

# Tax Evasion Dynamics via Ising Model Spin $S = 1$

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## Abstract

In this work, we study the problem of tax evasion on Erdős R nyi random graphs. Here, we consider that the agents may be in three different states, namely honest tax payers, tax evaders, and undecided that are individuals in an intermediate class among honests and evaders. Every individual can change his/her state following an Ising model dynamics with spin  $S = 1$ . In addition, we consider the punishment rules of the Zaklan econophysics model, for which there is a probability  $p_a$  of an audit each agent is subject to in every period and a length of time  $k$  detected tax evaders remain honest. The dynamic of temporal evolution of the Zaklan model was studied initially via the equilibrium Ising model with two opinions ( $-1$  and  $+1$ ), and recently via a non-equilibrium three-state kinetic agent-based model on a fully-connected population. Here, through Monte Carlo simulations, we study the problem of the tax evasion fluctuations using an Ising model with spin  $S = 1$  ( $-1$ ,  $0$ , and  $+1$ ) on Erdős R nyi random graphs in the dynamic of the temporal evolution of the Zaklan model. Then, we found that the Ising model with spin  $S = 1$  is as efficient as the Ising model and non-equilibrium three-state kinetic agent-based model in controlling the tax evasion fluctuations. This control is even better when we use strong punishment values  $k$  even for low audit probabilities  $p_a$ .

## Keywords

Econophysics, Sociophysics, Majority Vote, Ising Model

## 1. Introduction

In several branches of science, it is common to formulate problems using structures in networks (graphs). This approach considers the forming parts as well as the interactions of these parts and is able to explain the emergence of effects

arising from emergent behavior. In economic sciences, they are present in a system of interaction between consumers and producers, a network of suppliers for the synthesis of high technology and high added value products. Research in complex networks benefits from the increase of processing power, which allows the analysis of real network data. Initial research of real networks was guided by the models' randomness from Erdős & R enyi (1959). An Erdős R enyi (ER) random graph is a set of  $N$  vertices (sites or nodes) connected by  $c$  links (bonds) (Erdős et al., 1959, 1960, 1961). The probability  $p$  that a given pair of sites is connected by a bond is  $p = 2c/N(N-1)$ . The connectivity of a site is defined as the total number of bonds connected to it, that is  $b_i = \sum_j k_{ij}$ , where  $k_{ij}$  assumes values 1 if there is a link between the sites  $i$  and  $j$  and  $k_{ij} = 0$  otherwise. Networks are completely characterized by the mean number of bonds per site or the average connectivity  $z = p(N-1)$ . The distribution of connectivities is given by the Poisson distribution when  $N \rightarrow \infty$ .

In social systems, humans tend to adopt herding behavior and follow the crowd because they feel more comfortable when their decisions are supported by the like decisions of other people. In financial markets, this behavior is reinforced by so-called noise traders agents, who follow trends and over-react to both good and bad news when they buy and sell. In contrast, some agents find that following the global minority brings the best return, i.e., they buy when noise traders depress prices and sell when noise traders push prices up. These contrarian traders are also called fundamentalists, sophisticated traders or  $\alpha$ -investors (Lux, 1996, 1999, 2000; Bornholdt, 2001; Kaizoji et al., 2002; Takaishi, 2005; Long et al., 1990; Day et al., 1990; Mukherjee & Chatterjee, 2016). They base their decisions not on market euphoria but on rational expectation, and they push prices toward fundamental values. Another classification of agents in terms of two basic financial market strategies has also been investigated using realistic models. These models considered the presence of two subgroups containing individuals who are optimistic or pessimistic about the future development of the market (Lux et al., 1999, 2000).

Does Econophysics make sense? (Stauffer, 1999) Econophysics is an approach to quantitative economy using ideas, models, conceptual and computational methods of statistical physics (Ausloos, 2013). The name econophysics, a hybrid of economy and physics, was coined to describe applications of methods of statistical physics to economy in general. In practice, majority of the research concerned the finances. In such a way, physicists entered officially and scientifically the field of financial engineering. On top of similar statistical methods used by financial mathematicians, physicists concentrated on the analysis of experimental data using tools borrowed from the analysis of real complex systems.

Bloomquist (2006), F ollmer (1974), Andreoni (1998), Lederman (2003), Slemerod (2007), Wintrobe & G erxhani (2004) through an analysis of the social and economic behavior of a community of people fulfilled indicate that tax evasion in a community is a major cause of concern for governments and through em-

pirical evidence Gächter (2006), Frey & Torgler (2006) have provided that the group members or neighborhood of tax evaders are important in deciding whether or not to pay taxes.

In 2008 physicist D. Stauffer and economists Zaklan et al. (2008a, 2008b) and Westerhoff (2008) developed an economics model to study the problem of tax evasion dynamics on a people community using the equilibrium Ising model with spin  $S = 1/2$  (two states  $+1$  and  $-1$ ) on a square lattice (SL) and Monte-Carlo simulations with the Glauber algorithms. On SL the Ising model presents a phase transition well-defined second-order phase transition at critical temperature  $T_c = 2.269\dots$ .

The problem of tax evasion fluctuations can be better described as a non-equilibrium than by an equilibrium system. So, different than Zaklan et al. (2008a, 2008b), Oliveira (1992), Lima (2010) proposed the use of the non-equilibrium Majority-Vote Model (MVM) to study the tax evasion in any complex network (Lima, 2012a). He points out that people do not live in a social equilibrium, and any social noise can drive to a government or market chaos. Then, it is reasonable to expect that a non-equilibrium model (MVM) explains better events of non-equilibrium. In order to show that the problem of tax evasion fluctuations is better described by a non-equilibrium model than by an equilibrium model, Lima (2012b, 2012c, 2015, 2016) studied the Zaklan model (ZM) on Apollonian, Opinion-Dependent, Solomon, and Small-World Networks. In all these cases the tax evasion problem was analyzed using the two-state version of MVM, where the honest agent was rated  $+1$  and the evaded value  $-1$ .

In 2014, Crokidakis (2014) studied the problem of tax evasion via ZM on a fully-connected population. In his work, the agents may be in three different states ( $-1$ ,  $0$ , and  $+1$ ), namely honest taxpayers ( $+1$ ), tax evaders ( $-1$ ) and undecided ( $0$ ). The undecided agents are individuals in an intermediate class among honests and evaders. Each agent can change their state following a kinetic exchange opinion dynamics, where the agents interact by pairs with negative (probability  $p$ ) and positive ( $1 - p$ ) affinity parameters  $\mu_{ij}$ , representing agreement/disagreement between pairs of agents.

Using Monte Carlo simulations, Lima (Lima, 2012d) has studied the Ising model with spin  $S = 1/2$  and  $1$  on ER random graphs, with  $z$  neighbors for each spin. In the case with spin  $S = 1/2$ , the ER graphs present a spontaneous magnetization in the universality class of mean field theory, where on ER random graphs the model presents a spontaneous magnetization at  $p = z/N$  ( $z = 2, 3, \dots, N$ ), but no spontaneous magnetization at  $p = 1/N$  which is the percolation threshold. For ER graphs with spin  $S = 1$ , he finds a first-order phase transition for  $z = 4$  and  $9$  neighbors.

Here, we study the behavior of the tax evasion on an agents community of honest citizens, tax evaders, and undecided where the agents are positioned on sites of ER random graphs, but now using an Ising model with spin  $S = 1$  for the temporal evolution of ZM.

The article is organized as follows: We present, in the next section, the model and some of the Monte Carlo simulation details. In Section 3, the results are presented, and in the final section, some concluding remarks are discussed.

## 2. ZM via Equilibrium Dynamics of Ising Model Spin $S = 1$

We use the ZM via ferromagnetic spin  $S = 1$  Ising model to study the tax evasion fluctuations on a community of homogeneous agents located on ER random graphs. In every time period, each ER random graphs site is inhabited by an agent (individuals) with opinion  $\sigma_i$ , who can either be an honest taxpayer  $\sigma_i = +1$ , a cheater or evader taxpayer  $\sigma_i = -1$ , and undecided or indifferent  $\sigma_i = 0$ . It is assumed that initially, everybody is honest. Each period the agents can rethink their behavior and have the opportunity to become the opposite type of agent they were in the previous period or undecided. In the same way, an undecided individual may become an honest or evader in the next time step. Then, each agent's social on ER random graphs may either prefer tax evasion, reject it or remain indifferent.

The evolution in time of these systems is given by a single spin-flip like dynamics with a probability  $P_i$  given by

$$P_i = 1 / [1 + \exp(2E_i/k_B T)], \quad (1)$$

where  $T$  is the temperature,  $k_B$  is the Boltzmann constant, and  $E_i$  is the energy of the configuration obtained from the Hamiltonian

$$H = -J \sum_{\langle i, j \rangle} \sigma_i \sigma_j, \quad (2)$$

where the sum runs over all neighbor pairs of sites and the spin  $S = 1$  variables  $\sigma_i$  assume values  $\pm 1$  and 0. In the above equation,  $J$  is the exchange coupling. The spin  $S = 1$  case is well known in the literature (Onsager, 1944; Kaufmann, 1949; Baxter, 1982).

The ZM presents an enforcement mechanism that consists of two components: a probability of an efficient audit  $p_a$ ; and a number  $k$  of periods. Then once the tax evasion is detected, the tax evaders,  $\sigma_i = -1$ , can become honest individuals  $\sigma_i = +1$  or undecided  $\sigma_i = 0$  in the presence of an audit probability  $p_a$ , for a number  $k$  of periods. One time unit is one sweep through the entire system. The ordering in the system is measured by the quantity, namely magnetization (average opinion) defined by

$$m = \left\langle \frac{1}{N} \left| \sum_{i=1}^N \sigma_i \right| \right\rangle, \quad (3)$$

where  $\langle \dots \rangle$  denotes configurational average taken at steady states.

The fraction of tax evaders is

$$\text{tax evasion} = \left| \frac{N - N_{\text{honest}}}{N} \right|, \quad (4)$$

where  $N$  is the total number and  $N_{\text{honest}}$  is the honest number of agents. The

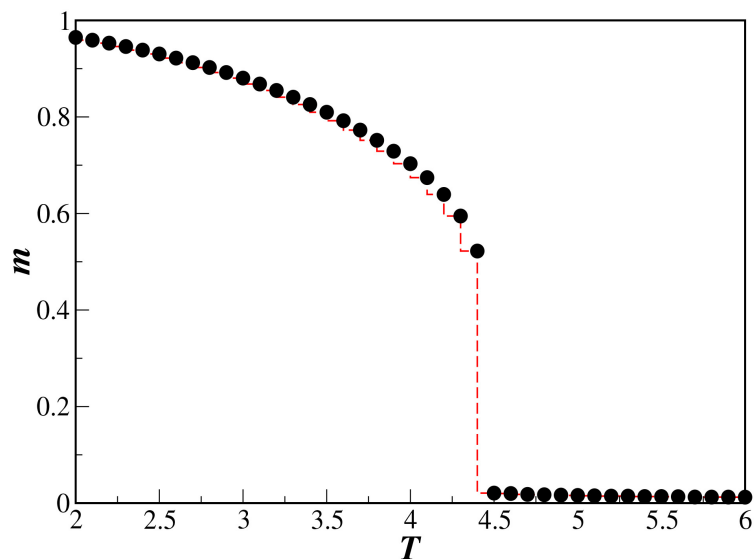
tax evasion is calculated at every time step  $t$  of system evolution. Here  $N = 10^4$  agents.

### 3. Results and Discussion

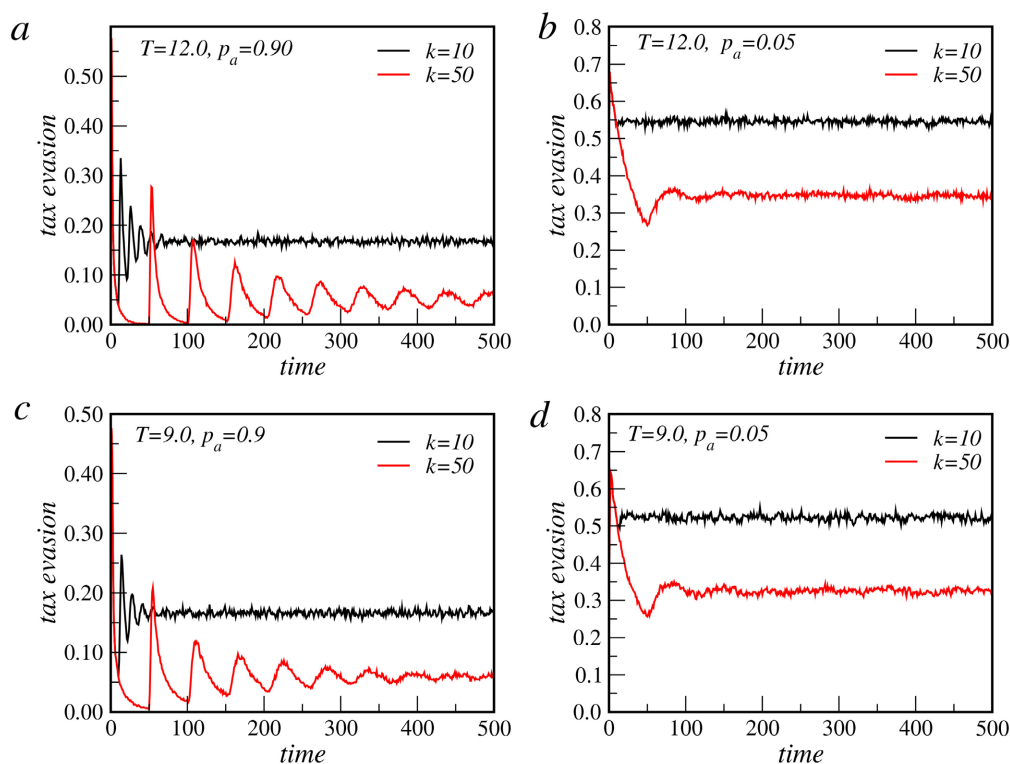
Following the ZM, we start with a population of  $N = 10^4$  individuals on ER random graphs, where initially everyone is considered honest  $\sigma_i(t=0) = +1$ . Then we apply the rules of the ZM as the punishment period  $k$  and the audit probabilities  $p_a$ .

In **Figure 1**, we show the magnetization per spin  $m$  of the Ising model spin  $S = 1$  (Lima, 2012d) as a function of the temperature  $T$ . The system undergoes an order-disorder phase transition of first-order at  $T_c = 4.5(1)$ , with a paramagnetic disordered phase defined by the coexistence of the three states  $\sigma = +1, -1$  and  $0$  with equal fractions ( $1/3$  for each one). The population size is  $N = 10^4$ , and on ER random graphs, each individual or agent has  $z = 9$  neighbors.

In **Figure 2**, we show the tax evasion as a function of time via the Ising model with three states ( $-1, 0$ , and  $+1$ ) for two different values of  $T > T_c = 4.5(1)$ , namely  $T = 12.0$  **Figure 1(a)** and **Figure 1(b)** and  $T = 9.0$  ((c) and (d)). In this case, for  $T > T_c$ , we will have the baseline case ( $k = 0$  and  $p_a = 0$ ), i.e., the dynamics defined by Equations (1) and (2) lead the system to a disordered state with an equal fraction  $1/3$  (30%) for  $\pm 1$  and  $0$  states. Thus, from **Figure 1** that if the audits are efficient ( $p_a = 90\%$ ) the tax evasion can be considerably



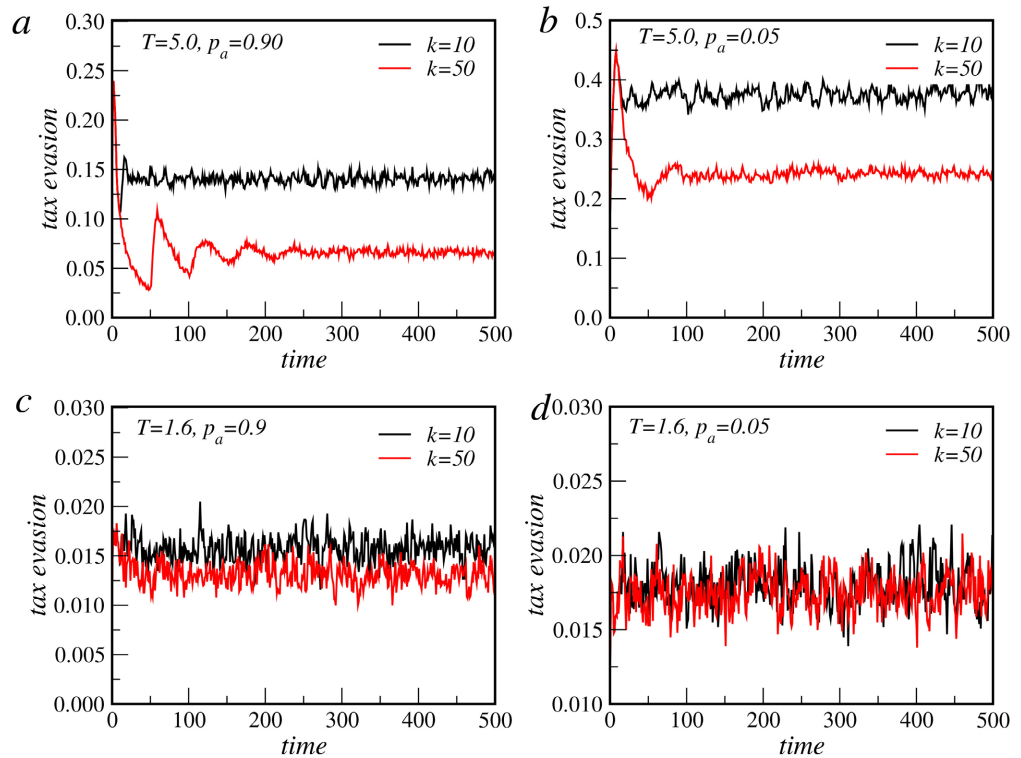
**Figure 1.** (Color online) Magnetization per spin  $m$  of the of Ising model spin  $S = 1$  of (Lima, 2012d) as a function of the temperature  $T$ , where no punishment rules were considered. The system undergoes an order disorder phase transition of first-order at  $T_c = 4.5(1)$ , with a paramagnetic disordered phase defined by the coexistence of the three states  $\sigma = +1, -1$  and  $0$  with equal fractions ( $1/3$  for each one). The population size is  $N = 10^4$ , the circles are numerical results averaged over 100 independent simulations and the dashed line is just a guide to the eyes.



**Figure 2.** (Color online) Time evolution of tax evasion via Ising model spin  $S = 1$  for different values of degrees of punishment  $k = 10$  and  $50$  and two distinct audit probabilities  $p_a = 0.9$  ((a) and (c)) and  $p_a = 0.05$  ((b) and (d)). The results are for  $T = 12.0$  ((a) and (b)) and  $T = 9.0$  ((c) and (d)). For realization of each curve, we use a population of size  $N = 10^4$  individuals at  $T = 0.95T_c$ .

reduced to  $\approx 18\%$  for  $k = 10$  and to  $\approx 5.0\%$  for  $k = 50$ . This behavior of fluctuations of tax evasion is identical to that reported in ZM on regular lattices and networks (Zaklan, 2008a, 2008b; Lima, 2010, 2012). For the cases where  $p_a = 5\%$ , considered more realistic by the literature, the punishment is more efficient when the penalty duration is high as  $k = 50$ . In this case, the tax evasion can be reduced for values around 30%. Notice that when the value of  $T$  decrease, the fraction of tax evaders also decreases, this is, for high  $T$ , the fraction of opinions  $-1$  is greater than in the cases of lower values of  $T$ .

In **Figure 3(a)** and **Figure 3(b)**, we present results for  $T = 5.0$ , this is, another value of  $T > T_c$  which is very close to the critical point  $T_c$ . See that for a high audit probability  $p_a = 90\%$  the tax evasion can be dramatically reduced for both punishment periods  $k = 10$  and  $k = 50$ . See also that even for  $p_a = 5\%$  considered here as the realistic value in a community of tax evaders, the application of severe punishments as  $k = 50$  can lead the evasion to low levels like 22%, see **Figure 3(b)**. Still, in the **Figure 3**, we show results for  $T < T_c$ , **Figure 3(c)** and **Figure 3(d)**. Then for  $T = 1.6$ , the time dynamics of the Ising model spin  $S = 1$  drive the system to a steady-state with a large number of honest individuals ( $O\sigma_i = +1$ ). For  $p_a = 90\%$  (**Figure 3(c)**) the tax evasion is reduced to 1.7% as  $k = 10$  and to 1.5% as  $k = 50$  and as  $p_a = 5\%$  (**Figure 3(d)**) the tax

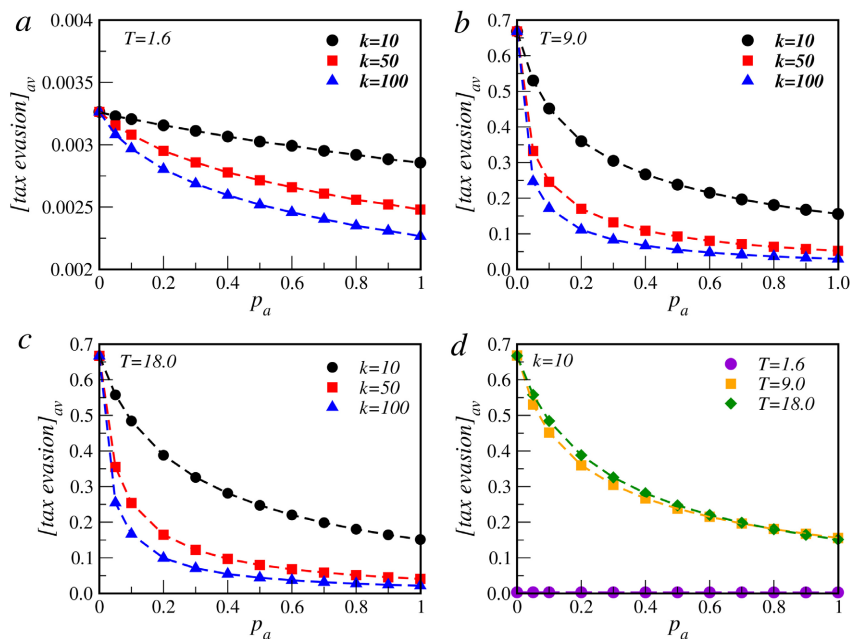


**Figure 3.** The same of the **Figure 1**, but now to  $T = 5.0$  ((a) and (b)), and  $T = 1.6$  ((c) and (d)).

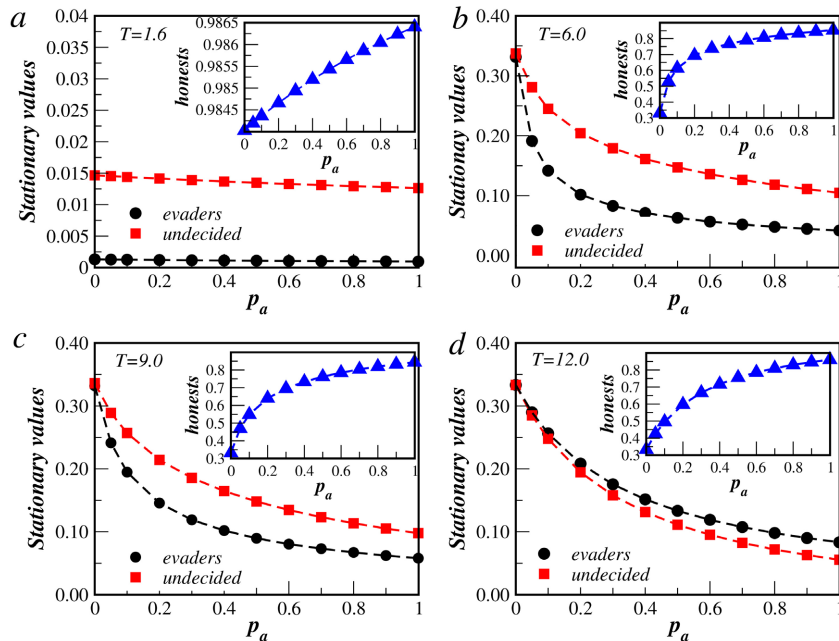
evasion is now reduced to 1.75% regardless of the values of the punishment periods  $k = 10$  and 50.

In **Figure 4**, we exhibit the average tax evasion in the stationary states as a function of the audit probability  $p_a$ . **Figures 4(a)-(c)** display the results for  $T = 1.6, 9.0$  and  $18.0$ , respectively, and for punishment period values of  $k = 10, 50$  and  $100$ . For  $T = 1.6$ , i.e.,  $T < T_c$ , the average tax evasion is small enough and decreases with the growth of  $p_a$ . This decreases to 0.22% for high punishment period values like  $k = 100$ . Otherwise, the tax evasion for  $k = 100$  and values of  $T > T_c$  as  $9.0$  and  $18.0$  decrease for 1% and 0.5%, respectively. In **Figure 4(d)**, we show the behavior of the tax evasion for  $k = 10$  and for  $T = 1.6, 9.0$ , and  $18.0$ . One can see that for  $T < T_c$  the tax evasion remains constant for all values of  $p_a$ . For values of  $T > T_c$  the tax evasion is reduced to  $\approx 15\%$  for both  $9.0$  and  $18$ .

In **Figure 5**, we exhibit the stationary fractions of the three classes of agents as functions of the audit probability  $p_a$  for  $k = 10$  and typical values of  $T$ . One can see that, in general, the stationary fraction of undecided individuals is greater than the stationary fraction of tax evaders, and the former is always  $> 1.5$  of the population. The evaders in the long-time limit are the majority in comparison with undecided agents only for large densities of interactions  $T$ , like  $T = 12.0$  [see **Figure 5(d)**]. Even in this case, the honests are the majority of the population. For increasing values of  $p_a$ , the fraction of honest agents grows slowly for  $T < T_c$  [see **Figure 5(a)**] and fastly for  $T > T_c$  [see **Figures 5(b)-(d)**].



**Figure 4.** (Color online) Average stationary tax evasion as a function of the audit probability  $p_a$ . In the Figures (a), (b) and (c), we show the results for  $T=1.6, 9.0$  and  $18.0$ , respectively, and values of degrees of punishment  $k=10, 50, 100$ . In the last Figure (d) it is shown the results for  $k=10$  and different fractions of interactions  $T$ . Each point is averaged over 100 independent simulations for population size  $N=10^4$ , and the dashed lines are just guides to the eye.



**Figure 5.** (Color online) Average stationary fractions of evaders (circles), undecided (squares) and honests (triangles, in the insets) as functions of  $p_a$  for  $k=10$  and typical values of  $T$ , namely  $T=1.6$  (a),  $t=6.0$  (b),  $T=9.0$  (c) and  $T=12.0$  (d). Each point is averaged over 100 independent simulations for population size  $N=10^4$ , and the dashed lines are just guides to the eye.



## 4. Conclusion

Zaklan et al. (2008a) using Monte Carlo Simulation and the Ising model in two-dimensional with two states ( $-1$  and  $+1$ ) (equilibrium model) performed the first numerical attempt to model the fluctuations tax evasion problem. Then Ising model was used as a tool for the dynamic evolution of the Zaklan model. However, the Ising model cannot be used in some geometries, such as Apollonius and Barabasian networks due to the lack of phase transition. Lima (2010) overcame this obstacle by proposing to use the majority vote model (non-equilibrium model) as a temporal evolution of the Zaklan model. Both proposals were successful in their particular topologies.

Here, we studied the behavior of the tax evasion via the Ising model with spin  $S = 1$  (equilibrium model) that presents three states  $-1$ ,  $0$ , and  $+1$ . As we saw above,  $-1$  represents evaders,  $0$  undecided, and  $+1$  the honests. We have also verified that the fraction of undecided agents favors the reduction of tax evasion. This fact, together with the control given by the public policies may lead to low levels of evasion.

The behavior of the Ising model with spin  $S = 1$  in relation to controlling the tax evasion fluctuation is very similar to those found by the Ising and majority vote models. Then, its model is another useful tool for studying the tax evasion problem in a community. We still verify that our results are in agreement with the results of Crokidakis (2014) for the non-equilibrium model with three different states ( $-1$ ,  $0$ , and  $+1$ ) disagreement between pairs of agents.

Therefore, we found the plausible result that tax evasion is diminished by stronger punishment  $k$  and audit probability  $p_a$ .

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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