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A Formulation of Investor Sentiment of Cryptocurrencies and Cryptocurrency Futures and Options

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Abstract

This study presents the mathematical formulations of investor sentiment for investors in cryptocurrencies. We assume that bitcoin prices are driven by investor sentiment measured in terms of Google search volume and social media posts. The current generation of retail investors uses non-traditional methods such as social media posts and Google searches to obtain information so that an increase in posts and searches on 'bitcoin,' indicate positive or negative investor sentiment. Mathematical formulations describe investor sentiment separately for risk-averse, moderate risk, and risk-taking investors. Risk-averse investors are considered to be aberrant in their investment in cryptocurrencies as they are naturally resistant to high-risk investments such as cryptocurrencies. Only risk-taking investors capture the fullest extent of irrational exuberance that prevailed in the cryptocurrency markets. However, risk-takers with very high-risk tolerance, such as hedge funds, trade in investments with volatility to capitalize upon the highest market prices for cryptocurrencies. Their behavior is modeled in cryptocurrency futures and cryptocurrency call options, and cryptocurrency put options. The insight provided by this paper is that the history of cryptocurrency prices is stored in a Laplace transform so that investor sentiment is based on the trajectory of past prices for cryptocurrencies and cryptocurrency futures. For cryptocurrency options, the history of volatility of prices is embedded in the Laplace transform, with increasing volatility embedded in call option prices, and decreasing volatility embedded in put option prices.

Keywords

Cryptocurrency Prices, Investor Sentiment, Google Search Volume, Twitter Post Sentiment, Levy Jump Process, Levy-Khintchine Formula, Cryptocurrency Futures, Cryptocurrency Options

1. Introduction

Cryptocurrency investments including investments in bitcoin, etherium, and other cryptocurrencies bypass central banks by being the currency of person-to-person transactions recorded on the blockchain ledger. Their rise was meteoric until about 2014, when the principal cryptocurrency, bitcoin, lost 80% of its value in a single year. Prices have risen in the 2016-2023 period (See **Figure 1** for a trend chart showing price movements from 2010-2024). However, price changes have been characterized by sharp increases and sharp decreases, suggesting the existence of severe volatility.

Figure 1 is a trend chart showing the rise in cryptocurrency prices until 2014, falling from 2014-2016, and rising from 2016-2024.

Retail investors are individuals who purchase securities for their own portfolios. They are contrasted with institutional investors who are pension funds and life insurance companies with billion-dollar portfolios, experienced research staff, and direct contact with firms in which they invest. Individual investors have small portfolios, self-manage investments, and rely on public information about security returns, dividends, and share repurchases. Investor sentiment of such investors has found an inverted size effect, suggesting that investors become increasingly optimistic about returns as firms increase in size. However, for very large firms, investor sentiment becomes pessimistic (Li, 2000). Perhaps, investors achieve confidence in large cash flows and financial stability as firms grow in size, but question if very large firms are agile and responsive to stimuli in the market. Cryptocurrencies appeal to retail investors as a new and exciting vehicle for investment. Lacking sophisticated tools for tracking security prices, retail investors rely on word-of-mouth to receive information about future prices. If peers have overly optimistic cryptocurrency valuations, retail investors will bid up prices of cryptocurrencies to unrealistically high levels in an exercise in irrational exuberance. If macroeconomic conditions result in a large decline in cryptocurrency prices, retail investors will lower price expectations resulting in final cryptocurrency prices achieving equilibrium at a low level.

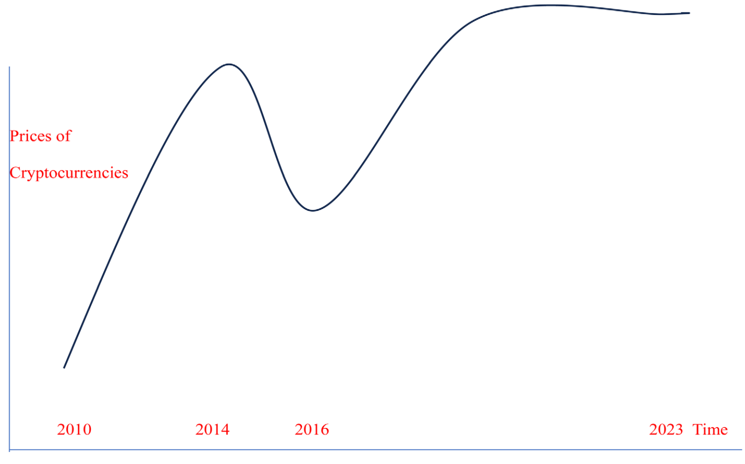

Figure 1. Trend chart of cryptocurrency prices.

Existing literature provides a limited review of cryptocurrency pricing based on investor sentiment. Abraham and El-Chaarani (2022) created a mathematical model to value etherium, the second prominent cryptocurrency after bitcoin. They valued ether separately for risk-averse retail investors, moderate risk-takers, and risk-takers. They employed utility theory to create investor sentiment functions. The risk-averse investor had an aberrant sentiment function as investing in risky cryptocurrencies does not fit with the psychological profile of the risk-averse investor. The risk-taker had a Bessel investor sentiment function, while the moderate risk-taker's function was a combination of that of the risk-taker and the risk-averse investor. Levy jump processes intersected with these investor sentiment functions to generate prices for etherium. This study valued ether futures, as well. Ether futures have a current spot price and a delayed term price about three months in the future during which ether prices fluctuate before reaching an equilibrium final price. The final price is based upon the term premium, or the Levy jump process approximation of price fluctuations together with skewness and kurtosis of price changes.

There are a limited number of recent empirical studies of cryptocurrency valuation. Nasir et al. (2019) and Burgrraf et al. (2021) considered investor sentiment to dominate in predictions of bitcoin prices. Both studies found that investors reallocated their holdings based on sentiment. Sentiment itself used traditional market measures. The breakthrough in measurement of investor sentiment occurred with the Burgrraf et al. (2021) study which reconceptualized investor sentiment in bitcoin. They recognized that modern retail investors search for information using search engines such as Google. They interact with friends, family, and social groups on social media platforms such as Twitter. In these conversations, individuals encourage each other to purchase cryptocurrencies. Alternatively, pessimists may discourage purchases of cryptocurrencies. This study also recognized that macroeconomic conditions and microeconomic conditions provide input into measures of investor sentiment. Da et al. (2011) had first recognized the value of including economic conditions in cryptocurrency prices leading to Burgrraf et al. (2021) incorporating these economic variables in their pessimistic measure of investor sentiment termed FEARS. FEARS index values were based upon Google search volumes and Twitter post volumes along with macroeconomic indicators and microeconomic indicators of a pessimistic nature that forecasted decreases in security returns. All three measures of investor sentiment, i.e. investor sentiment based on Google volume, investor sentiment based on macroeconomic conditions, and investor sentiment based on microeconomic conditions significantly reduced security returns.

We sense a research gap in that there is no current study with a mathematical formulation of bitcoin prices based upon modern measures of investor sentiment including Google search volumes and social media posts. The existing usage of these measures has been in empirical studies. Bitcoin futures and bitcoin options have started trading recently. We find a critical shortage of both empirical examinations and mathematical formulations of bitcoin derivatives. The only exception is the aforementioned Abraham and El-Chaarani (2022) study, which addresses etherium futures, while excluding bitcoin futures.

The purpose of this study is to create mathematical formulations of general cryptocurrency prices and cryptocurrency derivatives. This study advances knowledge in two ways. It pioneers the mathematical conceptualization of cryptocurrency prices as being a function of modern, informal person-to-person communication. It moves beyond the premise that trading volumes of cryptocurrencies effectively capture investor sentiment. It comprehends that modern investors use search engines to seek out cryptocurrency information, and post their opinions of cryptocurrency prices on Twitter and other social media sites. The second addition to knowledge stems from the inclusion of cryptocurrency futures and cryptocurrency options in the price formulations. This exercise shines light on these novel and very risky commodities attached to cryptocurrencies.

The novelty of this study emerges in three areas. This is the first paper presenting mathematical formulations of cryptocurrencies using price assumptions from the post-2016 period. The paper employs current measures of cryptocurrency trading volume such as Google search volume, Wikipedia search volume, and Twitter post volume which have not been used in mathematical formulations of cryptocurrencies. This is the first study to value crypto derivatives using Laplace transforms to model price revisions from additional information.

2. Literature Review

2.1. Research in Investor Sentiment of Cryptocurrencies

Miller (1977) presented a model of investors having heterogeneous expectations of security prices. Certain investors have irrational expectations of excessively high returns. Others are pessimists who expect lower returns. As heterogeneous expectations increase, pessimists will not short sell securities due to high short sale costs. This leaves the market to be dominated by optimists who bid up security prices. This theory may be applied to the price bubbles of cryptocurrencies. Retail investors may be optimists or pessimists about cryptocurrency prices. Given positive word-of-mouth and recommendations from friends, investors may have unrealistic expectations of future prices leading to price bubbles. The empirical question is identifying the most optimal measure of cryptocurrency volume.

Kristoufek (2013) measured investor sentiment in terms of Google search query volume and Wikipedia search query volume. He found that investor sentiment significantly explained cryptocurrency returns for both Google search query volume and Wikipedia search query volume. Garcia et al. (2014) evaluated price bubbles during which cryptocurrency trading volumes rise with the expectation of abnormal price gains, and then fall upon the bursting of the price bubble. In a subsequent study, Garcia and Schweitzer (2015) employed positive and negative Twitter posts as measures of cryptocurrency buy volume and cryptocurrency sell volume. Twitter posts as measures of investor sentiment were as effective as Google search volume in proxying investor sentiment's effect on cryptocurrency returns. Eom et al. (2019) added to these findings by using Google search volume as a measure of volatility of cryptocurrency prices. This finding was supported by a recent study in which Verma et al. (2023) observed that bitcoin volumes, ether volumes, and ripple volumes measured by Google trading volume significantly predicted price fluctuations during periods of irrational optimism.

Saha (2023) compared the relative ability of five machine learning algorithms to predict cryptocurrency prices. They include quadratic discriminant analysis, k-nearest neighborhood, logit model, decision tree, and neural network. The logit model showed the highest accuracy in predicting cryptocurrency prices across all datasets and time periods. Logistic regression uses categorical accept/reject dependent variables. Therefore, it measures the strength of independent variables in explaining whether cryptocurrency prices will increase (dependent variable with value = 1), or if cryptocurrency prices will decrease (dependent variable with value of 0).

Abraham and El-Chaarani (2022) created a mathematical model that described the value of ether as a late mover in comparison to bitcoin. Accordingly, the risk-averse investor's utility function was based on the ability of ether as a later mover to supersede bitcoin, the first mover. Additional variables were added to account for cryptocurrencies being a transformative process, and expectations of the performance of ether. The moderate risk-taker had an alt-Weibull distribution of investor sentiment (Merovci et al., 2016). The risk-taker purchases ether in each round of continuously rising prices. The asymptotic expansion of the Bessel function describes rising prices. To this distribution the paper added 1) a gamma distribution to describe the transformative nature of cryptocurrencies, and 2) a Legendre integral of expectations of ether superseding bitcoin. The risk-taker purchases ether in each round of continuously rising prices. Levy jump processes were used for the three types on investors to describe cryptocurrency prices. The intersections of investor sentiment distributions with Levy jump processes yielded final ether prices.

2.2. Research in Cryptocurrency Futures and Options

2.2.1. Cryptocurrency Futures

The futures price of a commodity is the combination of current prices, termed spot price, along with the term price upon delivery of the commodity three months in the future. Fama (1984) found that spot prices exerted a more powerful influence on final currency prices for 11 currencies. Conversely, Gorton et al. (2012) observed that commodity prices were dependent upon price fluctuations during the delivery period. For cryptocurrencies, we may theorize that pricesdepend heavily on theprice at the end of the delivery period, as this is a period of fluctuations in prices, with high price volatility. Purchasing commodities during periods of high prices, and short selling commodities during periods of low prices yield gains. Kaur (2023) observed that bitcoin futures prices lead bitcoin spot prices, suggesting that volumes of bitcoin traded at the end of the delivery period effectively forecasted bitcoin spot prices during the subsequent period. To sum, the empirical studies suggest that bitcoin futures prices depend upon bitcoin spot prices, and bitcoin delivery period prices, and that there is a relationship wherebybitcoin delivery period prices forecast bitcoin spot prices for bitcoin futures.

Abraham and El-Chaarani (2022) constructed a mathematical model in which ether prices were dependent upon current spot prices, delivery period prices, skewness of ether prices, and kurtosis of ether prices. They recognized that cryptocurrency price distributions are nonnormal in that they may be skewed positively for rising ether prices,and skewed negatively for declining ether prices.Further, observations of flat, thick-tailed leptokurtic distributions suggest significant kurtosis (Sebastiao & Godinho, 2020), so that they added a function for kurtosis.

2.2.2. Cryptocurrency Options

Early models of cryptocurrency options were created prior to the trading of this crypto-asset. Abraham (2018) priced cryptocurrency options as foreign currency options combined with a riskless bond. In the Abraham and El-Chaarani (2022) model, call options were presented as having value with rising ether prices. With falling ether prices, the investor would permit the call option to expire as it continuously loses value. The investor would gain on the returns and rising values of the short-term bond. The risk of declining ether values was transferred to the riskless short-term bond. With the approval of crypto-options for trading, empirical research in cryptocurrency option valuation has commenced.

Madan et al. (2019) created a stochastic volatility model of bitcoin index option prices correlated with price jumps. They collected data on bitcoin options prices based on several unregulated exchanges, finding that their model's volatility estimate outperformed the price volatility of the traditional Black-Scholes call option pricing model. Zulfiqar and Gulzar (2021) supported the volatility estimate of Madan et al.'s (2019) model, recommending the NewtonRhapson forecasting method to solve for the volatility of bitcoin options. Venter et al. (2020) created a GARCH option pricing model, which presents bitcoin option prices as the result of the absorption of symmetric news. The change in estimated bitcoin option prices matched market bid-ask spreads, suggesting that the options estimates accurately supported price discovery. Hu et al. (2021) followed up with an option pricing model with an underlying crypto-asset index. This is a current mathematical formulation with a martingale and Esscher transform for the bitcoin option pricing function. As a premier mathematical model following bitcoin options trading, this model must be supplemented with additional models that consider option pricing strategies, such as combining call and put options. It is to this objective that this study is directed.

3. Research Design and Methodology

3.1. Research Design

The study employs a descriptive research design. Five different formulations are considered for investors of three different levels of risk, cryptocurrency futures and cryptocurrency options. Investor sentiment is differentiated in terms of investor attitude toward risk. Risk-averse investors are very reluctant to accept risk. They only accept the risk of high-risk cryptocurrency investments upon perceiving significant increase in returns. Therefore, this study models the investor sentiment of risk-averse investors as aberrant behavior. Aberrant behavior is uncharacteristic behavior, as it is uncharacteristic to assume that risk-averse investors will purchase high-risk crypto-assets. The next type of investor is the moderate risk-taker. Moderate risk-takers are contemplative about their investments. They are likely to make an investment, evaluate it, then make another investment, and evaluate again. Successive evaluations suggest a slow upward progression of expectations of higher prices, optimally represented by a gamma distribution.

Risk-takers seek high returns from expectations of continually rising prices. Yet, they have some desire to take existing prices, revise them, and expect new, higher prices. This study models the evaluation process as a Laplace transform. Laplace transforms lend themselves to revisions of prices, assuming a smooth upward trajectory of prices over time. Yet, the high volatility of crypto derivatives is different from the high prices of cryptoassets. Therefore, the Laplace transforms for crypto derivatives contain revisions of volatility, while those of cryptocurrencies contain revisions of prices.

3.2. Research Methodology

The price distribution of cryptocurrencies and cryptocurrency derivatives is modeled as a Levy process for risk-averse investors, the Levy-Khintchine formula for risk-takers, and the Ito decomposition of the Levy-Khintchine formula for investors in crypto derivatives. Differential distributions are employed to account for differences in risk. A linear programming model was created for each of the five formulations. The objective function was the maximization of investor sentiment of higher returns. The constraint was the price distribution. The objective function and constraint were combined using Lagrange multipliers as the coefficient of the price distribution constraint. The final Lagrangian function was differentiated twice. The first derivative became the necessary condition for the maximization of crypto currency prices or crypto futures and crypto options prices. The second derivative became the sufficient condition for the maximization of crypto currency and crypto derivative prices.

4. Findings and Analysis

4.1. The Risk-Averse Cryptocurrency Investor

The sentiment of risk-averse investors is contained in these measures. 1) Risk-averse investors use Google searches, Twitter posts, and Wikipedia searches to obtain more information about cryptocurrencies. Therefore, the total volume of theses searches is a measure of investor sentiment and interest in cryptocurrencies. 2) Risk-averse investors are not naturally inclined to purchase high-risk investments such as cryptocurrencies. Their interest in these investments suggests aberrant behavior, so that we include a function representing aberrancy. 3) Cryptocurrencies have leptokurtic distributions with few observations in the center, and a large number of observations at the end points (tails). The risk may be excessively high if observations cluster in the upper tail or excessively low if observation cluster in the lower tail. A gradient vector is added to the investor sentiment function to reduce tail risk, as risk-averse investors will not tolerate such high levels of unpredictable risk.

1) The function based on search volume is contained in the investor's aberrancy sentiment expression,

$$(GV_{ij}+TV_{ij}+W_{ij})[(0.5\rho_1 s^2)-(1-6p^3s^4)]-[p^2-3(mean)(mean) + \frac{\rho(mean)(mean)s^4}{24\rho^5}]-\alpha[s-\frac{1}{6\rho^2 s^3}+\frac{\rho(mean)}{8\rho^4})s^4 = H \tag{1}$$

where,

GV_{ij} = Google search volume using the word 'cryptocurrency,' in the search engine,

TV_{ij} = Volume of Twitter posts speculating on cryptocurrency prices,

W_{Ij} = Volume of Wikipedia searches using the word 'cryptocurrency' as keyword.

s = arc length of the secant line,

$\boldsymbol{\rho}$ = radius of curvature of the line of aberrancy at the point of tangency.

Schot (1978) created the aberrancy expression in Equation (1) to describe uncharacteristic behavior as the secant to expectations of cryptocurrency prices.

2) A gradient vector is added to reduce tail risk.

$$v(v+1)-mean/(1-e)w \tag{2}$$

v = gradient vector in a leptokurtic distribution of cryptocurrency prices,

mean = mean of a leptokurtic distribution,

w = investor expectations of tail risk,

e = a measure of tail risk.

3) Risk-averse investors expect to gain from continuously rising cryptocurrency prices which may be approximated by an exponential distribution.

$$(1+\lambda)[1-\exp(-\alpha\gamma x)]-\lambda[1-\exp(-\alpha\gamma x)\alpha>0,[\lambda\leq 1] \tag{3}$$

λ = the exponentials of the exponential distribution, or the amount by which cryptocurrency prices increase,

$\alpha\gamma$ = constants in the generalized transmuted exponential distribution,

x = the price of cryptocurrency in the generalized transmuted exponential distribution.

4) Price changes of the cryptocurrency distribution are represented as a Levy jump process. The Levy-Khintchine expression is used as it describes the movement of the cryptocurrency as a Brownian motion with a deterministic amount (drift) and a stochastic amount consisting of a random variable following a Poisson process. Jumps are included in the Poisson process as cryptocurrency prices. The formula is presented below (Zolotarev, 1986).

$$\begin{aligned}&\int R(0)e^{10x}-1-i\theta i\theta x_{1_x}<1)\prod(dx)\\&=\prod(R)(-1,1))\int R(-1)vdx+\int R(e^{10x}e^{i\theta x}-1-i\theta x)\mu(dx)\end{aligned} \tag{4}$$

$\int R(0)e^{10x}-1-i\theta x_{1_x}<1)\prod(dx)$ = characteristic function of the Poisson process,

$\prod(R)(-1,1)$ = intensity of the Poisson process,

$i\theta x)\mu(dx)$ = pure jump process,

We collect these terms to create an objective function that maximizes positive investor sentiment as contained in Equation (1) and Equation (2). Equation (3) and Equation (4) form constraints. Figure 2 depicts the relationships, Figure 2 describes the investor sentiment of the risk-averse investor as Schot's (1978) aberrancy function. This sentiment intersects with the Levy jump process. It shows point C, the optimization point of maximum return for the risk-averse investor.

The objective function is,

Max

$$(GV_{ij}+TV_{ij}+W_{ij})[(0.5\rho_1 s^2)-(1-6p^3s^4)]-[p^2-3(mean)(mean)$$
$$+\frac{\rho(mean)(mean)s^4}{24\rho^4}]-\alpha[s-\frac{1}{6\rho^2 s^3}+\frac{\rho(mean)}{8\rho^4})s^4+v(v+1)-mean/(1-e)w$$

Subject to

$$\begin{aligned}&(1+\lambda)[1-\exp(-\alpha\gamma x)]-\lambda[1-\exp(-\alpha\gamma x)\alpha>0,[\lambda\le 1]\\&\int R(0)e^{10x}-1-i\theta x_{1_x}<1)\prod(dx)\\&=\prod(R)(-1,1))\int R(e^{10x}-1)vdx+\int Re^{i\theta x}(e^{i\theta x}-1-i\theta x)\mu(dx)\end{aligned} \tag{5}$$

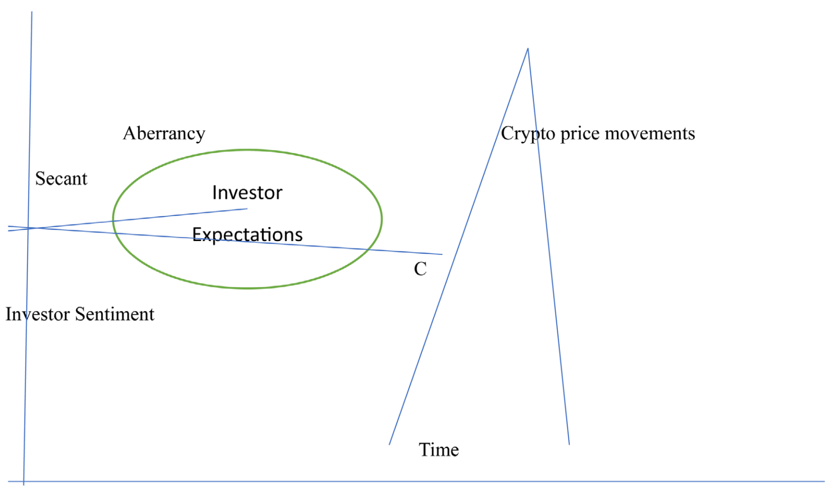

Figure 2. Investor sentiment function of the risk-averse investor.

Collecting constraints into Lagrangian functions to form a single objective function,

Max

$$
\begin{aligned}
&(GV_{ij}+TV_{ij}+W_{ij})[(0.5\rho_1 s^2)-(1-6p^4 s^4)]-[p^2-3(mean)(mean)\\
&+\frac{\rho(mean)(mean)s^4}{24\rho^5}]-\alpha[s-\frac{1}{6\rho^2 s^3}+\frac{\rho(mean)}{8\rho^4})s^4+v(v+1)-\frac{mean}{(1-e)w}\\
&-L_1(1+\lambda)[1-\exp(-\alpha\gamma x)]-\lambda[1-\exp(-\alpha\gamma x)\alpha>0,[\lambda\le 1]\\
&-L_2[\int R(0)e^{10x}-1-i\theta x_{1x}<1)\prod(dx)\\
&-\prod(R)(-1,1))\int R(e^{10x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu(dx)]
\end{aligned}
\tag{6}
$$

The first derivative of function (6) provides the change in investor sentiment with rising cryptocurrency prices.

$$
\begin{aligned}
&(GV_{ij}+TV_{ij}+W_{ij})[(\rho_1 s)-(1-48p^2 s^3)]-[p-3(mean)(mean)\\
&+\frac{4(mean)(mean)s^3}{96\rho^4}]-4\alpha[s-\frac{1}{36\rho s^2}+\frac{(mean)}{32\rho^3})s^3+v(v+1)-\frac{mean}{(1-e)w}\\
&-L_1(1+\lambda)[1-\exp(-\alpha\gamma x)]-\lambda[1-\exp(-\alpha\gamma x)\alpha>0,[\lambda\le 1]\\
&-L_2[R(0)e^{10x}-1-i\theta x_{1x}<1)\prod'(dx)\\
&-\prod'(R)(-1,1))R(e^{10x}-1)vdx+R(e^{i\theta x}1-i\theta x)\mu(dx)]
\end{aligned}
\tag{7}
$$

Expression (8) is thesecond derivative of the investor sentiment function (6) that maximizes returns from cryptocurrency investment for the risk-averse investor,

$$
\begin{aligned}
&(GV_{ij}+TV_{ij}+W_{ij})[1-288ps^2]-[1-3(mean)(mean)+\frac{12(mean)(mean)s^2}{384\rho^3}]\\
&-12\alpha[1-\frac{1}{726s}+\frac{(mean)}{64\rho^2})s^2+v(v+1)-\frac{mean}{(1-e)w}\\
&-L_1(1+\lambda)[1-\exp(-\alpha\gamma x)]-\lambda[1-\exp(-\alpha\gamma x)\alpha>0,[\lambda\le 1]\\
&-L_2[R(0)e^{10x}-1-i\theta x_{1x}<1)\prod''(dx)\\
&-\prod''(R)(-1,1))R(e^{10x}-1)v'dx+R(e^{i\theta x}-i\theta)\mu'(dx)]
\end{aligned}
\tag{8}
$$

4.2. The Moderate Risk-Taker

The moderate risk-taker will only invest in risky cryptocurrencies if returns are sufficiently high to exceed a psychological threshold of return. In other words, the sentiment of this investor is that excessive risk without returns about a psychological barrier is unjustified. This paper sets forth that a gamma distribution approximates the sentiments of the moderate risk-taker. A gamma distribution is a gradual upward-sloping function that permits the investor to make an investment, assess returns, returns rise up the step, and then make another investment which repeats the process. As the distribution is likely to be a leptokurtic distribution with fat tails and a small center, a gradient vector must be added to reduce tail risk.

The objective function and constraints are listed below, Max

$$(GV_{ij}+TV_{ij}+WV_{ij})*x^{\alpha-1}e^{\beta z}\beta^{\alpha}\psi(\alpha)^{-1}v(v+1)-mean/(1-e)w \tag{9}$$

Where,

$GV_{ij}+TV_{ij}+WV_{ij}$ = Good volume + Twitter feed volume + Wikipedia search volume;

$x^{\alpha-1}e^{\beta z}\beta^{\alpha}\psi(\alpha)^{-1}$ = probability density function of a gamma distribution with shape parameter k, and scale parameter θ, scale parameter $\beta = 1/\theta$, and gamma distribution of $\psi(x)$.

$v(v+1)-mean/(1-e)w$ = gradient vector to reduce tail risk.

Subject to

Returns < threshold rate of return,

$$\begin{aligned}&\int R(0)e^{10x}-1-i\theta x_{1x}<1)\prod(dx)\\&-\{\prod(R)(-1,1))\int R(e^{10x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu(dx)\}<H\end{aligned} \tag{10}$$

$\int R(0)e^{10x}-1-i\theta x_{1x}<1)\prod(dx)$ = characteristic function of the Poisson process,

$\prod(R)(-1,1)$ = intensity of the Poisson process,

$i\theta x)\mu(dx)$ = pure jump process,

H = threshold rate of return.

Collecting the objective function in Equation (9) and the constraint in Equation (10) into a Lagrangian function L,

Max

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*x^{\alpha-1}e^{\beta z}\beta^{\alpha}\psi(\alpha)^{-1}v(v+1)-\frac{mean}{(1-e)w}\\&-[L\int R(0)e^{10x}-1-i\theta x_{1x}<1)\prod(dx)\\&-\{\prod(R)(-1,1))\int R(e^{10x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu(dx)\}-H]\end{aligned} \tag{11}$$

The necessary condition for optimization is the first derivative of the Lagrangian function (11),

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*(\alpha-1)x^{\alpha-2}e^{\beta z}\alpha\beta^{\alpha-1}\psi(\alpha)^{-2}v'(v)-\frac{mean}{(1-e)w}\\&-[L[R(0)e^{i\theta x}-1-i\theta x_{1x}<1)\prod(dx)\\&-\{\prod(R)(-1,1))]R(e^{10x}-1)vdx+R(e^{i\theta x}-1-i\theta x)\mu(dx)\}-H]]\end{aligned} \tag{12}$$

The sufficient condition is the optimal point of maximum return which is the second derivative of the Lagrangian function (12)

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*(\alpha-1)(\alpha-2)x^{\alpha-3}e^{\beta z}\alpha(\alpha-1)\beta^{\alpha-2}\psi(\alpha)^{-3}v''(v)\\&-\frac{mean}{(1-e)w}-[L[R(0)e^{10x}-1-i\theta x_{1x}<1)\prod(dx)\\&-\{\prod'(R)(-1,1))]R(e^{10x}-1)v'dx+R(e^{i\theta x}-1-i\theta x)\mu'(dx)\}]]\end{aligned} \tag{13}$$

Figure 3 graphically depicts the investor sentiment of the moderate risk-taker.

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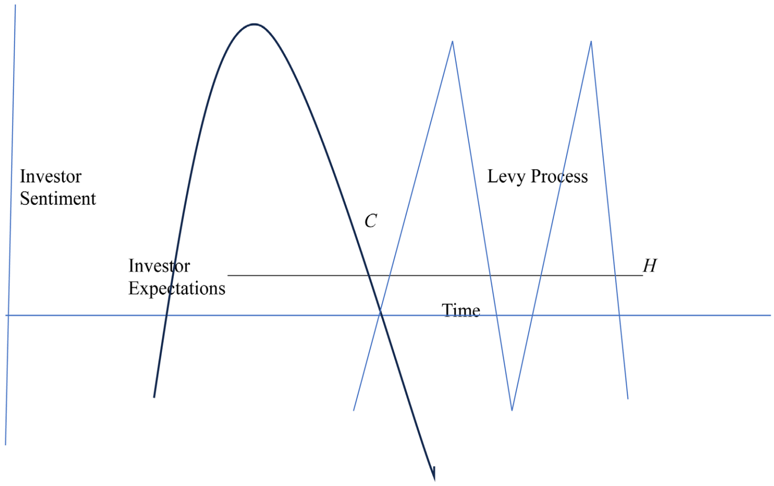

Figure 3. The optimization function of the moderate risk-taker investing in cryptocurrencies.

Figure 3 describes the optimization function of the moderate risk-taker. The investor sentiment is a gamma distribution. The figure depicts C, the optimal point of maximum return at which investor expectations meet cryptocurrency prices. H is the barrier above which returns must be achieved for the investor to consider cryptocurrency investments.

4.3. The Risk-Taker

The risk-taker wishes to earn continuously higher returns from investments in cryptocurrencies at multiple points in time. The Google Search volume, Twitter post volume, and Wikipedia volume of this investor contain a record of buying or selling based on incremental historic price changes. The incremental historic price changes are contained in a Laplace transform, listed below (Mikusinski, 2014).

Max

$$(GV_{ij} + TV_{ij} + WV_{ij}) * [(n-k)n^n F^n(\theta) - \sum k = 1\, to\, n\, \alpha^{n-k} F(k-1)^{-1}(0^-)] \quad (14)$$

Where,

$n - k$ = incremental price changes in cryptocurrencies,

$*[(n-k)n^n F^n(\theta) - \sum k = 1\, to\, n\, \alpha^{n-k} F(k-1)^{-1}(0^-)]$ = Laplace transform of change in

cryptocurrency prices,

Subject to

The Levy-Ito decomposition is used. With the moderate risk-taker and the risk-averse investor, cryptocurrency prices were assumed to have a gradual increase, so were modeled as a Brownian motion with drift, as these investors are satisfied with modest returns from small price increases in cryptocurrencies. Risk-takers desire higher returns so that a Levy-Ito decomposition is an appropriate representation as it adds Poisson random variables that capture the excess volatility of future cryptocurrency prices. **Figure 4** shows the investor sentiment with sharp upward expectations of cryptocurrency prices.

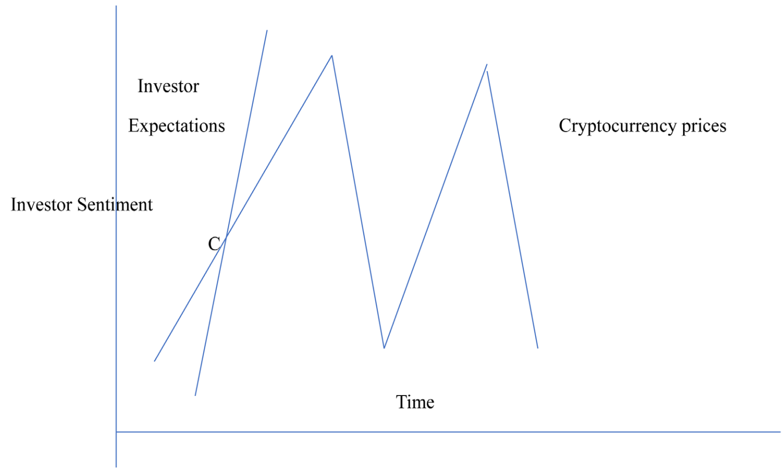

Figure 4. The risk-taker's optimization function.

$$\begin{aligned}&\int R(0)(e^{i\theta x}-1-i\theta x_{1x}<1)\prod(dx)\\&=\prod(R(-1,1))\int R(e^{i\theta x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu dx\end{aligned} \tag{15}$$

$\prod(R(-1,1))\int R(e^{i\theta x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu dx$ = Levy processwith incremental price changes θ with v random variables with mean μ

The complete objective function is a Lagrangian that combines Equation (14) and Equation (15),

Max

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*[(n-k)n^{n}F^{n}(\theta)-\sum k=1\,to\,n\,\alpha^{n-k}F(k-1)^{-1}(0^{-})]\\&\int R(0)(e^{i\theta x}-1-i\theta x_{1x}<1)\prod(dx)\\&-L[\prod(R(-1,1))\int R(e^{i\theta x}-1)vdx+\int R(e^{i\theta x}-1-i\theta x)\mu dx]\end{aligned} \tag{16}$$

Taking first derivatives, the necessary condition for increasing returns by considering past price changes is obtained in Equation (17),

Max

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*[(n-k)nn^{n-1}F^{n}(\theta)\\&-\sum k=1\,to\,n\,(n-k)\alpha^{n-k-1}F'(k-1)^{-1}(0^{-})]\\&-L(e^{i\theta x}-1-i\theta x_{1x}<1)\prod L'\prod(R(-1,1)R(e^{i\theta x}-1)v'+R(e^{i\theta x}-1-i\theta x)\mu\end{aligned} \tag{17}$$

Taking second derivatives, we obtain the point of maximization of shareholder wealth,

$$\begin{aligned}&(GV_{ij}+TV_{ij}+WV_{ij})*[(n-k)n(n-1)n^{n-2}F^{n}(\theta)\\&-\sum k=1\,to\,n\,(n-k)(n-k-1)\alpha^{n-k-2}F'(k-1)^{-1}(0^{-})]\\&-L(e^{i\theta x}-i\theta x_{1x}<1)\prod L'\prod(R(-1,1)R((e^{i\theta x}-1)v'+R(e^{i\theta x}1-i\theta x)\mu'\end{aligned} \tag{18}$$

Figure 4 shows the investor sentiment of risk-takers of a Laplace transform that revises historical cryptocurrency prices. A Levy-Ito decomposition is used to portray the price distribution. The Laplace transform intersects with a Levy-Ito decomposition to yield the optimal cryptocurrency price at point *C*.

4.4. Investing in Cryptocurrency Derivatives

An investor in cryptocurrency derivatives may select either futures or options. An investment in either of these derivatives that assumes the right to purchase the cryptocurrency if there is a rise in cryptocurrency prices. The difference is that for cryptocurrency futures, prices are binding so that purchase must occur at the futures price. For cryptocurrency options, prices are nonbinding so that purchase (exercise) will occur as long as the market cryptocurrency price >the exercise price. The objective function of both futures and options will reflect the willingness to benefit from long-term volatility which drives cryptocurrency prices to a higher level.

Cryptocurrency Futures

The Laplace transform of higher long-term volatility is given below,

Max

$$\int 0 \; to \; \alpha e^{-\alpha t} dt - \int 0^{-} \; to \; \alpha e^{\alpha t} / -\alpha f'(t) dt \tag{19}$$

where,

α = incremental change in volatility over time period, t,

Subject to,

$$\begin{aligned} &(GV_{ij} + TV_{ij} + WV_{ij}) * \int R(0)(e^{i\theta x} - 1 - i\theta x_{1x} < 1) \prod (dx) \\ &= \prod (R(-1,1)) \int R(e^{i\theta x} - 1) v dx + \int R(e^{i\theta x} - 1 - i\theta x) \mu dx = H \end{aligned} \tag{20}$$

where,

$*\int R(0)(e^{i\theta x} - 1 - i\theta x_{1x} < 1) \prod (dx) = \prod (R(-1,1)) \int R(e^{i\theta x} - 1) v dx + \int R(e^{i\theta x} - 1 - i\theta x) \mu dx = H$ is a binding price distribution of cryptocurrency futures with purchase of cryptocurrency at price, H.

The final objective function that combines Equation (19) and the constraint in Equation (20) with a Lagrangian is listed below,

$$\begin{aligned} &\int 0 \; to \; \alpha e^{-\alpha t} dt - \int 0^{-} \; to \; \alpha e^{\alpha t} / -\alpha f'(t) dt \\ &-L[(GV_{ij} + TV_{ij} + WV_{ij}) * \int R(0)(e^{i\theta x} - 1 - i\theta x_{1x} < 1) \prod (dx) \\ &= \prod (R(-1,1)) \int R(e^{i\theta x} - 1) v dx + \int R(e^{i\theta x} - 1 - i\theta x) \mu dx] - H \end{aligned} \tag{21}$$

The necessary condition for the rate of change in volatility approaching the maximum cryptocurrency price is the first derivative of Equation (21),

$$\begin{aligned} &e^{-\alpha t} - e^{-\alpha t} / -\alpha f'(t) - L[(GV_{ij} + TV_{ij} + WV_{ij}) * R(0)(e^{i\theta x} - i\theta x_{1x} < 1) \prod' \\ &- \prod (R(-1,1)) R(e^{i\theta x} - 1) v' dx + R(e^{i\theta x} - 1 - i\theta x) \mu' dx] \end{aligned} \tag{22}$$

The optimal price of the cryptocurrency future which is the maximum point of prices is the second derivative of Equation (21),

$$\begin{aligned} &e^{-\alpha t} - e^{-\alpha t} / -\alpha f''(t) - L[(GV_{ij} + TV_{ij} + WV_{ij}) * R(0)(e^{i\theta x} - i\theta x_{1x} < 1) \prod'' \\ &- \prod' (R(-1,1)) R(e^{i\theta x} - 1) v'' dx + R(e^{i\theta x} - 1 - i\theta x) \mu'' dx] \end{aligned} \tag{23}$$

Figure 5 shows the relationship between investor sentiment *OAB* and crypto currency futures prices, *CDEFG*. The optimal cryptocurrency futures price is at point *H*.

Figure 5 shows the investor sentiment for crypto futures investments as a Laplace transform of revised volatility. The price distribution is a Levy-Ito decomposition. The investor sentiment intersects the optimal cryptocurrencyfutures price at point *H*, the intersection of investor sentiment, *OAB*, and cryptocurrency futures prices, *ABCDEFG*

Cryptocurrency Options

An investor may select cryptocurrency call options or cryptocurrency put options. Purchasers of call options are optimistic that prices will continue to increase, with $S > X$, where S = the market price and X = exercise price. As long as S stays above X, the call buyer will not exercise the option and purchase the cryptocurrency. The buyer seeks to benefit from the volatility of price movements above the exercise price, purchasing at the point of maximum volatility. The incremental changes in crypto volatility are revised over time to converge to the maximum point in the second derivative of a Laplace Transform.

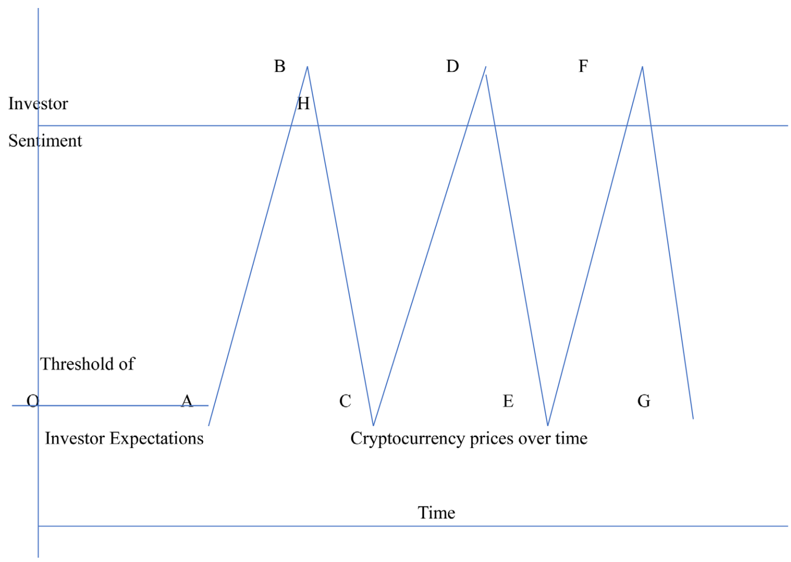

Figure 5. The Optimization Function for Cryptocurrency Futures Prices.

This specifies the following objective function,

Max

$$s^2F'(s) - s\int(0^-) - f'(0^-) \tag{24}$$

Where s = incremental change in crypto volatility.

Subject to

The Laplace transform of the derivative of the crypto-asset's volatility follows an exponential distribution for $S\text{-}X > 0$ for a call option. The Laplace transform of the derivative is given on the left side of Equation

(25),

$$(S - X)[\frac{f(0^-)}{-s}] + \frac{1}{sL\{f'(t)\}} > 0 \quad (25)$$

Using a Lagrangian function to combine the objective function and constraint,

Max

$$s^2 F'(s) - s\int(0^-) - f'(0^-) - L[(S - X)[\frac{f(0^-)}{-s}] + \frac{1}{sL\{f'(t)\}}] - 0 \quad (26)$$

The necessary condition for the achievement of higher gain is the rate of change in volatility, or the derivative of Equation (26)

$$2sF''(s) - s'(0^-) - f'(0^-) - L'[(S' - X')[\frac{f'(0^-)}{-s'}] + \frac{1}{s'L'\{f''(t)\}}] - 0 \quad (27)$$

Figure 5 shows the optimal price for a call buyer. The sufficient condition for the achievement of higher gain is the maximum volatility, or derivative of Equation (27),

$$2F''(s) - s''(0^-) - f''(0^-) - L''[(S'' - X'')[\frac{f''(0^-)}{-s''}] + \frac{1}{s''L''\{f''(t)\}}] - 0 \quad (28)$$

Figure 6 shows the optimal price for a put buyer. Purchasers of cryptocurrency put options will follow the identical pricing procedure as the call option, with the inclusion of *(X-S)* as the gain. Aput option gains with falling cryptocurrency prices so that gain will be realized as long as the market price, *S*, is below the exercise price, *X*. Equation (28) is modified with *X-S* replacing *S – X*, in Equation (29).

$$2F''(s) - s''(0^-) - f''(0^-) - L''[(S'' - X'')[\frac{f''(0^-)}{-s''}] + \frac{1}{s''L''\{f''(t)\}}] - 0 \quad (29)$$

Figure 6 shows the investor sentiment and optimal price for a cryptocurrency call buyer.

Figure 6 contains the investor sentiment for a crypto call buyer with revised volatility estimates of crypto prices above the exercise price. It shows point *H,* the point of maximum volatility and maximum return on the cryptocurrency call investment. This point is at the intersection of the risk-taking investor's sentiment, OA, and cryptocurrency prices, CD.

Figure 7 shows the optimal crypto options price for a crypto put buyer.

Figure 7 is a function of investor sentiment with revised volatility estimates. The price distribution shows sharply falling crypto prices. The point *H,*is the point of maximum volatility and maximum return on the cryptocurrency put investment. This point is at the intersection of the risk-taking investor's sentiment, *OA*, and cryptocurrency prices, *CD.*

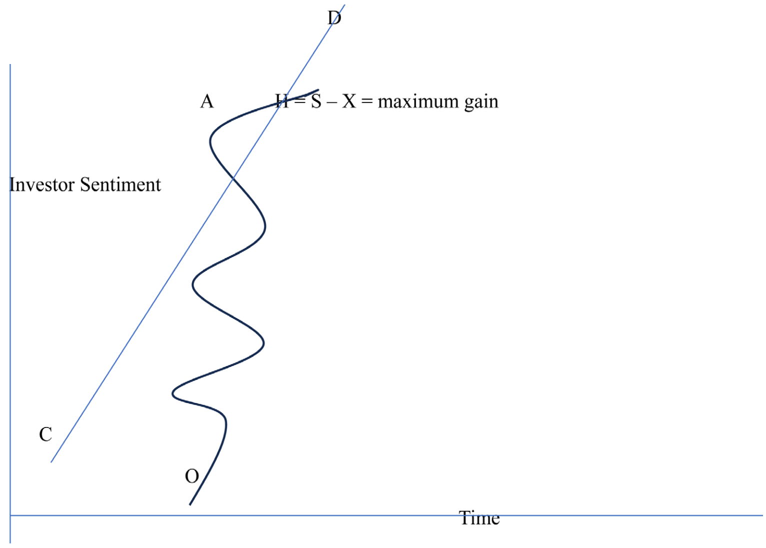

Figure 6. Optimal price for a cryptocurrency call buyer.

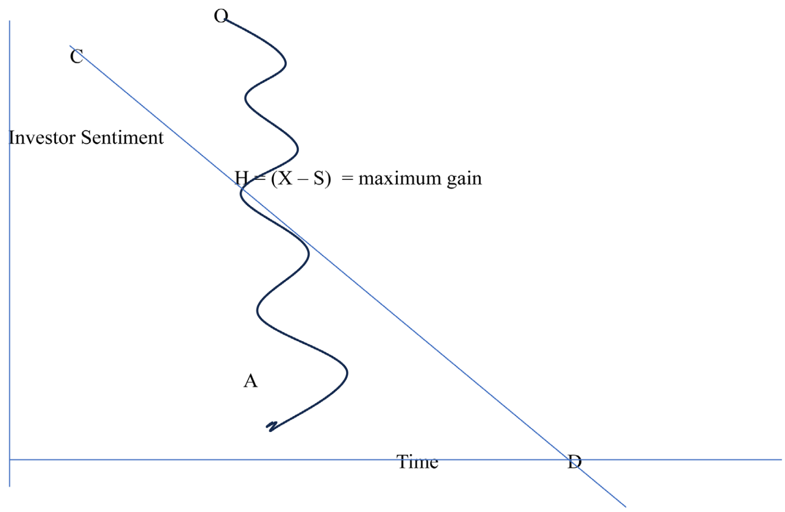

Figure 7. Optimal price for a cryptocurrency put buyer.

5. Conclusion

5.1. Discussion of Findings

This paper is a pioneering formulation of cryptocurrency prices driven by investor sentiment that differs widely among investors of different character. There are six formulations of investor sentiment in cryptocurrencies, arriving at optimal prices through linkage with Levy jump processes and the Levy-Ito decomposition for cryptocurrency prices. Laplace transforms described investor sentiment for crypto-asset futures, and crypto-asset options. We recognize three types of risk-takers. The first type of risk-taker is the risk-averse investor. Risk-averse investors usually abhor risk. Yet they may be willing to accept the high risk of cryptocurrency investment if returns are sufficiently high. This paper models their behavior as aberrant, as it is not characteristic of their normal risk propensities. The second type of investor is a moderate risk-taker. This study views them as capitalizing on the gains from modest increases in cryptocurrency prices. The Levy-Khintchine formula provides an approximation of gradual price movements in crypto prices favored by these investors, who do not take undue risk.

The third type of investor is the risk-taker. The risk-taker wishes to gain from steep price increases. Yet, the risk-taker is mindful of the history of price increases and price declines of cryptocurrencies. This study employs Laplace transforms to incorporate this information in investor sentiment so that future investor expectations are based on realistic knowledge of past returns. The two other types of risk-takers invest in cryptocurrency derivatives. They are very sophisticated investors, such as hedge funds, whose desire is to achieve maximum return with disregard of risk. They trade on volatility so that past volatility estimates and volatility revisions are contained in the Laplace transforms describing investor sentiment.

The study employed Google search volumes, Twitter post volumes, and Wikipedia search volumes as proxies for volumes of cryptocurrencies demanded. This is a novel measure of volume brought about by retail investors using modern methods of searching for cryptocurrency investments. This study takes the position that the sum total of all three volumes is needed to predict the demand for cryptocurrencies, as retail investors have diverse preferences in choice of social media, i.e. some may use Twitter, others may use Google searches, etc.

5.2. Recommendations for Future Research

Future research should isolate specific cryptocurrencies, such as bitcoin, ether, and dogecoin, to form similar formulations. The existing literature (Abraham & El-Chaarani, 2022) does not address the use of Laplace transforms with price revisions, so that new mathematical formulations may take the existing Laplace transforms and add to their complexity by including probabilities, and third and fourth derivatives to capture the most incremental price changes.

It is possible that since there is a three-month delay in closing the final price of both crypto-asset futures and crypto-asset options, that macroeconomic factors such as GDP and inflation, and the microeconomic factor of tariffs may influence final cryptocurrency prices. Formulations including economic variables must be developed.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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