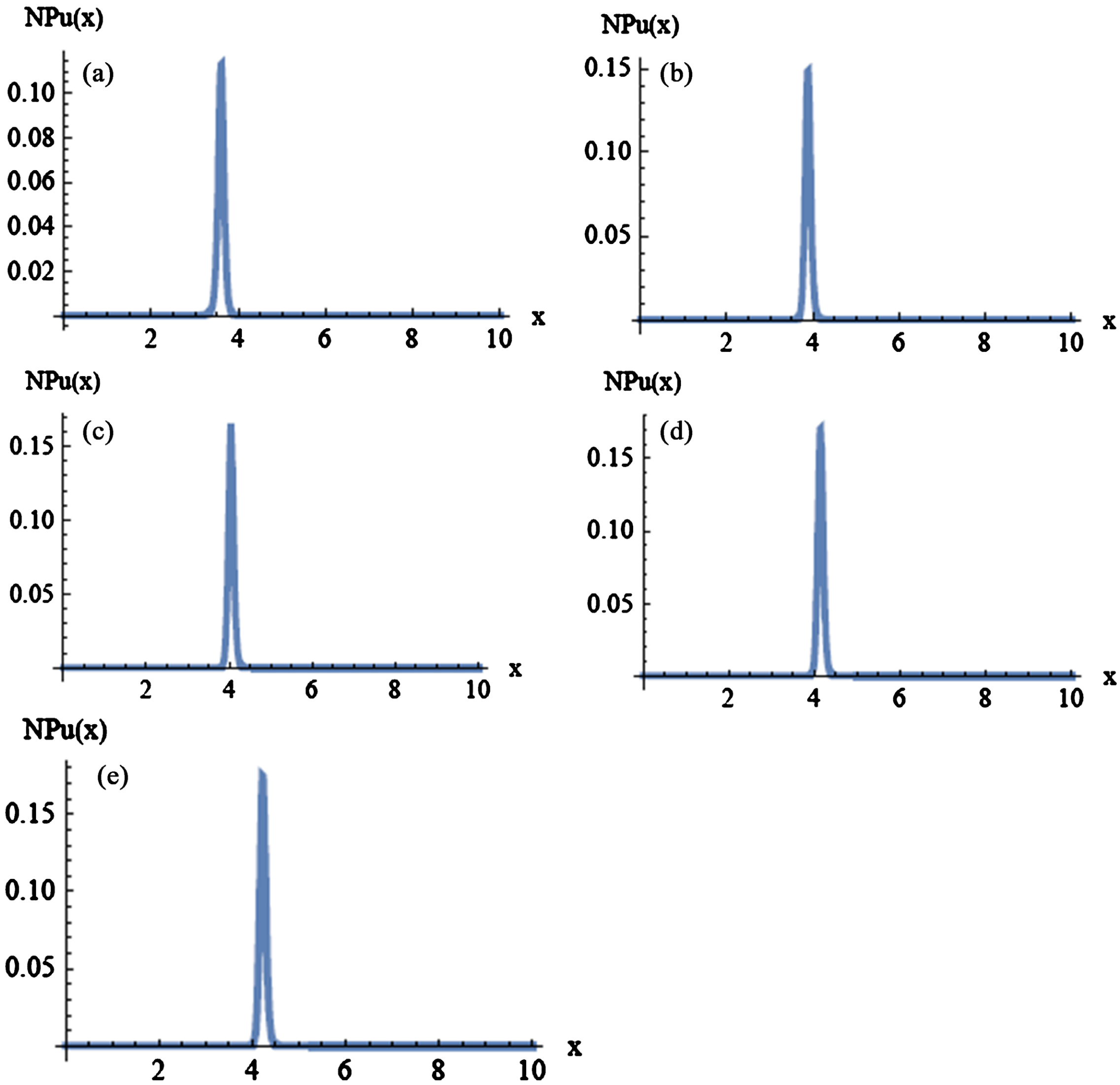

Figure 5. Kinetics of the density of plutonium 239 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of plutonium 239 nuclei on the spatial coordinate $N^*_{Pu}(x)$ for the time point of calculation $t = 30$ days; (b) $N^*_{Pu}(x)$ for $t = 60$ days; (c) $N^*_{Pu}(x)$ for $t = 90$ days; (d) $N^*_{Pu}(x)$ for $t = 120$ days; (e) $N^*_{Pu}(x)$ for $t = 150$ days).

steady-state wave burning rate, which is approximately equal to 0.39×10^{-5} cm/sec.

Note that in [1] for the second numerical experiment, we could not make this estimate because of the short simulation time of 48 days.

Figures 6-10 show the calculation results for the same kinetic system, the same basic calculation constants (see (2)), and the same initial and boundary conditions, with the exception of the value for the neutron flux density of the external source equal to 10^{15} cm^{-2}·s^{-1}, and the length the fissile medium in which the wave of neutron-nuclear burning propagates is 1000 cm, the total simulation time is $t = 150$ days, the time step is $\Delta t = 5$ minutes, and the step along the spatial coordinate $\Delta x = 1$ cm.

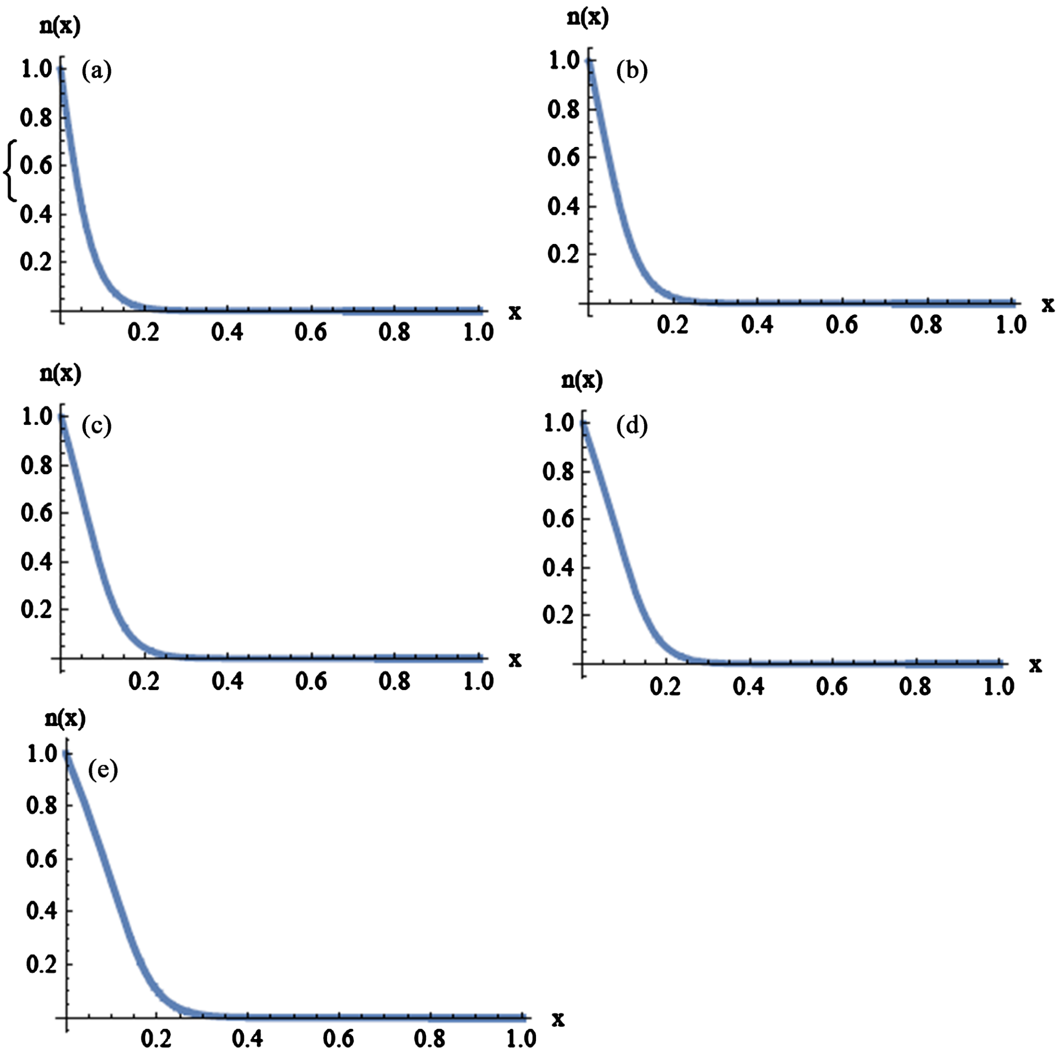

Figure 6. The kinetics of neutrons in the wave neutron-nuclear combustion of natural uranium (a) the dependence of the dimensionless neutron density on the spatial coordinate $n^*(x)$ for the time point of calculation $t = 30$ days; (b) $n^*(x)$ for $t = 60$ days; (c) $n^*(x)$ for $t = 90$ days; (d) $n^*(x)$ for $t = 120$ days; (e) $n^*(x)$ for $t = 150$ days).

From the presented results, for example, in Figure 10 it can be seen that after 60 days the amplitude of the concentration of plutonium 239 reaches a steady-state maximum and the maximum of plutonium during the simulation equal to 90 days has shifted by 60 cm, which allows us to estimate the rate of steady-state wave burning, which is approximately equal to 0.77×10^{-5} cm/c.

A comparative analysis of the results of these two numerical experiments shows that with a decrease in the neutron flux density of the external source from 10^{23} cm^{-2}·s^{-1} to 10^{15} cm^{-2}·s^{-1} with all other constants unchanged (in the first calculation, only the calculation time step was 10 min, and in the second 5 min) the wave combustion rate increased approximately twofold, and the wave half-width also increased significantly (evaluation was carried out using plutonium 239) from 10 cm to 100 cm.

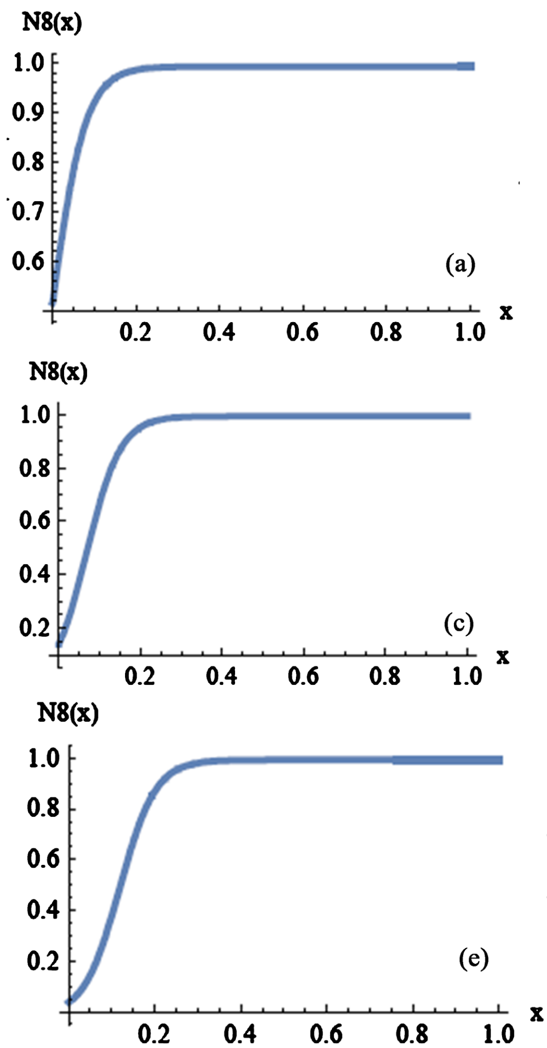

Figure 7. Kinetics of the density of uranium 238 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of uranium 238 nuclei on the spatial coordinate $N_8^*(x)$ for the time point of calculation $t = 30$ days; (b) $N_8^*(x)$ for $t = 60$ days; (c) $N_8^*(x)$ for $t = 90$ days; (d) $N_8^*(x)$ for $t = 120$ days; (e) $N_8^*(x)$ for $t = 150$ days).

An explanation of the change in the parameters of the combustion regime, such as the phase velocity and the width of the wave burning zone, when only the flux density of the external source is changed, we can give, if we recall the assumption of a quantum-mechanical analogy for the diffusion equation of neutrons, that is, the need to fulfill the Bohr-Somerfeld quantization rule for solutions of this equation put forward by Feoktistov L.P. in [2], which was studied in [3], and later developed by V. D. Rusov into the quantum-statistical Wigner distribution for phase velocities of wave combustion, for example [4] [5].

Indeed, this allowed us to write in [4] [5] the critical condition for steady-state wave burning in the form:

$$I = \int \sqrt{\frac{n^{Pu239}}{n_{crit}^{Pu239}} - 1}\, \mathrm{d}x = \frac{\pi}{2}, \tag{3}$$

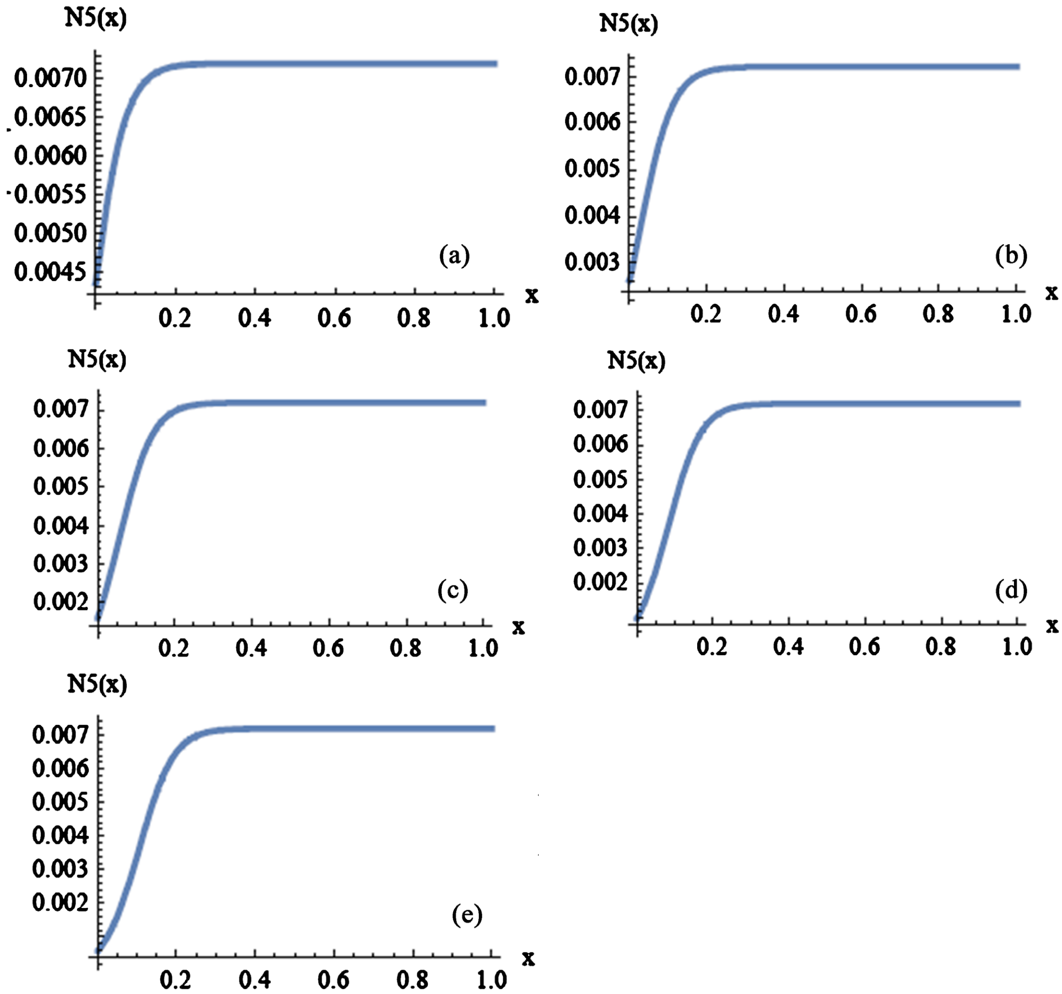

Figure 8. Kinetics of the density of uranium 235 nuclei during wave neutron-nuclear burning of natural uranium (a) the dependence of the dimensionless density of uranium 235 nuclei on the spatial coordinate $N_5^*(x)$ for the time point of calculation $t = 30$ days; (b) $N_5^*(x)$ for $t = 60$ days; (c) $N_5^*(x)$ for $t = 90$ days; (d) $N_5^*(x)$ for $t = 120$ days; (e) $N_5^*(x)$ for $t = 150$ days).

where the integral is taken over the supercritical region

$$(n^{Pu239} > n_{crit}^{Pu239})$$

The simulation results show that even when only the flux density of the external source changes in our simplified kinetic system, the width of the burning region changes, and, therefore, according to (3), the ratio between the quantities n^{Pu239} and n_{crit}^{Pu239} should change, since the value of the integral should remain constant.

From this we can conclude that the equilibrium-stationary and critical concentration of plutonium 239 changes, which are included as parameters in the Wigner quantum-statistical distribution for phase velocities of wave combustion [4] [5], which causes a change in the wave burning velocity.

Note that the presented simulation results clearly demonstrate the influence of the parameters of an external neutron source on the parameters of the burning

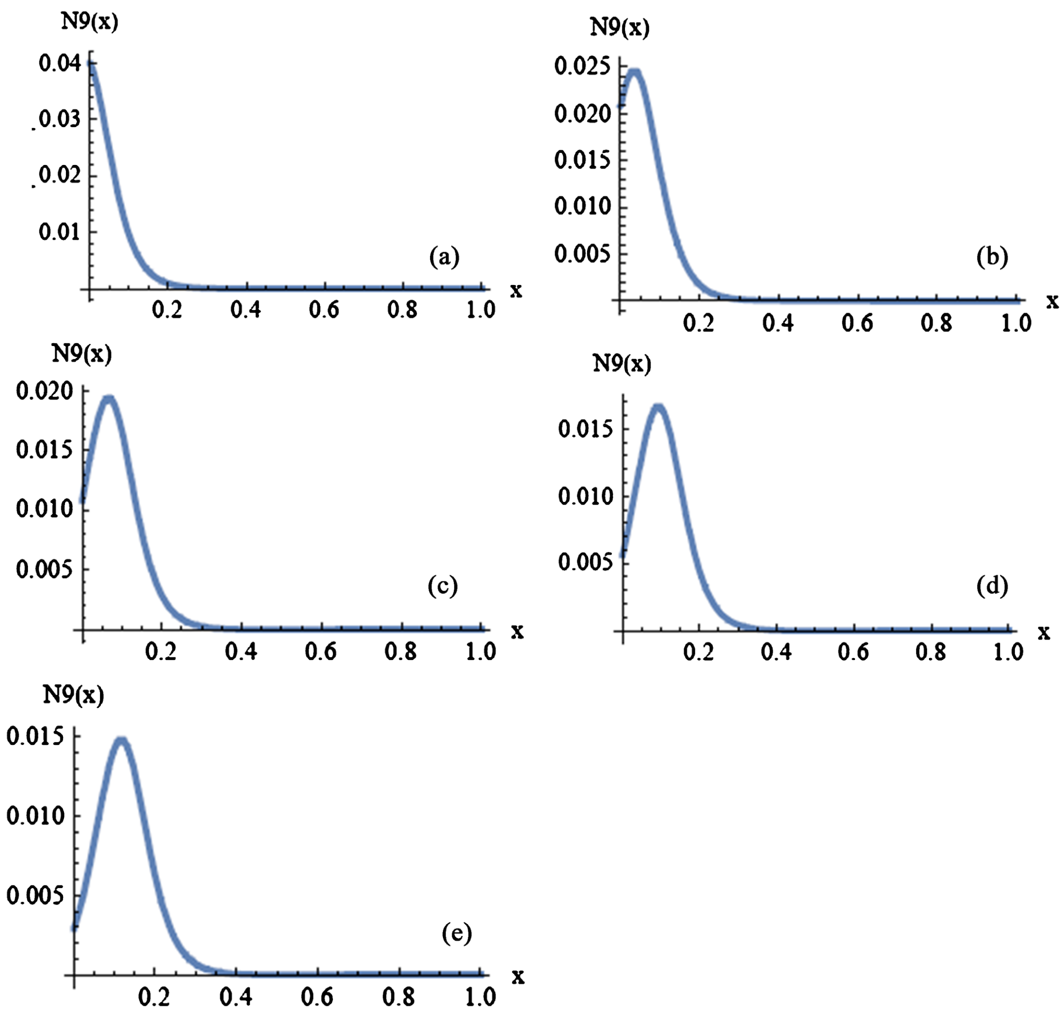

Figure 9. Kinetics of the density of uranium 239 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of uranium 239 nuclei on the spatial coordinate $N_9^*(x)$ for the time point of calculation $t = 30$ days; (b) $N_9^*(x)$ for $t = 60$ days; (c) $N_9^*(x)$ for $t = 90$ days; (d) $N_9^*(x)$ for $t = 120$ days; (e) $N_9^*(x)$ for $t = 150$ days).

regime even for the simplified model under consideration. In a real process of nuclear burning, not only the flux density of an external neutron source, but also its energy spectrum will have a significant effect on the combustion regime.

This is in good agreement with the physical mechanisms that ensure the reproduction coefficient and the effect of the parameters of an external neutron source on the wave burning regime necessary for the implementation of the autowave mode of burning and with the general provisions of nonlinear nonequilibrium thermodynamics presented [6].

According to the theory of a soliton-like neutron wave of slow nuclear burning, developed on the basis of the theory of quantum chaos in [4] [5], neutron-nuclear burning rates must satisfy the Wigner quantum-statistical distribution. The phase velocity u of a soliton-like neutron wave of nuclear burning is determined by the following approximate equality:

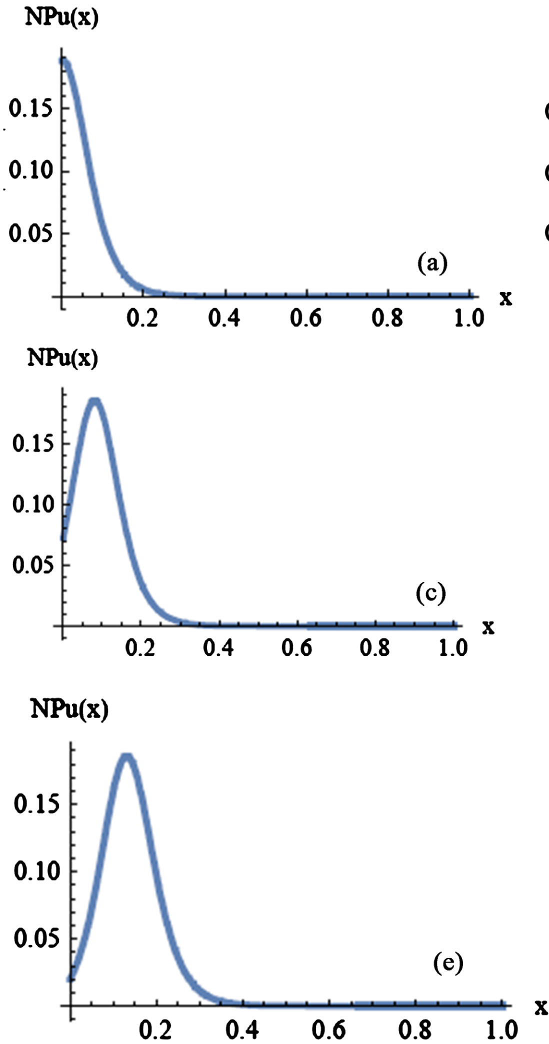

Figure 10. Kinetics of the density of plutonium 239 nuclei upon wave neutron-nuclear burning of natural uranium (a) dependence of the dimensionless density of plutonium 239 nuclei on the spatial coordinate $N_{Pu}^*(x)$ for the time point of calculation $t = 30$ days; (b) $N_{Pu}^*(x)$ for $t = 60$ days; (c) $N_{Pu}^*(x)$ for $t = 90$ days; (d) $N_{Pu}^*(x)$ for $t = 120$ days; (e) $N_{Pu}^*(x)$ for $t = 150$ days).

$$\Lambda(a_*) = \frac{u\tau_\beta}{2L} \cong \left(\frac{8}{3\sqrt{\pi}}\right)^6 a_*^4 \exp\left(-\frac{64}{9\pi}a_*^2\right), \quad a_*^2 = \frac{\pi^2}{4}\cdot\frac{N_{crit}^{Pu}}{N_{eq}^{Pu} - N_{crit}^{Pu}} \qquad (4)$$

where $\Lambda(a_*)$ is a dimensionless invariant, depending on the parameter a_*; N_{eq}^{Pu} and N_{crit}^{Pu} are the equilibrium and critical concentrations of Pu^{239}, L is the mean free path of neutrons, τ_β is the delay time, connected with birth of an active (fissile) isotope and equal to the effective period of the β, decay of compound nuclei in the Feoktistov uraniumeplutonium cycle.

To check the correspondence between the Wigner distribution (4) and the phase velocity of the slow neutron-nuclear burning of natural uranium in the epithermal region of neutron energies, which is obtained by numerical simulation, we will make the corresponding estimates of the parameter a_*^2 and invariant $\Lambda(a_*)$.

For this, we can use the data of numerical simulation at an external source flux density of 10^{15} cm^{-2}·s^{-1}, shown in **Figure 10(e)** for plutonium 239.

From **Figure 10(e)**, one can find $N_{eq}^{Pu} \approx 0.19 \times 4.8 \times 10^{22}\ \text{cm}^{-3}$ (maximum on the concentration curve for plutonium 239) and $N_{crit}^{Pu} \approx 0.10 \times 4.8 \times 10^{22}\ \text{cm}^{-3}$ (inflection point on the concentration curve for plutonium 239).

Then, according to (4), for the parameter a_* and invariant $\Lambda(a_*)$ we obtain the following estimates:

$$a_* = \sqrt{\frac{\pi}{4} \times \frac{0.10 \times 10^{22}\ \text{cm}^{-3}}{0.19 \times 10^{22}\ \text{cm}^{-3} - 0.10 \times 10^{22}\ \text{cm}^{-3}}} = \sqrt{\frac{\pi}{4} \times \frac{0.10}{0.09}} \approx 1.66\,,\quad \Lambda(a_*) \approx 0.30 \quad (5)$$

An estimate of the parameter a_* and invariant $\Lambda(a_*)$ obtained for the slow neutron-nuclear burning of natural uranium in the epithermal region of neutron energies (1.0 - 7.0 eV) is shown in **Figure 11**.

An estimate of the phase velocity of the neutron-nuclear burning of natural uranium in the epithermal region of neutron energies, obtained from the above numerical simulation results for an external source flux density of 10^{15} cm^{-2}·s^{-1}, as already noted above, is approximately equal to:

$$u_{calc} \approx 60\ \text{cm}/(90 \times 24 \times 3600\ \text{s}) \approx 0.77 \times 10^{-5}\ \text{cm/s}\,. \quad (6)$$

The average mean free path for neutrons of the indicated epithermal region of neutron energies *i*.

$$L = \frac{1}{\Sigma_a} = \frac{1}{\bar{\sigma}_c^9 N_8(t=0)} \approx \frac{1}{4.68 \times 10^{-24}\ \text{cm}^2 \times 4.8 \times 10^{22}\ \text{cm}^{-3}} \approx 4.45\ \text{cm}\,. \quad (7)$$

According to expression (4), we also obtain an estimate for the phase velocity of burning:

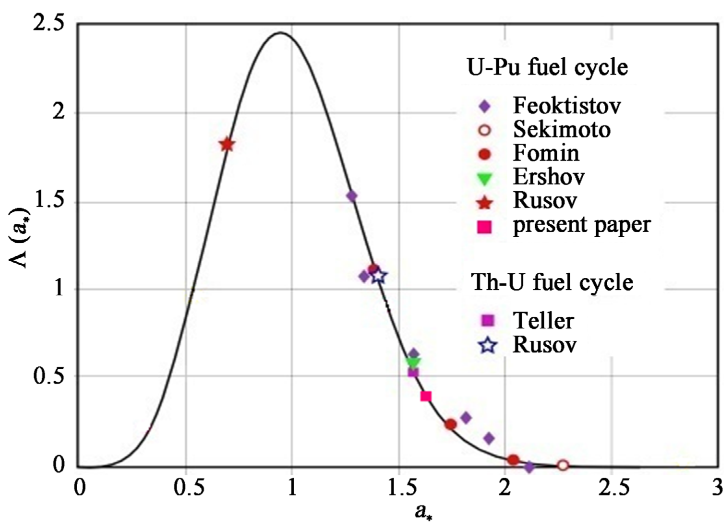

Figure 11. Theoretical (solid line) and calculated (points) dependence for the phase velocities of neutron-nuclear burning $\Lambda(a_*) = u\tau_\beta/2L$ on the parameter a_*, presented in [4] [5] and supplemented by the estimate obtained in this work for slow wave burning of natural uranium in the epithermal region of neutron energies (1.0 - 7.0 eV).

$$u = \frac{2 \cdot \Lambda \cdot L}{\tau_\beta} \approx \frac{2 \times 0.30 \times 4.45 \text{ cm}}{2.85 \times 10^5 \text{ s}} \approx 0.94 \times 10^{-5} \text{ cm/s} . \quad (8)$$

A comparison of the obtained values of (6) and (8) allows us to conclude that they are in good agreement.

Based on the presented simulation results, it is possible to obtain an estimate of the thermal power emitted in the reactor from the active zone from natural uranium in the form of a cylinder with a diameter of 50 cm if it implements a wave burning regime that coincides with the regime obtained in the simulation for an external source with a neutron flux density of 10^{15} $\text{cm}^{-2}\cdot\text{s}^{-1}$.

Indeed, given that the derivative of the concentration of plutonium 239 with respect to time and the derivative with respect to the coordinate for the mode of wave burning are related by the following relation:

$$\frac{\mathrm{d}N^{Pu239}}{\mathrm{d}t} = u \frac{\mathrm{d}N^{Pu239}}{\mathrm{d}x} , \quad (10)$$

where u is the wave burning rate, according to the simulation results presented in **Figure 10(e)**, it is possible to calculate the specific thermal power P_{spec} :

$$\begin{aligned} P_{spec} &\approx 210.3 \text{ MeV} \times \frac{\mathrm{d}N^{Pu239}}{\mathrm{d}t} = 210.3 \text{ MeV} \times u \frac{\mathrm{d}N^{Pu239}}{\mathrm{d}x} \\ &\approx 210.3 \text{ MeV} \times 0.77 \times 10^{-5} \times 0.48 \times 10^{22} \frac{1}{\text{cm}^3 \cdot \text{s}} \approx 77.73 \times 10^{17} \frac{\text{MeV}}{\text{cm}^3 \cdot \text{s}} \\ &\approx 77.73 \times 10^{23} \times 1.6 \times 10^{-19} \frac{\text{J}}{\text{cm}^3 \cdot \text{s}} \approx 1.24 \frac{\text{MW}}{\text{cm}^3} \end{aligned} \quad (11)$$

The estimation of the volume of the burning zone for such a reactor is equal to:

$$V_{region\ burning} \approx \pi \cdot r_{A3}^2 \cdot L \approx 3.14 \times (25 \text{ cm})^2 \times 4.45 \text{ cm} \approx 8733.13 \text{ cm}^3 \quad (12)$$

The thermal capacity of the reactor can now be estimated:

$$P = P_{spec} \cdot V_{region\ burnig} \approx 1.24 \frac{\text{MW}}{\text{sm}^3} \times 8733.13 \text{ sm}^3 \approx 10829.08 \text{ MW} \quad (13)$$

Thus, we obtained an estimate of the thermal power of the considered wave reactor, which is 3.6 times higher than the thermal power of the VVER-1000 reactor.

It should be noted that presented on **Figure 1** and **Figure 6**, the results of the kinetics for the neutron density, as well as in [1], do not demonstrate a neutron wave, in contrast to the previously published results (for example, [4] [5]) of neutron-nuclear burning of uranium 238 for fast neutrons (neutron energy ~ 1 MeV).

The authors, as in our work [1], believe that one of the possible explanations for this may be the following. The system of differential equations was numerically solved with respect to the dimensionless variables according to relations (1), and when dimensionless, the neutron density was divided by the flux density of the external source, which was specified by a specially overestimated value in order to reduce the calculation time and equal to $\Phi_0 = 10 \times 10^{23}$ $\text{cm}^{-2}\cdot\text{s}^{-1}$ (modeling .calculations in [1] and the results of calculations in **Figures 1-5** in this paper). Therefore, the difference in the scales of the flux density of the external source and the neutron flux density in the nuclear combustion region in the regime of steady-state wave burning does not allow us to see the neutron wave.

Although for the results of the second numerical simulation presented in this work in **Figures 6-10**, the neutron flux density was significantly lower compared to other calculations of burning in the epithermal region and was set equal to 10^{15} cm^{-2}·s^{-1}, however, as can be seen from the simulation results for neutrons presented in **Figure 6**, the neutron wave is still not visible.

It is possible that in this case as well the reason is the difference in the scale of the quantities.

To clarify this problem with neutrons, we carried out several numerical simulations, in which the power of an external source gradually decreased after wave burning reached a steady state. The formulation of such a numerical experiment was based on the principles of nonlinear nonequilibrium thermodynamics for open physical systems, which was discussed in [6].

Indeed, in nonlinear nonequilibrium open physical systems, stationary states can be realized under constant boundary conditions and when the Prigogine criterion is fulfilled.

In our problem, an external neutron source acts on the boundary and its shutdown can destroy the regime of steady-state wave burning.

The following modeling calculations were carried out, and all the constants remained the same as in the above numerical calculations and only the flux density of the external source changed.

In the first calculation of this kind, the initial neutron density of the external source was 10^{23} cm^{-2}·s^{-1}, the simulation time was 150 days, the time step was $\Delta t = 10$ minutes, the step in spatial coordinate $\Delta x = 1$ см, information was output every 30 days of the calculation.

At the junction of the second and third months, the neutron flux density of the external source decreased smoothly by an order of magnitude from the initial value.

In the obtained graphs for neutrons and nuclides, nothing qualitatively changes in comparison with the results presented in **Figures 1-5**, but for plutonium 239, the peak of plutonium itself becomes slightly higher and reaches a value $0.2 \times 4.8 \times 10^{22}$ cm^{-3}.

In the second calculation, the initial neutron density of the external source was 10^{23} cm^{-2}·s^{-1}, the simulation time was 720 days, the time step was $\Delta t = 60$ minutes, the step in spatial coordinate $\Delta x = 1$ см, information was output every 60 days of the calculation.

At the junction of the second and third months, the neutron flux density of the external source decreased smoothly by 2 orders of magnitude from the initial value.

In the obtained graphs for neutrons and nuclides, nothing qualitatively changes in comparison with the results presented in **Figures 6-10**, only the peak of plutonium 239, as in the previous calculation, becomes slightly higher and reaches a value $0.2 \times 4.8 \times 10^{22}$ cm^{-3}.

Apparently, such a decrease in the neutron flux density is not enough to detect a neutron wave.

It is also possible that the wave burning of plutonium 239 is not visible in the figures for the neutron density, since the burning of uranium 235 is superimposed on it.

Indeed, the results of modeling the kinetics of the density of uranium 235 nuclei presented in **Figure 8** show that the burn-up region of uranium 235 is ahead of the burning region of plutonium 239 and that uranium 235 is almost completely burned out.

Here, the superposition of the burning processes of uranium 235 and plutonium 239 occurs, which can cause some new phenomena.

Recently, another problem has emerged that needs to be resolved.

There are scientific groups which, similarly to how L.P. did it Feoktistov in his work [2], investigate the wave burning process by searching for a stationary solution of a simplified kinetic system in the auto wave approximation, that is, a solution asymptotic in time (or coordinate) is sought, for example, [7] [8].

In this case, a zero external neutron source is specified at the boundary.

For the stationary solutions found, it was found that a critical state is maintained in the burning zone, that is, there is no transition through the critical state.

This contradicts the physical model of the burning process, which is the basis of almost all works on wave neutron-nuclear burning and involves the transition of the burning region to the supercritical state, for example, works [1] [2] [5] [9] [10] [11] [12] and the present work.

In this connection, the results of [12] are interesting, in which the dynamics of the system reaching the stationary regime during wave burning with fast neutrons is studied.

Based on the results obtained by the Monte Carlo method and presented in **Figure 12**, which shows the dependence of k_{eff} (excluding feedbacks) on time during the initialization of the system and entering the self-regulating burning mode, the authors conclude that the process of going to the hospital takes 20 years, and the stationary period is 24 years (before the waves reach cylinder ends), during which the k_{eff} value remains almost constant.

The authors also study the stability of the wave-burning mode and conclude that only with positive values of the parameter ρ (in the presence of a reactivity margin in the system) is the stability of this mode ensured.

It is clear that in order to answer all these questions, it is necessary to carry out new modeling calculations with varying input calculation parameters, for example, initial concentrations of the nuclide composition of the fuel medium, cross sections of nuclides taking into account the neutron energy spectrum (multi-group approximation), and also the derivation of not only information about

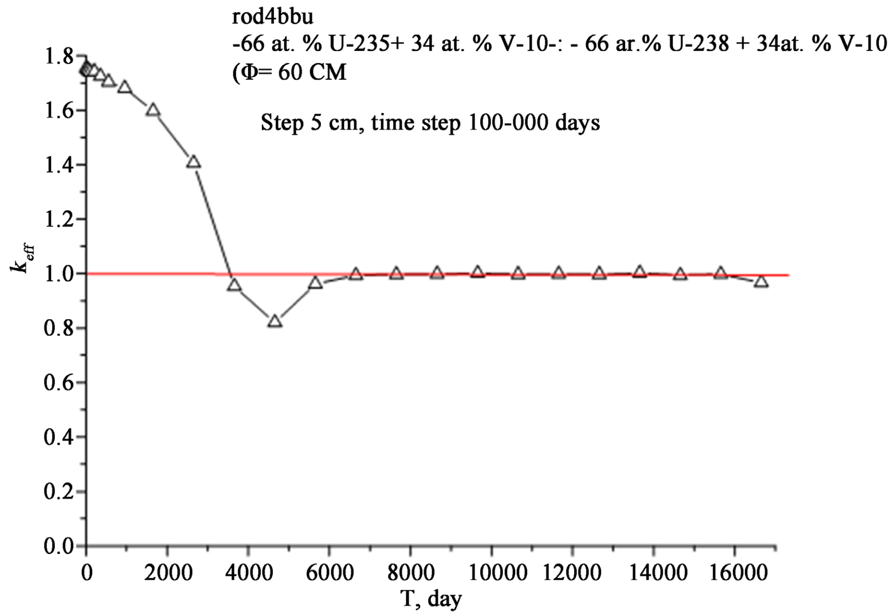

Figure 12. The dependence of k_{eff} on time [12].

the kinetics neutrons and nuclides, but also about the kinetics of the critical concentration of plutonium in the burning zone and the kinetics of the density of the neutron source.

Of course, work is needed to search for the composition of the core and its structure, in which the epithermal neutron spectrum is realized.

As an example of works in which similar studies are carried out, we can cite the work [13].

And also, it is necessary to carry out simulation of wave burning taking into account heat transfer [4] [5] [14].

4. Conclusions

New results of two computer experiments on modeling the epithermal neutron-energy complex are presented.

Note that the presented simulation results clearly demonstrate the influence of the parameters of an external neutron source on the parameters of the burning regime even for the simplified model under consideration.

In a real process of nuclear burning, not only the flux density of an external neutron source, but also its energy spectrum will have a significant effect on the burning regime.

This is in good agreement with the physical mechanisms that ensure the reproduction coefficient and the effect of the parameters of an external neutron source on the wave burning regime necessary for the implementation of the autowave mode of burning and with the general provisions of nonlinear nonequilibrium thermodynamics presented [6].

Based on the results of wave burning simulation, an estimate was obtained of the thermal power of the epithermal wave nuclear reactor with a cylindrical homogeneous core with a diameter of 50 cm from natural uranium and a moderator, assuming that it implements a wave burning regime that matches the simulation results. The resulting estimate of the power of such a reactor is 10,829.08 MW, which is 3.6 times higher than the thermal power of the VVER-1000 reactor.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Rusov, V.D., Tarasov, V.A., Eingorn, M.V., Chernezhenko, S.A., *et al.* (2015) Ultraslow Wave Nuclear Burning of Uranium-Plutonium Fissile Medium on Epithermal Neutrons. *Progress in Nuclear Energy*, **83**, 105-122. https://doi.org/10.1016/j.pnucene.2015.03.007

[2] Feoktistov, L.P. (1989) Neutron-Fission Wave. *Reports of the Academy of Sciences*, **309**, 864-867.

[3] Ershov, A.P. and Anisichkin, V.F. (2003) Natural Neutron-Fission Wave. *Combustion, Explosion and Shock Waves*, **39**, 226e231. https://doi.org/10.1023/A:1022925403499

[4] Rusov, V.D., Linnik, E.P., Tarasov, V.A., Zelentsova, T.N., Vaschenko, V.N., Kosenko, S.I., Beglaryan, M.E., Chernezhenko, S.A., *et al.* (2011) Traveling Wave Reactor and Condition of Existence of Nuclear Burning Soliton-Like Wave in Neutron-Multiplicating Media. *Energies*, **4**, 1337-1361. https://doi.org/10.3390/en4091337

[5] Rusov, V.D., Tarasov, V.A., Sharf, I.V., Vaschenko, V.M., Linnik, E.P., Zelentsova, T.N., Beglaryan, M.E., Chernegenko, S.A., *et al.* (2015) On Some Fundamental Peculiarities of the Traveling Wave Reactor Operation. *Science and Technology of Nuclear Installations*, **2015**, Article ID 703069. https://doi.org/10.1155/2015/703069

[6] Rusov, V.D., Tarasov, V.A., Eingorn, M.V., Chernezhenko, S.A., Kakaev, A.A., Smolyar, V.P., *et al.* (2020) Simulation of the Traveling Wave Burning on Epithermal Neutrons on the Year Time Scale. arXiv preprint arXiv:2003.11820

[7] Pavlovich, V.N., Khotayintsev, V.N. and Khotayintseva, E.N. (2008) Physical Basis of the Nuclear Combustion Wave Reactor. I. *Nuclear Physics and Energetics*, **2**, 39-49.

[8] Pavlovich, V.M., Khtoyintsev, V.M. and Khtoyintseva, O.M. (2010) Reactor on a Nuclear Combustion Wave: Control of Wave Parameters.

[9] van Dam, H. (2000) Self-Stabilizing Criticality Waves. *Annals of Nuclear Energy*, **27**, 1505-1521. https://doi.org/10.1016/S0306-4549(00)00035-9

[10] Sekimoto, H. (2006) Light a Candle. An Innovative Burnup Strategy of Nuclear Reactors. 2*nd International Conf. on Quantum Electrodynamics and Statistical Physics* (*QEDSP*2006), Kharkov Ukraine, 19-23 September, 2006, 306-317. http://www.nr.titech.ac.jp/coe21/eng/index.html

[11] Osborne, A.G. and Deinert, M.R. (2013) Comparison of Neutron Diffusion and Monte Carlo Simulations of a Fission Wave. *Annals of Nuclear Energy*, **62**, 269-273. https://doi.org/10.1016/j.anucene.2013.06.023

[12] Gann, V.V., Abdulaev, A.M. and Gann, A.V. (2010) Benchmark of the Traveling Wave Reactor Using MCNPX Code. NSC "Kharkov Institute of Physics and Tech-

nology", Kharkov, 24 c.

[13] Cerullo, N., Chersola, D., Lomonaco, G. and Marotta, R. (2013) The Use of GFR Dedicated Assemblies in the Framework of Advanced Symbiotic Fuel Cycles: An Innovative Way to Minimize Long-Term Spent Fuel Radiotaxicity. 12*th Information Exchange Meeting on Actinide and Fission Product Partitioning and Transmutation*, *OECD* 2013, Pisa, 10 p.

[14] Rusov, V.D., Tarasov, V.A., Vaschenko, V.M., Linnik, E.P., Zelentsova, T.N., Beglaryan, M.E., Chernegenko, S.A., *et al.* (2013) Fukushima Plutonium Effect and Blow-Up Regimes in Neutron-Multiplying Media. *World Journal of Nuclear Science and Technology*, **3**, 9-18. https://doi.org/10.4236/wjnst.2013.32A002