

Preface

The division by zero has a long and mysterious history all over the world (see, for example, [14, 90] and the Google site with the division by zero) with its physical viewpoint since the document of zero in India in AD 628. In particular, note that Brahmagupta (598 -668 ?) established four arithmetic operations by introducing 0 and at the same time he defined as $0/0 = 0$ in Brāhmasphuṭasiddhānta. We have been, however, considering that his definition $0/0 = 0$ is wrong for over 1300 years, but, we will see that his definition is right and suitable.

The division by zero $1/0 = 0/0 = z/0$ itself will be quite clear and trivial with several natural extensions of fractions against the mysteriously long history, as we can see from the concept of the Moore-Penrose generalized inverse to the fundamental equation $az = b$, whose solution leads to the definition of $z = b/a$.

However, the result (definition) will show that for the elementary mapping

$$W = \frac{1}{z},$$

the image of $z = 0$ is $W = 0$ (**should be defined from the form**). This fact seems to be a curious one in connection with our well-established popular image for the point at infinity on the Riemann sphere ([2]). As the representation of the point at infinity of the Riemann sphere by the zero $z = 0$, we will see some delicate relations between 0 and ∞ which show a strong discontinuity at the point of infinity on the Riemann sphere. We did not consider any value of the elementary function $W = 1/z$ at the origin $z = 0$, because we did not consider the division by zero $1/0$ in a good way. Many and many people consider its value by limiting like $+\infty$ and $-\infty$ or the point at infinity as ∞ . However, their basic idea comes from **continuity** with the common sense or based on the basic idea of Aristotele. – For the related Greek philosophy, see [121, 122, 123]. However, as the division by zero we will consider the value of the func-

tion $W = 1/z$ as zero at $z = 0$. We will see that this new definition is valid widely in mathematics and mathematical sciences, see ([50, 60]) for example. Therefore, the division by zero will give great impacts to calculus, Euclidean geometry, analytic geometry, differential equations, complex analysis at the undergraduate level and to our basic idea for the space and universe.

We have to arrange globally our modern mathematics at our undergraduate level. Our common sense on the division by zero will be wrong, with our basic idea on the space and universe since Aristotele and Euclid. We would like to show clearly these facts in this book. The content is at an undergraduate level.

Close the mysterious and long history of division by zero that may be considered as a typical symbol of the stupidity of the human race and open the new world since Aristotele - Euclid.

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