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# Uniform Flow of Molten Metals in Rectangular Open Channels

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## Abstract

The flow of liquids in open channels has been studied since ancient Rome. However, the vast majority of published reports on flow in open channels are focused on the transport of drinking water and sewage disposal. The literature on the transport of molten metals in open channels is quite scarce. In this work, the uniform flow of pig iron and molten aluminum in rectangular open channels is studied. Specific energy curves are constructed and critical heights are analytically determined. The transition from subcritical to supercritical flow is analyzed as a function of the angle of inclination of the channel and the roughness of its walls. Manning's equation is applied to the pig iron flow using data reported in the literature for molten aluminum. The need to correct the roughness coefficient for pig iron is observed in order to obtain results consistent with those previously reported.

## Keywords

Critical Height, Froude Number, Molten Metal Flow, Open Channel, Rectangular Channel, Uniform Flow

## 1. Introduction

An essential aspect of metallurgical processes is fluid flow. The flow of molten metals, molten slag and hot gases undoubtedly determine the efficiency, feasibility and effectiveness of metallurgical equipment and processes [1]. The pig iron from blast furnaces is transported to the torpedo ladles, and the molten aluminum is transported from the scrap melting furnaces to the degassing unit through open channels. Despite the widespread use of open channel transport of molten metals in the iron and aluminum metallurgical industries [2] [3], only a very limited number of publications have been published enabling open channel design and flow characterization.

Flow in prismatic open channels has been studied since ancient Rome [4]. In a prismatic channel the cross section is equal along its length, and the slope of its bottom remains constant. However, the vast majority of published books and papers on flow in prismatic open channels are focused on the transport of drinking water and sewage disposal, e.g. [5] [6] [7] [8]. The flow of molten metals in open channels under the influence of electromagnetic fields has been studied for several years, e.g. [9]-[15]. Unfortunately, these reports are part of the field of Lorentz force velocimetry, so they are focused on the development of flow meters based on the exposure of molten metal to an external electromagnetic field, and not on the characterization of the type of flow either in the design of channels for metallurgical applications.

An early application of the flow of molten metals in open channels can be found in [16], where the flow of molten pig iron from a blast furnace in a rectangular channel is discussed. In uniform flow the velocity of the transported liquid remains constant over time and the length of the channel. Applying a balance between the forces of gravity and the shear stresses on the channel walls, an expression was obtained by the author to calculate the volumetric flow of molten iron as a function of the roughness of the surface, the hydraulic diameter, and the component of gravity in the direction of the channel angle. In a recent work [17] a report on the flow of pig iron in an open channel is presented aimed at studying the metal-slag separation using multi-phase Computational Fluid Dynamics simulations. The authors consider a trapezoidal channel in which natural convection occurs, and conclude that a precise control of the height difference between the iron dam and the slag port is of paramount importance for maintaining the effectiveness of the metal-slag separation during the blast furnace tapping.

The purpose of the investigation of the present work is the characterization of the uniform flow of molten pig iron in a rectangular channel of industrial type. The curves of specific energy of the molten metal against its height are determined in terms of the slope of the channel and the actual roughness of the channel walls. In addition, the Froude number, the critical height and the volumetric flow rate are determined for different values of the angle of inclination and the wall roughness. Besides, the conditions under which the Manning equation is applicable for the case of molten metals are discussed.

## 2. Mathematical Description of Uniform Flow

A uniform flow in an open channel is one in which the flow area and bottom slope remain constant. Similarly, the average velocity is constant both in time and along the length of the channel. For a rectangular channel, a force balance can be formulated as follows [16]:

$$AL\rho g \sin\alpha = \tau_w L\left(2H + b\right) \tag{1}$$

where $A$ is the flow area, $L$ is the channel length, $\rho$ is the density of the molten metal, $g$ is the gravity acceleration, $\alpha$ is the channel inclination angle, $\tau_w$ is the shear stress on the walls, $H$ is the molten metal height, and $b$ is the channel width. For a rectangular channel $A = Hb$. Besides,

$$\tau_w = 2\rho u_m^2 f_f \tag{2}$$

where $u_m$ is the mean velocity and $f_f$ is the is the friction factor, which can be determined from the following expression:

$$f_f = \frac{1}{8}\left(\frac{e_r}{D_h}\right)^{1/3} \tag{3}$$

According to [16], the friction factor is independent of the Reynolds number as long as the Reynolds number is greater than 5000. In the above equation $e_r$ is the actual roughness of the channel walls and $D_h$ is the hydraulic diameter. The latter is determined as follows:

$$D_h = \frac{4A}{P} \tag{4}$$

where $P = 2H + b$ is the wetted perimeter. The hydraulic radius is defined as $R_h = A/P$, so that $D_h = 4R_h$.

Finally, combining the previous equations, in [16] the following expression is reported to determine the volumetric flow rate of molten metal (Q) in the rectangular open channel:

$$Q = AD_h^{2/3} e_r^{-1/6} \left(g \sin\alpha\right)^{1/2} \tag{5}$$

Given that $u_m = Q/A$, from Equation (5) it is obtained that

$$u_m = D_h^{2/3} e_r^{-1/6} \left(g \sin\alpha\right)^{1/2} \tag{6}$$

In an open channel the flow of molten metal occurs only by gravity, so the specific energy of the metal ($E_s$) can be obtained from Bernoulli's equation:

$$E_s = H + \frac{u_m^2}{2g} \tag{7}$$

Given that $u_m = Q/A = Q/(Hb)$, then

$$E_s = H + \left(\frac{Q^2}{2gb^2}\right)\frac{1}{H^2} \tag{8}$$

Differentiating H, letting $\mathrm{d}E_s/\mathrm{d}H = 0$ and solving for $H$ gives

$$H_c = \left(\frac{Q^2}{gb^2}\right)^{1/3} \tag{9}$$

In Equation (9) $H_c$ is the critical height of the molten metal flowing in a rectangular open channel. The critical height of the flow is defined as the condition for which the Froude number is equal to one and in which the specific energy is minimum. The critical height can also be determined from the Froude number, which is defined as

$$Fr = \frac{u_m}{\sqrt{gH}} \tag{10}$$

The critical height is the value of the height of the metal when $Fr = 1$. If $Fr < 1$ (or $H > H_c$) the flow is called subcritical. In this case the role of gravitational forces is more pronounced, so the flow has a low velocity and is considered "calm". If $Fr > 1$ (or $H < H_c$) the flow is called supercritical. In this case the inertial forces are dominant, so the flow has a high velocity and is considered "fast" [5] [18].

## 3. Uniform Flow of Molten Metals

For the uniform flow analysis, molten pig iron from a blast furnace is considered flowing in a rectangular channel $L = 20$ m long, with a width of $b = 0.2$ m [16]. The height of the molten metal was kept at $H = 0.15$ m. The calculated values for the hydraulic diameter, the hydraulic radius, and the flow area are $D_h = 0.24$ m, $R_h = 0.06$ m and $A = 0.030$ m$^2$, respectively. The standard values are $e_r = 5 \times 10^{-4}$ m for the wall roughness and $\alpha = 5$ degrees for the inclination angle. To calculate the Reynolds number, the density of pig iron was considered $\rho = 7100$ kg/m$^3$ and its viscosity $\mu = 5 \times 10^{-3}$ kg/(m.s). The above data yields a reference value for the volumetric flow rate of $Q = 0.038$ m$^3$/s [16]. In the subsequent calculations five values of the angle of inclination were considered, namely 1, 5, 10, 15, and 20 degrees, and five values of the wall roughness were also considered, namely $1 \times 10^{-5}$, $5 \times 10^{-5}$, $1 \times 10^{-4}$, $5 \times 10^{-4}$, and $1 \times 10^{-3}$ m.

**Figure 1** shows the specific energy of the molten metal as a function of its height in the channel for several values of the inclination angle. It was built using Equation (8) considering a wall roughness of $5 \times 10^{-4}$ m and calculating the volumetric flow rate with Equation (5). The critical height was determined using Equation (9), and is shown in **Figure 1** as an ascending red line. This means that the critical height increases as the inclination angle of the channel increases. **Table 1** shows that for an angle of 1 degree the Froude number is less than 1 and therefore

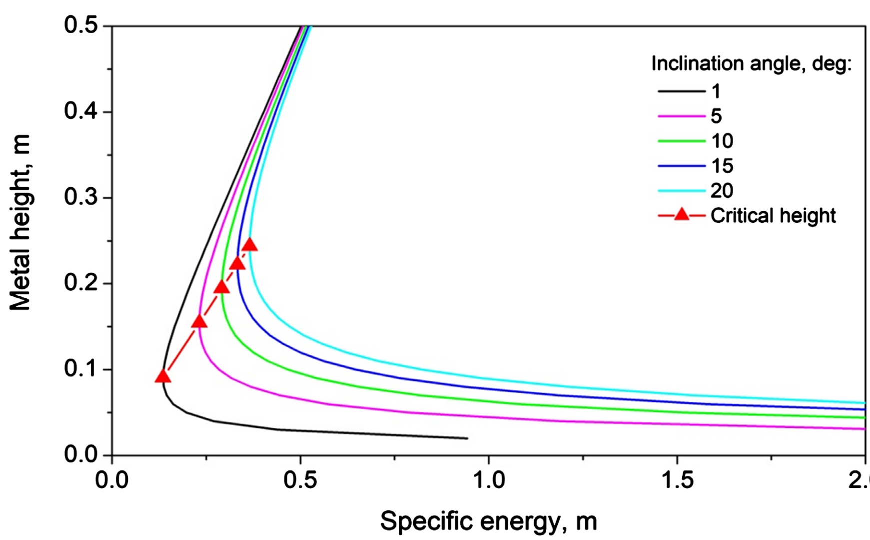


Figure 1. Specific energy of the molten metal as a function of the height of the metal for different values of the angle of inclination of the channel.

the flow is subcritical. For an angle of inclination of 5 degrees and above, the flows are supercritical since their Froude numbers are greater than unity. Table 1 also shows that the volumetric flow rate increases as the angle of inclination of the channel increases. Similarly, in this Table it can be seen that the Reynolds number is always greater than 5000, so that the validity condition of Equation (3) is maintained regardless of the angles of inclination considered. Figure 2 shows the variation of the critical height as a function of the angle of inclination of the channel. An exponential growth of the critical height is observed as the angle of inclination increases. A least squares fit yields the following expression:

$$H_c(\alpha) = 0.26595 - 0.19297\mathrm{e}^{-\alpha/9.7402} \tag{11}$$

The effect of channel wall roughness on critical height and flow type was analyzed. The angle of inclination was kept constant at five degrees, and five roughness values were considered, as can be seen in Figure 3. In this Figure, the behavior of the critical height as a function of roughness is shown with a descending red line. The numerical values of the critical height for each roughness value can be consulted in Table 2, where the corresponding values of the Froude number,

Table 1. Characterization of the uniform flow of molten metal as a function of the angle of inclination of the channel.

| Angle, deg | *Fr* | $H_c$, m | Flow | *Re* | *Q*, m³/s |
|---|---|---|---|---|---|
| 1 | 0.4676 | 0.0904 | Subcritical | $1.93330 \times 10^5$ | $1.7016 \times 10^{-2}$ |
| 5 | 1.0449 | 0.1545 | Supercritical | $4.3196 \text{ x}10^5$ | $3.8026 \times 10^{-2}$ |
| 10 | 1.4749 | 0.1944 | Supercritical | $6.0976 \text{ x}10^5$ | $5.3675 \times 10^{-2}$ |
| 15 | 1.8001 | 0.2220 | Supercritical | $7.4441 \text{ x}10^5$ | $6.5529 \times 10^{-2}$ |
| 20 | 2.0700 | 0.2436 | Supercritical | $8.5541 \text{ x}10^5$ | $7.3529 \times 10^{-2}$ |

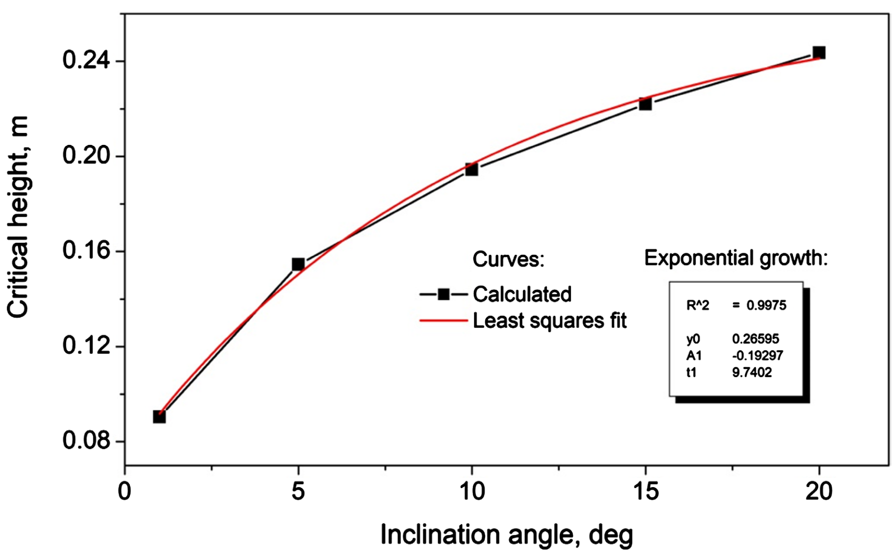


Figure 2. Critical height of the molten metal as a function of the inclination angle of the channel. The red curve shows a least squares fit.

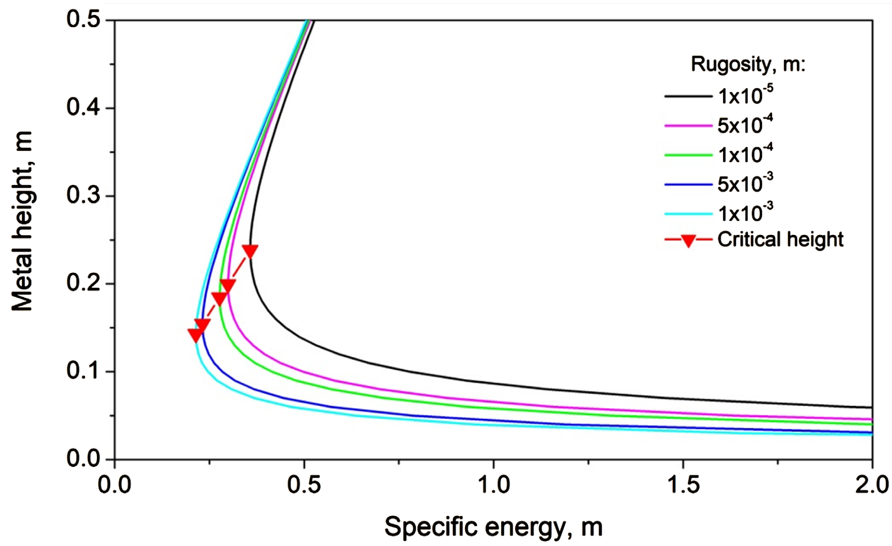


**Figure 3.** Specific energy of the molten metal as a function of the height of the metal for different values of the rugosity of the channel walls.

**Table 2.** Characterization of the uniform flow of molten metal as a function of the rugosity of the channel walls.

| Rugosity, m | *Fr* | $H_c$, m | Flow | *Re* | *Q*, m$^3$/s |
|---|---|---|---|---|---|
| $1 \times 10^{-5}$ | 2.0056 | 0.2386 | Supercritical | $8.2913 \times 10^5$ | $7.2986 \times 10^{-2}$ |
| $5 \times 10^{-5}$ | 1.5337 | 0.1995 | Supercritical | $6.3406 \times 10^5$ | $5.5815 \times 10^{-2}$ |
| $1 \times 10^{-4}$ | 1.3664 | 0.1847 | Supercritical | $5.6488 \times 10^5$ | $4.9725 \times 10^{-2}$ |
| $5 \times 10^{-4}$ | 1.0449 | 0.1545 | Supercritical | $4.3196 \times 10^5$ | $3.8026 \times 10^{-2}$ |
| $1 \times 10^{-3}$ | 0.9309 | 0.1430 | Subcritical | $3.8483 \times 10^5$ | $3.3877 \times 10^{-2}$ |

the type of flow, the Reynods number, and the volumetric flow rate are also shown. Figure 3 and Table 2 show that the critical height of the molten metal in the channel decreases as the roughness increases. The same inverse behavior has the volumetric flow rate, since increasing the roughness increases the resistance to flow. Table 2 also shows that for roughness values above $5 \times 10^{-4}$ the flow goes from supercritical to subcritical. Due to the resistance to flow imposed by the roughness of the channel walls, for high values of roughness the inertial forces lose importance compared to the gravitational forces and the flow becomes "slow". On the other hand, Figure 4 shows the dependence of the critical height on the roughness of the walls. Using a least squares fit, it can be seen that this dependence follows a second-order exponential decay pattern:

$$H_c(e_r) = 0.13866 + 0.06307e^{-e_r/2.77405\times10^{-5}} + 0.05737\mathrm{e}^{-e_r/3.87764\times10^{-4}} \quad (12)$$

Uniform flow in open channels has long been analyzed using the Manning equation:

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2} \quad (13)$$

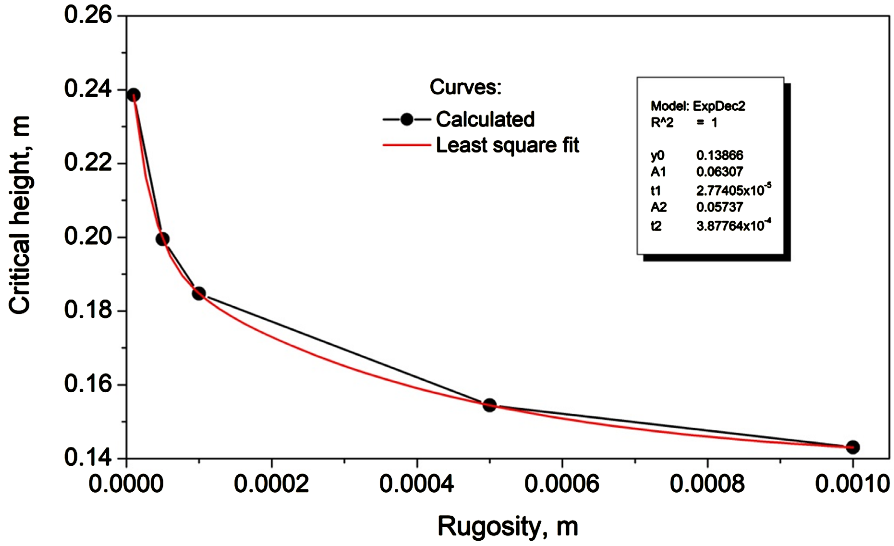


**Figure 4.** Critical height of the molten metal as a function of the rugosity of the channel walls. The red curve shows a least squares fit.

where $n$ is the roughness coefficient of the channel walls and $S_0$ is the slope of the channel. The coefficient $n$ is basically a function of the roughness and nature of channel walls. In the International System of Units $n$ has units of [$m^{-1/3}$ s]. Due to its simplicity and acceptable degree of accuracy, the semi-empirical Manning equation is perhaps the most widely used formula for uniform flow calculations today [19]. In a recent work [15] Manning's equation was used to determine the velocity and depth of molten aluminum flowing in a rectangular open channel, in which the aluminum is exposed to a static magnetic field. Manning equation is used in practice mainly for studying the water flow in natural or artificial channels. The roughness parameter $n$ varies from 0.01 for artificial channels to 0.08 for rivers. To date no available values about the $n$ parameter for liquid metals are reported in the literature. In the above work [15] it is reported that higher values of $n$ for liquid metals are required compared with values for water flow in artificial channels. A value of $n$ equal to 0.1 is considered in [15] to numerically obtain the velocity and depth of molten aluminum in a rectangular channel.

In this work, the volumetric flow rate of the pig iron was obtained using Equation (5). The calculations show that $Q$ = 0.038 $m^3/s$, as reported in [16], for values of the angle of inclination of 5 degrees and wall roughness of $5 \times 10^{-4}$ m. If the value of $n = 0.1$ suggested in [15] is used in Equation (13), the Manning equation yields a value for $Q = 0.0136$ $m^3/s$, along with $S_0$ = tan (5) = 0.08749, which is significantly lower than that reported in [16]. Only by using a value of $n$ = 0.036 in the Manning equation do Equation (5) and Equation (13) give the same result for the volumetric flow rate. Therefore, the value of $n = 0.1$ for molten aluminum reported in [16] does not provide adequate results for molten pig iron. This could be due to the different physical properties (mainly density and viscosity) of pig iron and aluminum.

## 4. Conclusions

The uniform flow of molten pig iron and molten aluminum in rectangular open channels was studied. From the obtained numerical results the following conclusions can be drawn:

1) The critical height of the pig iron in the channel presents an exponential growth with respect to the angle of inclination of the channel. The critical height of the pig iron in the channel exhibits a second-order exponential decay with respect to the roughness of the channel walls.

2) The pig iron flow presents a transition from subcritical to supercritical as the angle of inclination of the channel is increased. From five degrees of inclination onwards the flow is already supercritical.

3) The pig iron flow presents a transition from supercritical to subcritical as the roughness of the channel walls is increased. From a roughness of $1 \times 10^{-3}$ m onwards, the flow becomes subcritical.

4) The volumetric flow rate of molten aluminum and its depth in an open channel can be determined by Manning's equation. However, for pig iron, an adjustment in the roughness coefficient of the Manning equation is required to obtain results similar to those reported in the literature.

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## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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