

Chapter 1:

Introduction

In the study of the field equations, the problem of extension of integrable systems whose differential operators can be field ramifications of a connection defined in holomorphic vector bundles establish the possibility of consider a method that exhibits the kernels of the differential operators corresponding to the images of cohomological classes whose resolutions are given by the respective spectral resolutions (as the Leray or Vitories) having in account the sheaves of the germs corresponding to the holomorphic bundles on which are constructed moduli stacks of the different physics ant their picture in geometry.

The Penrose transform generates conformally differential operators. Let \mathcal{P} , the Penrose transform that give solution to the field equations modulo a flat conformally connection [1] [15]-[17]. Then the spectral resolution of complex sheaves to the corresponding set of equations of massless fields, to certain class seated in the projective space \mathbb{P}^1 can be written as

¹We consider the Penrose transform [1] [9] through the correspondence

$$\begin{array}{ccc} & F & \\ \nu \swarrow & & \searrow \pi \\ \mathbb{P} & & M \end{array}$$

where $F = F_{1,2}(V)$ is the manifold of flags of dimension one and two, associated to 4-dimensional complex vector V . Let $\mathbb{P} = F_2(V)$, such that $F_1(V) \cong \mathbb{P}^3(\mathbb{C})$, (complex lines in \mathbb{C}^4) and let $M = F_2(V)$, such that $F_2(V) \cong G_{2,4}(\mathbb{C})$, (the Grassmannian manifold of 4-dimensional complex spaces) with $M \cong \mathbb{R}^4 \otimes_{\mathbb{R}} \mathbb{C}$, where explicitly $M = \{z \in \mathbb{C}^4 \mid z = (z_1, z_2, z_3, z_4), \forall z_i = x_i + jy_i, \forall x_i, y_i \in \mathbb{R}\}$, is the 4-dimensional complex compactified Minkowski space. The projections of F are given for: $\nu(L_1, L_2) = L_1$, and $\pi(L_1, L_2) = L_2$, where $L_1 \subset L_2 \subset V$ are complex subspaces of dimension one and two, respectively, defining an element (L_1, L_2) of F defined this as: $F = \{(L_1, L_2) \in V \times V \mid L_1 \subset L_2 \subset V, \nu(L_1, L_2) = \pi(L_1, L_2) = L_2\}$. If M is compactified Minkowski space then $\{\text{set of equations of massless fields}\} \cong \{dF = 0, dF^* = j, W \circ \delta = 0, R^j = 0, R^j - g^j R = 0, \dots\}$.

$$\mathcal{O}_P^0(h) \rightarrow \dots \rightarrow \mathcal{O}_P^i(h) \rightarrow \mathcal{O}_P^{i+1}(h) \rightarrow \dots \rightarrow 0, \quad (1.1)$$

Then the formalism of the double fibration associated to the Penrose transform is used to represent the holomorphic solutions of the generalized wave equation [17], with the helicity parameter established in the total resolution (1.1) to different interaction phenomena:

$$\square_h \phi = 0, \quad (1.2)$$

on some open sets (in the space-time M) $U \subset M$, in terms of cohomological classes of line bundles on $\underline{U} = \nu(\pi^{-1}(U)) \subset P$ (P is the super-projective space). Likewise, is necessary to mention that these cohomological classes are the conformal classes that are searched to be determined to solve the phenomenology of the space-time to diverse interactions studied in gauge theory [1] and can to construct a general solution of the general cohomological problem of the space-time.

We consider $h(k) = (1+k/2)$, $\forall x \in P$, and $x = \pi(\nu^{-1}(x))$. Then a result that establishes the equivalences on the cohomological classes of a line bundle on \underline{U} , and the solutions family of the massless fields equations on the Minkowski space M , with helicity h is given by the following theorem:

Theorem 1.1. (with classic Penrose Transform). *Let $U \subset M$ be an open subset such that $U \cap \underline{x}$ is connect and simply connect $\forall x \in \underline{U}$. Then $\forall k < 0$, the associated morphism to the twistor correspondence given in the page foot 1, which map a 1-form on \underline{U} to the integral along the fibers of π , of their inverse image for ν induces an isomorphism of cohomological classes:*

$$H^1(\underline{U}, \mathcal{O}_P(k)) \cong \ker(U, \square_{h(k)}), \quad (1.3)$$

Proof. [33].

But arise the following questions respect to the possibility of an extension of the space of equivalences of the type (1.3), behold of the classes cohomologically represented on the holomorphic bundles used on the sheaves of the differential operators that can be defined. Then, which are the classes that are extensions of an equivalence space of the type (1.3)?

What version of the Penrose transform will be required?

Precisely these equivalences shape a classification given of the homogeneous vector bundles of lines [15] [18] [19] to differential operators classification [1].

Some important facts that demonstrate by the mean of the use of the Radon transform and their version of D -module transform (Radon-Schmid transform), is the necessity of include a result that establishes the regularity in the analytical sense of the Riemannian manifold, which shapes the space-time, and that allows the application of the involutive distribution theorem on integral submanifolds as solutions of the corresponding equations of massless fields on submanifolds isomorphic in the Kählerian model inside of the *Flat model* given on $G_{2,4}(\mathbb{C})$.

Of fact, an analogy in the obtaining of models of space-time (*under the same reasoning*) must be realized between *special Lagrangian submanifolds* and *m-folds of Calabi-Yau*. But to it, we need define the *complex micro-local structure* that define all the phenomena of strings and branes in microscopic level, which happen in the 6-dimensional component of the Universe (*6-dimensional compact Riemannian manifold*) with ratio of the order 10^{-33} cm (Max Plank longitude of a string). The *Penrose transform* is an integral geometric method that interprets elements of various analytic cohomology groups on open subsets of complex projective 3-space as solutions of linear differential equations on the Grassmannian $G_{2,4}(\mathbb{C})$. The motivation for such transform comes from the interpretation of this Grassmannian as the complexification of the conformal compactification of the Minkowski space and their differential equations being the massless field equations of various helicities $h(k) = (1 + k/2), \forall k \in \mathbb{Z}$.

Part of the object and goals of the followed research in these years, was centered in the extension of the space of equivalences of the type (1.3), under a more general context given through of the language of the D -modules, that is to say, we have wanted extend our classification of differential operators of the field equations to a context of the G -equivariant holomorphic bundles and a complete classification of all differential operators on the curved analogues of the Minkowski space of M .

In the curved analogue spaces to the Minkowski space on M , the n th-direct image of

sheaves $\nu^2 * O(p, r)[q]$, over M , are defined by:

$$\Gamma(U, \nu^2 * O(p, r)[q]) = H^n(U', O(p, r)[q]), \quad (1.4)$$

For example, if we consider the resolution on Γ , through the double fibration $P \rightarrow F \rightarrow M$, we have

$$0 \rightarrow \mu^{-1}O(-4, 0, 0) \rightarrow O(-4, 0)[0] \xrightarrow{\nabla} O(-3, 1)[-2] \rightarrow 0, \quad (1.5)$$

Then the cohomology space given by $H^1(U'', O(-4, 0, 0))$ is the kernel of a first order differential operator from $O_{(A'B')}[-1]$ to $O_{(A'A)}[-2]$.

Also we consider the following resolution

$$0 \rightarrow \mu^{-1}O(0, 1, 0) \rightarrow O(0, 0)[1] \xrightarrow{D_2} O(2, 2)[-4] \rightarrow 0, \quad (1.6)$$

where immediately we see that $H^0(U'', O(0, 1, 0))$ is isomorphic to the kernel of a second order conformally invariant differential operator from $O[1]$ to $O_{(AB)(A'B')}[-2]$. Then to D_2 is had that

$$H^0(U'', O(0, 1, 0)) = \{ \sigma \in \Gamma(U, O[1]) \mid (\nabla + \Phi)\sigma = 0 \}, \quad (1.7)$$

Until here, the classic Penrose transform (included some context in algebraic D -modules considered in the conformal geometry) obeys to rules and establishes a solution criteria in field theory to produce solution classes of type (1.3). However, what happen when the spectral sequence fails to converge in the first step to obtain a cohomology space that is isomorphic to a kernel space of the differential operator in question?

In this sense, is necessary we remount to the first questions made in the before (page 12), which could to bring light on the answer of this last question.

A way of answer to the first and second asks, is to analyze the origin of the structure of the complexes that define the microscopic physical phenomena. Through this way, and in