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## Preface

Hilbert-type inequalities including Hilbert's inequalities (built in 1908), Hardy-Hilbert-type inequalities (built in 1934) and Yang-Hilbert-type inequalities (built in 1998) played an important role in analysis and its applications, which are mainly divided three classes of integral, discrete and half-discrete. In recent twenty years, there are many advances in research on Hilbert-type inequalities, especially in Yang-Hilbert-type inequalities.

In this book, applying the weight functions, the parameterized idea and the techniques of real analysis and functional analysis, we provide three kinds of Hilbert-type and Hardy-type integral inequalities in the whole plane as well as their reverses with parameters, which are extensions of Hilbert-type and Hardy-type integral inequalities in the first quarter. The equivalent forms, the operator expressions and some equivalent statements of the best possible constant factors related to several parameters are considered. The lemmas and theorems provide an extensive account of these kinds of integral inequalities and operators.

There are seven chapters in this book. In Chapter 1, we introduce some recent developments of Hilbert-type integral, discrete and half-discrete inequalities. In Chapter 2-3, by using the weight function and real analysis, some new Hilbert-type and Hardy-type integral inequalities in the whole plane with the non-homogeneous kernel are given, and the cases of the homogeneous kernel are deduced. The equivalent forms and some equivalent statements of the best possible constant factors related several parameters are obtained. We also consider the operator expressions as well as the reverses. In Chapter 4-7, other two kinds of Hilbert-type and Hardy-type integral inequalities in the whole plane are also considered.

We hope that this monograph will prove to be useful especially to graduate students of mathematics, physics and engineering sciences.