Theoretical Economics Letters, 2023, 13, 1632-1638
https://www.scirp.org/journal/tel
ISSN Online: 2162-2086
ISSN Print: 2162-2078

Owner-Employee Compensation Game

Robert A. Agnew

Palm Coast, Florida, USA
Email: raagnew1@gmail.com

How to cite this paper: Agnew, R. A. (2023). Owner-Employee Compensation Game. *Theoretical Economics Letters, 13,* 1632-1638.
https://doi.org/10.4236/tel.2023.136093

Received: October 10, 2023
Accepted: December 24, 2023
Published: December 27, 2023

Abstract

A firm typically consists of an owner and capital provider plus key employees who together can create surplus value above individual outside opportunities. Our research approach is to model the firm as a cooperative game across these individual players with specific attention to the core and Shapley value. In our model, Shapley value splits surplus value 50 - 50 between the owner and the group of key employees. This seems intuitively fair but it is not dominant over other allocations in the core, particularly all surplus to the owner and all surplus to the employees. This is a recurring theme in cooperative game theory: Shapley value is a standard of distribution fairness but it is usually not uniquely dominant over other core solutions.

Keywords

Compensation, Cooperative Game Theory, Shapley Value

1. Introduction

Suppose that a firm, including its owner and key at-will employees, produces surplus value-added above what these individuals can obtain elsewhere. Employees have reserve wages (including benefits) in the labor market. The owner also has a reserve "wage" consisting of an available dividend if he redeploys his capital, as either investor or creditor. Just as his employees have outside opportunities, so does the owner in terms of alternative investments. However, the owner has a special role. He owns the firm and he hires the employees, but he can't obtain anything without at least some employees. Moreover, his current team is exceptional at creating value beyond total reserve wages and he has no guarantee that replacements would do as well. How should surplus value be allocated across the owner and his talented employees? What is a "fair" distribution?

We address this question with tools of cooperative game theory, with particular focus on the core and Shapley value. Our game structure assigns reserve wages to employees in coalitions without the owner but they enable value accumulation in coalitions with the owner, scaled by their reserve wages, i.e., their reserve wages reflect not only their alternate value in the labor market but also drive their incremental value contribution to the owner. In this setup, Shapley value gives one-half of surplus value to the owner while the other half is divided proportionately among the employees. However, Shapley value is only one such split in the core. Other splits (including all surplus to owner and all surplus to the employees) are in the core. Hence, there is no absolute answer, although Shapley value is intuitively appealing and it is widely viewed as an instrument for fair distribution.

There is a large game-theoretic literature on firm vs employees. Sungatullina and Sokolov (2015) portray administration vs employees as a zero-sum game. De Mesnard (2018) seeks a fair compensation distribution between senior managers and middle managers. Wan (2019) and Akinola (2021) independently evaluate game models for pay increases and promotions involving firm and employee strategies for high and low competence employees. Wu (2007) examines both noncooperative and cooperative models for firm-supported human capital investments by employees.

Our approach is focused on cooperative compensation bargaining amongst the select group of firm owner and key at-will employees. This follows developments in Stole and Zwiebel (1996a) (SZ) and Brugemann, Gautier, and Menzio (2019) (BGM). SZ developed a sequential (and probabilistic) firm-employee bargaining structure that they claimed led to Shapley value in the corresponding cooperative game. BGM corrected their bargaining structure to achieve that result. They both assumed identical employees although SZ indicated how that assumption could be relaxed. In a tandem publication, Stole and Zwiebel (1996b) examined implications of their findings in organizational design and decision-making for hiring, training, and capital investing. We differentiate employees based on their reserve wages and we don't worry about how bargaining actually occurs, only about what division of firm surplus seems fair.

In the next section, we review requisite elements of cooperative game theory utilizing concepts and notation from Owen (2001), a widely referenced game theory textbook. This is basic information but we include it for easy reference. The following section contains the structure of our employee compensation game, followed by a simple illustration and our conclusions.

2. Elements of Cooperative Game Theory

We have a finite set of players $N=\{1,\cdots,n\}$ and a superadditive characteristic function v defined on subsets of N with $v(\phi)=0$. Nonempty subsets of N are called coalitions. Superadditivity requires that $v(S\cup T)\geq v(S)+v(T)$ whenever $S\cap T=\phi$. In addition, a game is convex if $v(S\cup T)+v(S\cap T)\geq v(S)+v(T)$ for all $S,T\subset N$. We assume throughout that $n\geq 2$ and that v reflects transferable currency like the U.S. dollar.

DOI: 10.4236/tel.2023.136093

An imputation is a vector $x=(x_1,\cdots,x_n)$ such that $x_i \ge v(\{i\})$ and $\sum_{i=1}^{n} x_i = v(N)$. The idea here is that a player must receive at least what he can achieve on his own and that the grand coalition N will ultimately form so the issue is fair distribution of the total pie. There are various solution concepts for cooperative games in terms of imputations but we will focus on two, the core and Shapley value.

We say that imputation y dominates imputation z if $y_i > z_i$ for all i in some nonempty $S \subset N$ and $\sum_{i\in S} y_i \le v(S)$. The core $C(v)$ is the set of undominated imputations and is characterized as the set of imputations x satisfying $\sum_{i\in S} x_i \ge v(S)$ for all $S \subset N$ and $\sum_{i\in N} x_i = v(N)$. If the core is nonempty, imputations outside of it are inherently unstable.

Shapley value is the particular imputation $\phi[v]$ defined for characteristic function v by $\phi_i[v] = \sum_{\substack{S\subset N \\ i\in S}} \frac{(s-1)!(n-s)!}{n!}\left[v(S)-v(S-\{i\})\right]$ where $s=|S|$ = number of elements in set S and $\gamma(S)=\frac{(s-1)!(n-s)!}{n!}=\frac{1}{n\binom{n-1}{s-1}}$ depends only on the size of S. Shapley value is derived axiomatically but it has a heuristic expected value interpretation involving randomly permuted arrivals, all with the same probability $1/n!$. If player i arrives and finds coalition $S-\{i\}$ already there, he receives his marginal value $v(S)-v(S-\{i\})$. Shapley value $\phi_i[v]$ is the expected payoff to player i under this randomization scheme. Shapley value is widely viewed as distributionally fair. If the game is convex, then Shapley value is in the core $C(v)$.

3. Owner-Employee Compensation Game

We define a cooperative game with a firm owner and capital provider (Player 1) and $n-1$ at-will key employees (Players $2,\cdots,n$). These employees are leaders and innovators who are able to create incremental value. Remaining employees are simply expensed along with other normal business costs. For simplicity, we ignore corporate income taxes and we assume that profit flows through to the owner as dividends. Moreover, we ignore corporate growth and assume that the firm is in a profitable steady state.

Each of the individual employees has a reserve annual wage (including benefits) w_i that is available to him in the labor market. Likewise, the owner has a reserve annual dividend w_1 that is available to him if he redeploys his capital elsewhere as an investor or creditor. We assume that the current owner and key employees can together create annual value $V > W = \sum_{i=1}^{n} w_i$ but that the owner is indispensable to *any* surplus value creation by his key employees. Employees can create surplus, in conjunction with the owner, proportionately to their reserve wages. Hence, the CEO plays a bigger role than a VP, attorney, or engineer, but he can still be replaced (without his particular value increment). The owner, on the other hand, can't be readily replaced and he is critical to value creation by his key employees. Hence we define $v(S)=\sum_{i\in S} w_i$ if $1\notin S$ (i.e., employees can't gain anything without the owner) and $v(S)=w_1+\alpha\sum_{\substack{i\in S\\ i\neq 1}} w_i$ if $1\in S$ where $\alpha=\frac{V-w_1}{W-w_1}>1$ (i.e., surplus value is proportional to included employee reserve wages if the coalition includes the owner, but the owner can't gain anything without at least one key employee). It follows in particular that $v(\{i\})=w_i$ for all $i=1,\cdots,n$ and that $v(N)=w_1+\alpha(W-w_1)=V$. Once again, we assume that any key employee can be replaced by the owner at his reserve wage in the labor market but that this replacement doesn't contribute incremental value in the same way as the current incumbent. We proceed to examine how surplus value should be shared in the grand coalition. To simplify notation, we define the set-function $F(S)=\sum_{i\in S} w_i$ for all $S\subset N$ and $F(\phi)=0$ so that $v(S)=F(S)$ if $1\notin S$ and $v(S)=w_1+\alpha F(S-\{1\})$ if $1\in S$.

Proposition 1. The game v is convex.

Proof. Suppose that $S,T\subset N$. If $1\notin S$ and $1\notin T$, then $v(S\cup T)+v(S\cap T)=F(S\cup T)+F(S\cap T)=F(S)+F(T)=v(S)+v(T)$. If $1\in S$ and $1\in T$, then

$$\begin{aligned} v(S\cup T)+v(S\cap T)&=w_1+\alpha F((S\cup T)-\{1\})+w_1+\alpha F((S\cap T)-\{1\})\\ &=w_1+\alpha F(S-\{1\})+w_1+\alpha F(T-\{1\})\\ &=v(S)+v(T) \end{aligned}.$$

If $1\notin S$ and $1\in T$, then

$$\begin{aligned} &v(S\cup T)+v(S\cap T)-v(S)-v(T)\\ &=w_1+\alpha F((S\cup T)-\{1\})+F(S\cap T)-F(S)-w_1-\alpha F(T-\{1\})\\ &=\alpha\left(F((S\cup T)-\{1\})-F(T-\{1\})\right)-F(S)+F(S\cap T)\\ &=(\alpha-1)F(S-T)\geq 0 \end{aligned}.$$

Thus, the game is convex. ■

Definition. B is the set of imputations of the form $x_1=w_1+\beta(V-W)$ and $x_i=(\beta+(1-\beta)\alpha)w_i$ for $i>1$ where $\beta\in[0,1]$, i.e., the owner gets share β of surplus $V-W$ and the employees share $1-\beta$ of the surplus, each proportionately to his reserve wage. This subset B of imputations will be of particular focus for us.

Proposition 2. For an imputation x to be in the core, it is necessary that $w_1\leq x_1\leq w_1+V-W$ and $w_i\leq x_i\leq \alpha w_i$ for $i>1$. Moreover, $B\subset C(v)$, i.e., all of these beta-imputations are in the core. Hence, the core covers all degrees of surplus sharing between the owner and his employees.

Proof. If imputation $x\in C(v)$, then $\sum_{i\in S} x_i\geq v(S)$ for all $S\subset N$ and

$\sum_{i\in N} x_i = v(N)$. It follows that $x_i \ge w_i$ for all *i*,

$x_1 = v(N) - \sum_{i\ne 1} x_i \le V - F(N-\{1\}) = V - (W - w_1) = w_1 + V - W$, and for $i > 1$,

$x_i = v(N) - \sum_{j\ne i} x_j \le V - (w_1 + \alpha F(N - \{i,1\})) = \alpha(W - w_1) - \alpha(W - w_1 - w_i) = \alpha w_i$.

This proves the necessity part.

Now consider imputation *x* with $x_1 = w_1 + \beta(V - W)$ and

$x_i = (\beta + (1-\beta)\alpha) w_i$ for $i > 1$ where $\beta \in [0,1]$. For $1 \notin S$,

$\sum_{i\in S} x_i = (\beta + (1-\beta)\alpha) F(S) \ge F(S) = v(S)$. For $1 \in S$,

$$\begin{aligned}
\sum_{i\in S} x_i &= w_1 + \beta(V - W) + (\beta + (1-\beta)\alpha) F(S - \{1\}) \\
&= w_1 + \beta(\alpha - 1)(W - w_1) + (\beta + (1-\beta)\alpha) F(S - \{1\}) \\
&= w_1 + \beta(\alpha - 1)(F(S - \{1\}) + F(N - S)) + (\beta + (1-\beta)\alpha) F(S - \{1\}) \\
&= w_1 + \alpha F(S - \{1\}) + \beta(\alpha - 1) F(N - S) \\
&\ge w_1 + \alpha F(S - \{1\}) = v(S)
\end{aligned}$$

with equality if $S = N$. Hence, these imputations are in the core for every $\beta \in [0,1]$. ■

Proposition 3. Shapley value imputation *z* puts the surplus sharing between the owner and his employees at 50% with $z_1 = w_1 + (V - W)/2$ and $z_i = (1+\alpha) w_i / 2$ for $i > 1$. Hence, Shapley value is a particular element of *B* with equal surplus sharing between the owner and his employees.

Proof.

$$z_1 = \sum_{\substack{S\subset N \\ 1\in S}} \gamma(S)\left[v(S) - v(S - \{1\})\right] = \sum_{\substack{S\subset N \\ 1\in S}} \frac{w_1 + \alpha F(S - \{1\}) - F(S - \{1\})}{n\binom{n-1}{s-1}}$$

$$= w_1 \sum_{s=1}^{n} \frac{\binom{n-1}{s-1}}{n\binom{n-1}{s-1}} + (\alpha - 1) \sum_{s=2}^{n} \frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}} \sum_{j\ne 1} w_j$$

since there are $\binom{n-1}{s-1}$ ways to select a subset $S \subset N$ containing 1 and there are $\binom{n-2}{s-2}$ ways to select a subset *S* containing 1 and a particular $j \ne 1$.

Hence,

$$\begin{aligned}
z_1 &= w_1 + (\alpha - 1)(W - w_1) \sum_{s=2}^{n} \frac{(s-1)}{n(n-1)} = w_1 + (\alpha - 1)(W - w_1)/2 \\
&= w_1 + (V - w_1 - W + w_1)/2 = w_1 + (V - W)/2
\end{aligned}$$

For $i > 1$,

$$\begin{aligned}
z_i &= \sum_{\substack{S\subset N \\ i\in S}} \gamma(S)\left[v(S) - v(S - \{i\})\right] \\
&= \sum_{\substack{S\subset N \\ i\in S \\ 1\notin S}} \gamma(S)\left[v(S) - v(S - \{i\})\right] + \sum_{\substack{S\subset N \\ i\in S \\ 1\in S}} \gamma(S)\left[v(S) - v(S - \{i\})\right].
\end{aligned}$$

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\notin S}} \gamma(S)\left[v(S)-v(S-\{i\})\right]$$

$$= \sum_{\substack{S\subset N\\ i\in S\\ 1\notin S}} \frac{w_i}{n\binom{n-1}{s-1}} = w_i \sum_{s=2}^{n} \frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}} = w_i \sum_{s=2}^{n} \frac{(s-1)}{n(n-1)} = w_i/2 .$$

$$\sum_{\substack{S\subset N\\ i\in S\\ 1\in S}} \gamma(S)\left[v(S)-v(S-\{i\})\right]$$

$$= \sum_{\substack{S\subset N\\ i\in S\\ 1\in S}} \frac{w_1+\alpha F(S-\{1\})-w_1-\alpha F(S-\{i,1\})}{n\binom{n-1}{s-1}}$$

$$= \alpha w_i \sum_{s=2}^{n} \frac{\binom{n-2}{s-2}}{n\binom{n-1}{s-1}} = \alpha w_i \sum_{s=2}^{n} \frac{(s-1)}{n(n-1)} = \alpha w_i/2$$

. Thus, $z_i = (1+\alpha)w_i/2$. ■

4. Hypothetical Example

The numerical entries in **Table 1** were created by the author for illustration; they seem reasonable but they obviously don't correspond to any real-world enterprise. We have a steady-state firm whose total value-added, beyond normal expenses, is V = \$2,400,000. Reserve wages total W = \$1,800,000 with surplus $V-W$ = \$600,000 to be allocated across the owner and his employees above reserve wages. Shapley assigns half the surplus to the owner and the remaining half proportionately to the employees. This seems reasonable in view of the owner's somewhat higher bargaining power. However, once again, we note that all surplus splits between the owner and his talented employee team are in the core. As in many cooperative games, Shapley value conveys fairness but it is not uniquely dominant.

Table 1. Shapley compensation across firm owner and his key employees ($000).

Player	Reserve Wage	% of Total Reserve	Shapley Comp	Shapley % of $600 Surplus	Shapley % of Total Comp	Shapley % Over Reserve
Owner	$800	44.4%	$1100	50%	45.8%	37.5%
Empl1	$400	22.2%	$520	20%	21.7%	30.0%
Empl2	$300	16.7%	$390	15%	16.3%	30.0%
Empl3	$200	11.1%	$260	10%	10.8%	30.0%
Empl4	$100	5.6%	$130	5%	5.4%	30.0%
Total	$1800	100.0%	$2400	100%	100.0%	33.3%

5. Conclusion

We have modeled the owner-employee compensation game using the tools of cooperative game theory, particularly the core and Shapley value. We believe that our model captures the relative bargaining power of a firm owner and his key employees. Moreover, we conclude that Shapley value provides a fair distribution of surplus between the owner and his employees, specifically 50% to the owner and 50% to the employees. Nevertheless, Shapley value is not a uniquely dominant solution to this game. All surplus to the owner and all surplus to the employees are alternate solutions in the core. This is a recurring issue in cooperative game theory. Shapley value is considered a standard of fair distribution, but it is not dominant over other solutions in the core.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Akinola, A. T. (2021). Game Theory and Motivation among Enterprises and Employees, a Key to Human Resource Management. *International Journal of Economics and Management Sciences, 10,* 1-4.
https://www.hilarispublisher.com/open-access/game-theory-and-motivation-among-enterprises-and-employees-a-key-to-human-resource-management-83193.html

Brugemann, B., Gauther, P., & Menzio, G. (2019). Intra Firm Bargaining and Shapley Values. *Review of Economic Studies, 86,* 564-592.
https://doi.org/10.1093/restud/rdy015

De Mesnard, L. (2018). *Executive Compensation: The Teachings of Game Theory.* SSRN.
https://dx.doi.org/10.2139/ssrn.2584600

Owen, G. (2001). *Game Theory* (3rd ed.). Academic Press.

Stole, L. A., & Zwiebel, J. (1996a). Intra-Firm Bargaining under Non-Binding Contracts. *Review of Economic Studies, 63,* 375-410. https://doi.org/10.2307/2297888

Stole, L. A., & Zwiebel, J. (1996b). Organizational Design and Technology Choice under Intrafirm Bargaining. *American Economic Review, 86,* 195-222.
https://www.jstor.org/stable/2118263

Sungatullina, L. B., & Sokolov, A. Y. (2015). Applying Game Theory to Optimize Expenses for Employees' Remuneration. *Asian Social Science, 11,* 364-368.
https://doi.org/10.5539/ass.v11n11p364

Wan, L. (2019). Nash Equilibrium in the Game of Compensation and Promotion between Enterprises and Employees. *Advances in Economics, Business and Management Research, 91,* 463-468. https://www.atlantis-press.com/proceedings/edmi-19/125914991

Wu, A. (2007). An Analysis of Employee Investment in Specific Human Capital Based on Game Theory. *Journal of Contemporary Management Issues, 12,* 41-56.
https://hrcak.srce.hr/19158