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Hilbert-type inequalities include Hilbert's inequalities, Hardy-Hilbert-type inequalities and Yang-Hilbert-type inequalities, which are important in Analysis and its applications. They are mainly divided three kinds of integral, discrete and half-discrete. In recent twenty years, there are many advances in research on Hilbert-type inequalities, especially in Yang-Hilbert-type inequalities.

In this book, by using the way of weight functions, the parameterized idea and technique of Real and Functional Analysis, we introduce multi-parameters and provide three kinds of double Hilbert-type inequalities with the general measurable kernels and the best possible constant factors. The equivalent forms, the reverses and some particular inequalities are obtained. Furthermore, the operator expressions with the norm, a large number of examples on the norm, some composition formulas of the operators, and three kinds of compositional inequalities with the best possible constant factors are considered. The theory of double Hilbert-type inequalities and operators are almost built. The lemmas and theorems provide an extensive account of these kinds of inequalities and operators.

There are five chapters in this book. In Chapter 1, we introduce some recent developments of Hilbert-type integral, discrete and half-discrete inequalities, especially in Yang-Hilbert-type inequalities. In Chapter 2, some double Hilbert-type integral inequalities are obtained. The operator expressions with some composition formulas and a few compositional Hilbert-type integral inequalities with the best possible constant factors are deduced. In Chapter 3, by using Hermite-Hadamard's inequality and



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the decreasing property, some new results of double discrete Hilbert-type inequalities and operator expressions are built. In Chapter 4, summing up of the methods in Chapter 2 and Chapter 3, a number of half-discrete Hilbert-type inequalities including compositional inequalities with the best possible constant factors are given, the operator expressions, a few compositional formulas as well as some compositional inequalities are also considered. In Chapter 5, by means of the way of weight functions and technique of real analysis, three kinds of Hardy-Hilbert-type inequality with general measurable kernels and interval variables are provided, which are extensions of Hilbert-type inequalities (3.10), (4.14) and (2.6) with the best possible constant factors. The equivalent forms, the operator expressions with the norm, the reverses and some particular examples are also considered.

The intended readership of this book are large number of graduate students of mathematics, the scholars of mathematical inequalities and the pure and applied mathematicians.

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