Chapter 1

General

1.1. Periodic Motion

Harmonic Periodic motion $x(t)$ is when motion is repeated itself regularly, in equal intervals of time T (the period of oscillation) and is designated by the time function,

$$x(t) = x(t+T) = x = A \cdot \sin\left(\frac{2\pi}{T}\right) = A \cdot \sin wt = A \cdot \cos(kq - wt)$$

which is sinusoidal, where A is the amplitude of oscillation measured from equilibrium position and for repeated motion $t = T$. Quantity $\frac{2\pi}{T} = w = 2 \cdot \pi \cdot f$ is circular frequency, or $f = \frac{1}{T} = \frac{w}{2\pi}$, is the frequency and, k, is the wave number $k = \frac{2\pi}{\lambda}$ and the speed of a wave is $v = \lambda \cdot f$ or $w = v \cdot k$ and because of relation of angular velocity $\overline{v} = w \cdot r = w\left(\frac{1}{k}\right) \rightarrow w = v \cdot k$ then $r \cdot k = 1$.

Velocity

$$\overline{v} = \dot{x} = wA \cdot \cos wt = wA \cdot \sin\left(wt + \frac{\pi}{2}\right)$$

$$\overline{a} = \ddot{x} = w^2 A \cdot (-\sin wt) = w^2 A \cdot \sin(wt + \pi)$$

i.e. Velocity, $\dot{x}$, and Acceleration $\ddot{x}$ are also harmonic with the same frequency of oscillation, and when evaluated lead to the displacement, x, by $\pi/2$ and π radians respectively and the whole system reveals at $\ddot{x} = -w^2 A$, so that *acceleration In harmonic motion* to be proportional to the displacement and directed toward the origin, and because also Newton's second law of motion states that the acceleration is proportional to the force, then harmonic motion can be expected with force varying as kx. (which is Hook's law $F = kx$ and k, the stiffness coefficient, directed in centrifugal velocity vector $\overline{v}r$, *on the radius* r).

In Free vibration of monads $AB = q = [s + \overline{v}\nabla i]$ andbecause velocity vector is composed of the centrifugal velocity $\overline{v}r$, and the rotational velocity $\overline{v}q$, perpendicular to displacement, *x*, and because viscous damping represented by a dashpot, is described by a force proportional to the velocity as holds $F = c\dot{x}$ where, *c*, is the damping coefficient, it is a constant of transverse proportionality and this because $\dot{x} \perp dx$, then it is directional to transverse velocity $\overline{v}y = \dot{x}/dt$ and is holding the homogenous differential equation $m\ddot{x} + c\ddot{x} + kx = 0$.

For a flexible string of mass, *ρ*, per unit d*x* is stretched under Tension *T* and analyzing Newton Laws for tiny length, d*x*, then Net Force, $T\ddot{x} = \rho\overline{a}$ and the equation of motion is

$$\ddot{x} = \left[\frac{1}{v^2}\right]\overline{a} \text{ or } \frac{\partial^2 y}{\partial x^2} = \left[\frac{1}{v^2}\right] \cdot \frac{\partial^2 y}{\partial t^2} \tag{1}$$

where $v = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{T}{m}}$

The general solution of (1) is $y = F1(ct - x) + F2(ct - x)$ where *F*1, *F*2 are arbitrary functions and regardless of the type of function *F*, the argument $(ct \pm x)$ upon differentiation leads to the equation,

$$\frac{\partial^2 F}{\partial x^2} = \left[\frac{1}{v^2}\right] \cdot \frac{\partial^2 F}{\partial t^2} \tag{2}$$

where *F* = The tension, *v* = the velocity of wave propagation

Another general solution of (1) is that of separation of variables as

$$y(x,t) = Y(x) \cdot G(t) \tag{3}$$

where then (1) becomes $\rightarrow \frac{1}{Y}\frac{\partial^2 Y}{\partial x^2} = \left[\frac{1}{v^2}\right]\frac{1}{G} \cdot \frac{\partial^2 G}{\partial t^2}$ and because of independent variables *x*, *t* are both constant the general solutions are,

$$Y = A\sin(w/v)x + B\cos\left(\frac{w}{v}\right)x$$

$$G = A\sin wt + D\cos wt,$$

where arbitrary constants *A*, *B*, *C*, *D* depend on boundary conditions and for $y(0,t) = 0$ will require $B = 0$ and for $y(l,t) = 0$ the solution lead to equations,