CHAPTER 1

Introduction

1.1. Preface

The origin of celestial bodies is a quite old problem. We may say that, since the mankind knowing how to think, on the earth, there has been somebody thinking of where the star coming from. Therefore so many mythos and legends arose, and various hypotheses were developed. In 18th century, I. Kant and P. S. Laplace [1] proposed independently the nebular hypothesis: the stars were condensed from groups of nebulae by their own gravitation. Since then many other theories were proposed [2], some of them did not agree with the nebular hypothesis, such as catastrophic hypothesis, but most of them started from the nebular hypothesis and looked for dynamic bases or contraction models of it. In present, most scientists believe that the celestial bodies, including stars and galaxies, formed from nebulae. We know [3]-[6] that the major components of nebulae are atoms and molecules of hydrogen, the vestiges are molecules of helium and other gases and dusts, the its average density is about 10^{-22} g/cm^3. How can such a thin nebula condense under it own gravitation interaction? We know [7] [8] that the gravitation is the most weak interaction. Take the intensity of strong interaction for unity, the intensity of gravitation is only 10^{-39}, the electromagnetic interaction 1/137, and the weak interaction 10^{-5}. Therefore how stars form from nebulae really a puzzling problem.

1.2. Jeans Unstable Property Theory of Gravitation[1]

Sir J. H. Jeans introduces a concept of unstable property of gravitation in 1902, and discussed it in detail in 1929. He considered that if a density disturbance appears in a mass system in equilibrium state, because of the existence of gravitation, the disturbance may increase with time, and the equilibrium could not recover, then appears the gravitation unstable property. The celestial bodies, such as stars and galaxies, formed from nebulae when the gravitation unstable

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[1]Here the introduction about Jeans theory comes from "The Evolution of Solar System" (in Chinese) section 2.5, by Dai wensai, 1979.

property appeared in them.

When gravitation is neglected, a small disturbance of density in continuous medium is propagated in the form of sound wave. Then a group of fundamental complete equations describing the motion of system is expressed as following:

Continuity equation

$$\partial\rho/\partial t + \mathrm{div}\left(\rho v\right) = 0 \tag{1.2.1}$$

Motion equation

$$dv/dt = -\left(\mathrm{grad}P\right)/\rho \tag{1.2.2}$$

State equation

$$P = P\left(\rho\right) \tag{1.2.3}$$

where P, ρ, v are pressure, density, velocity of volume element of the medium (nebula), respectively. In equilibrium state, the medium is uniform and stationary, its density ρ_0 and pressure P_0 are constants, and $v_0 = 0$. If there is a disturbance in the medium, $\rho = \rho_0 + \rho_1$, $P = P_0 + P_1$, and $v \neq 0$, when the disturbance is very small, $\rho_1 \ll \rho_0$, $P_1 \ll P_0$, and v is small. Substituting them into Equations (1.2.1) and (1.2.2), and neglecting second order small quantities, we get the linear equations:

$$\partial\rho_1/\partial t + \rho_0 \mathrm{div} v = 0 \tag{1.2.4}$$

$$\partial v/\partial t = -\left(\mathrm{grad}P_1\right)/\rho_0 \tag{1.2.5}$$

From Equation (1.2.3), we can obtain

$$\mathrm{grad}P_1 = \left(\partial P/\partial\rho\right)_0 \mathrm{grad}\rho_1 = v_s^2 \mathrm{grad}\rho_1 \tag{1.2.6}$$

where

$$v_s^2 = (\partial P / \partial \rho)_0 \tag{1.2.7}$$

and v_s is the sound velocity. Differentiating Equation (1.2.4) to time, and substituting the result into Equations (1.2.5) and (1.2.6), we get

$$\partial^2 \rho_1 / \partial t^2 - v_s^2 \text{div grad} \rho_1 = \partial^2 \rho_1 / \partial t^2 - v_s^2 \nabla^2 \rho_1 = 0 \tag{1.2.8}$$

This is a typical wave equation. For linear equation, a sum of its special solutions also is its solution. The most convenient special solution is a mono-color plane wave. We know that no wave cannot be expressed as a sum of mono-color plane waves by Fourier series or integral, therefore the discussion about mono-color plane wave is fundamental.

We assume that a solution in the form of mono-color plane wave propagating along z direction is

$$\rho_1 = \rho_{10} \exp \left[i (\omega t - kz) \right] \tag{1.2.9}$$

where ρ_{10} is the amplitude of density disturbance, ω the circular frequency of wave, $k = 2\pi / \lambda$ the wave number and λ the wave length. Substituting the formal solution (1.2.9) into Equation (1.2.8), we get a color divergent equation:

$$\omega^2 - k^2 v_s^2 = 0 \tag{1.2.10}$$

While the phase velocity of the wave is

$$\omega / k = v_s \tag{1.2.11}$$

For pressure P and velocity v, some solutions similar to Equation (1.2.9) can be derived, but the change of corresponding amplitude is need.

When gravitation is counted, a gravitation term

$$\boldsymbol{g} = \operatorname{grad}\phi \qquad\qquad (1.2.12)$$

should be added to the right hand side of the motion Equation (1.2.2), then Equation (1.2.2) becomes

$$d\boldsymbol{v}/dt = \operatorname{grad}\phi - \left(\operatorname{grad}P\right)/\rho \qquad\qquad (1.2.13)$$

In Equation (1.2.12) ϕ is the gravitation potential, which satisfy Poisson equation:

$$\nabla^2\phi = -4\pi G\rho \qquad\qquad (1.2.14)$$

In this case, Equations (1.2.1), (1.2.3), (1.2.13) and (1.2.14) form a complete equation system.

With a process similar to that of getting sound wave Equation (1.2.8), for some small disturbances we can obtain a complete equation system:

$$\partial\rho_1/\partial t + \rho_0 \nabla\cdot\boldsymbol{v} = 0 \qquad\qquad (1.2.15)$$

$$\partial v/\partial t = \nabla\phi_1 - \left(\nabla P_1\right)/\rho_0 = \nabla\phi_1 - v_s^2\left(\nabla\rho_1\right)/\rho_0 \qquad\qquad (1.2.16)$$

$$\nabla^2\phi_1 = -4\pi G\rho_1 \qquad\qquad (1.2.17)$$

approximating to first order. Differentiating Equation (1.2.15) to time and substituting Equations (1.2.16) and (1.2.17) into the result, we have

$$\partial^2\rho_1/\partial t^2 - v_s^2\nabla^2\rho_1 - 4\pi GP_0\rho_1 = 0 \qquad\qquad (1.2.18)$$

This also is a wave equation. Substituting the formal solution (1.2.9) into

Equation (1.2.18), we can obtain a color divergent equation:

$$\omega^2 = k^2 v_s^2 - 4\pi G \rho_0 \qquad (1.2.19)$$

the phase velocity of the wave is

$$\omega/k = \sqrt{\left(v_s^2 - 4\pi G \rho_0 / k^2\right)} = \sqrt{v_s^2 - G \rho_0 \lambda^2 / \pi} \qquad (1.2.20)$$

Obviously, this wave is different from sound wave for that its phase velocity is dependent on its wave length. The gravitation effect (the second term in the symbol of root) makes the phase velocity decrease, the longer the wave length, the smaller the phase velocity. From Equation (1.2.19) we may see that when the gravitation term is larger than the sound velocity term, $\omega^2 < 0$. We write Equation (1.2.9) as

$$\rho_1 = \left(\rho_{10} \exp(-ikz)\right) \exp(i\omega t) \qquad (1.2.21)$$

When $\omega^2 < 0$, we write $\omega = \pm i\theta$, $\theta = \sqrt{\omega^2} > 0$, and then we have

$$\rho_1 = \left(\rho_{10} \exp(-ikz)\right) \exp(\pm \theta t) \qquad (1.2.22)$$

Therefore, for positive exponent, disturbance density ρ_1 increases with time in exponent form. Then a gravitation unstable property appears. From $\omega^2 < 0$ and Equation (1.2.19) we can get, when

$$k^2 < 4\pi G \rho_0 / v_s^2 \qquad (1.2.23)$$

i.e., when

$$\lambda > \lambda_J \sqrt{\pi / G \rho_0} v_s \qquad (1.2.24)$$

the gravitation unstable property appears. Equation (1.2.24) is called "Jeans cri-