PART 1
GENERAL INFORMATION. ELECTROMAGNETIC FIELDS AND WAVES. LAWS OF ELECTROMAGNETICS

The skill to evaluate books without reading can be attributed, to my mind, without doubts to the number of greatest discoveries, to which the human intellect recently comes.
G.K. Lichtenberg

CHAPTER 1

GENERAL DEFINITIONS AND RELATIONS OF ELECTRODYNAMICS

1.1. GENERAL DEFINITIONS

An electromagnetic field (EMF) is a unified process in a space and time, i.e. it exists in any space point ($\vec{r}$), in any medium and in each time moment (t). Its electrical "part" - *an electric field* – is characterized by electric vectors $\vec{E}=\vec{E}(\vec{r},t)$ and $\vec{D}=\vec{D}(\vec{r},t)$, accordingly, which are called as an *electrostatic field intensity* and its *induction*. In the SI- units system the electrostatic field intensity is measured in *Volts per meter* ($[V/m]$) (in honor of *Alessandro Volta*: 1745-1827)[*)], an induction – *in coulomb per square meter* $([Cl/m^2])$ (Charles-Augustin de Coulomb: 1736-1806). Similarly, its magnetic "part" – *a magnetic field* – is characterized by magnetic vectors $\vec{H}=\vec{H}(\vec{r},t)$ and $\vec{B}=\vec{B}(\vec{r},t)$, accordingly, which are called as the *magnetic field intensity* and its *induction.* The magnetic field intensity $\vec{H}$ is measured in *Ampere per meter* and has a dimension $([A/m])$ (André-Marie Ampère: 1775-1836), and its induction – in *Tesla* with dimension (Tl) (Nicola Tesla: 1856-1943).

Charges distributed in the space and time with the density $\rho=\rho\,(\vec{r},t)$ and *electric currents (conduction currents)* distributed with the density $\vec{j}=\vec{j}(\vec{r},t)$ are the *sources of an electromagnetic field*. Here we must define exactly *densities* of currents and charges because they can be attributed to each space point in the given time moment. A charge is measured in *coulombs* $([Cl])$, and a current – in *amperes* ([A]). If we are speaking about the charge density ρ and the current density j, then they have dimensions $[Cl/m^3]$ and $[A/m^2]$, accordingly. The current density $\vec{j}$ and the charge density ρ are mutually related by the *continuity equation*, which is the mathematical formulation of the *law of electric charge conservation*:

$$\partial\rho/\partial t+div\,\vec{j}=0. \qquad (1.1)$$

[*)] Detailed information about scientists, which contribution into physics and electrodynamics as well as into the other knowledge areas can be found in Appendix 2.

In other words, the electric charge cannot be annihilated and cannot be arisen from nothing, it can be redistributed only among the bodies at their contacts. The fundamental physical sense of this equation (1.1) is quite simple: the any charge variation (more exact the *charge density*) ρ in the time moment *t* ($\partial\rho/\partial t$) corresponds to the current density variation at the same space point and in the same time moment, which the second term in the last part of (1.1) describes. This is a *divergence*, which can be described, say, in the simplest case of rectangular coordinates (x, y, z) as: $div\,\vec{j} = dj_x/dx + dj_y/dy + dj_z/dz$. Here $j_x, j_{y,} j_z$ are components of the current vector $\vec{j}$ (sometimes, when it does not lead to misunderstanding, we shall use instead of the *current density* the simply a *current*).

From the *continuity equation* (1.1) it directly follows that the conduction current $\vec{j} = \vec{j}(\vec{r}, t)$ is caused by the free charge motion $\rho = \rho(\vec{r}, t)$ obeyed to the *electricity conservation law*, which can be related to Russian scientist *M. Lomonosov* (1711-1765).

An electromagnetic field affects to point (indefinitely small, concentrated in a point) charge q, moving in the space with a velocity $\vec{v}$ under force $\vec{F} = \vec{F}(\vec{r}, t)$ - *the Lorentz force* (Hendrik Antoon Lorentz: 1853-1928) equaled to

$$\vec{F} = q\left(\vec{E} + \left[\vec{v}, \vec{B}\right]\right). \qquad (1.2)$$

The force in the SI system is measured in *newtons* $([N])$ (Isaac Newton: 1642-1727). In the last equation (1.2) square brackets designate the *cross product* of vectors $\vec{v}$ and $\vec{B}$**.** Thus, at absence of a magnetic field ($\vec{B} = 0$; *electrostatics case*) the force $\vec{F}$ acting on the point charge q will be equal to the *scalar product* $\vec{F} = q\vec{E}$, i.e. vectors $\vec{F}$ and $\vec{E}$ will be parallel and the charge motion will coincide with the direction of the electric field vector $\vec{E}$. In another case, when an electric field is absent ($\vec{E} = 0$; *magnetostatics case*) the force $\vec{F} = q\left[\vec{v}, \vec{B}\right]$ is defined by the *cross product* of vectors $\vec{v}$ and $\vec{B}$. Hence, the force $\vec{F}$ in this case will be directed at right angle (normally) to the plane, in which vectors $\vec{v}$ and $\vec{B}$ are situated**.** At that, the triplet of vectors $\vec{F}, \vec{v}, \vec{B}$ will be *right-hand* (if to see from the end of the vector $\vec{F}$, the rotation from $\vec{v}$ to $\vec{B}$ should occur counter-clockwise).

An electromagnetic field induces (excites) in the medium or in the body with conductivity σ the electric current, which density $\vec{j}$ is related to the field electric component $\vec{E}$ as:

$$\vec{j} = \sigma\vec{E}\,. \qquad (1.3)$$

This is so-called *differential* form (i.e. related to the one point of a space or a surface) of the *Ohm law* (Georg Simon Ohm: 1789-1854). The conductivity σ is included in the equation (1.3), which is measured in *siemens per meter* (Sim/m) (Carl Heinrich von Siemens: 1816-1892). Depending on its value, media can be divided into conductors, when $\sigma \geq 10^4$ Sim/m, semiconductors if $10^{-10} < \sigma < 10^4$ Sim/m, and at last dielectrics, for which $\sigma < 10^{-10}$ Sim/m.

The differential form of the Ohm law is necessary when the current is non-uniformly distributed along the conductor section or in some volume as it can be seen in Figure 1.1,*a.* We can see that the current from two electrodes evidently non-uniformly spreads in the volume, and therefore, the current J in the mutual circuit can be determined by integration of its distribution by a section.

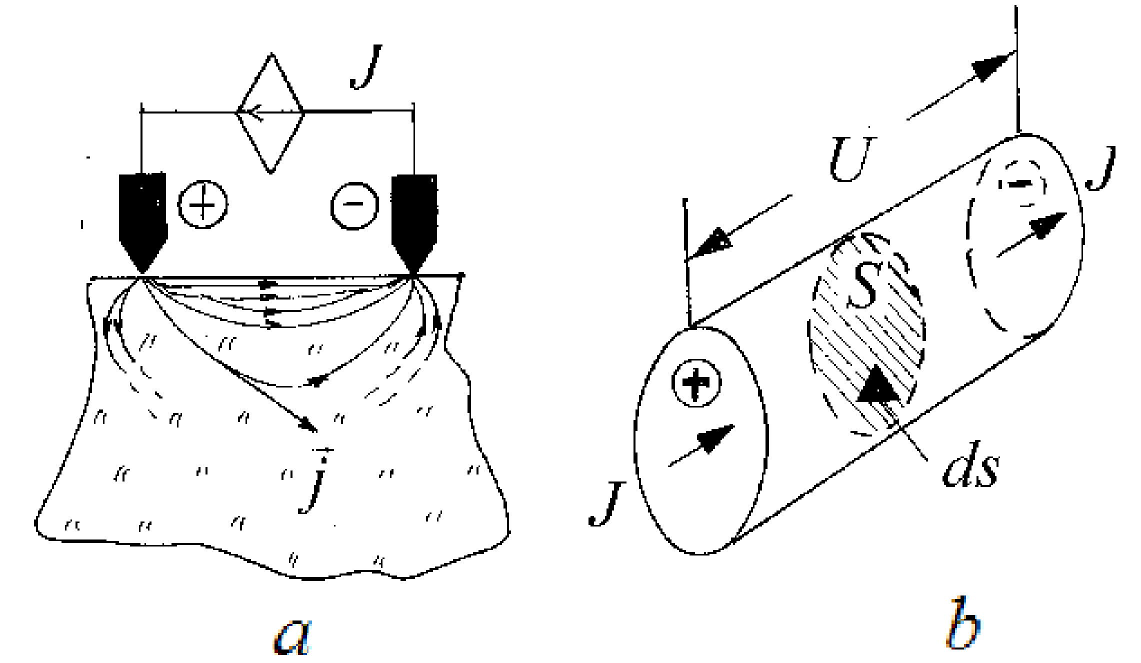

Figure 1.1. Patterns of current flowing in the conducting semi-space (*a*) and through the conductor segment (b)

The integrated form of the *Ohm law* is well known from the secondary school's physics, namely: if the voltage U is applied to the resistive circuit piece with resistance R (Figure 1.1,*b*), the current J will flow in this circuit:

$$J = U/R\,. \tag{1.4}$$

The last equation is the *Ohm law* in the more "usual" form.

In a number of cases, one has to use the equation (1.3), and we shall be sure more than once in this lecture course. For instance, if the current with the density $\vec{j}$ flows through the section S of some conductor with conductivity σ and this density is arbitrarily distributed over section S, the net current through this section S can be determined as

$$J = \oint_S \vec{j}\, d\vec{s}\,. \tag{1.5}$$

Here the elementary area in the section S is designated as $d\vec{s}$.

1.2. GENERAL LAWS OF CLASSICAL ELECTRODYNAMICS. SOURCES OF AN ELECTROMAGNETIC FIELD

Now, after relations mentioned in the pervious section, we are ready to speak about the general laws of electromagnetics. This point is not very simple in its form but it is deep in its sense. The reader must simply get accustomed to the somewhat unusual (at the first glance) form of electromagnetics laws. An electromagnetic field (EMF) is described by its intensities: the *electric field intensity* $\vec{E}$ and the *magnetic field density* $\vec{H}$. Vectors of the electromagnetic field $\vec{E}, \vec{D}, \vec{H}, \vec{B}$ and its sources $\vec{j}, \rho$ are related between each other by a system of the *Maxwell equations* (James Clerk Maxwell: 1931-1879) (in the *differential form*, i.e. concerned to the single point of a space in the given time moment), which can be written as:

$$rot\,\vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{j}, \tag{1.6}$$

$$rot\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \tag{1.7}$$

$$div\vec{D} = \rho, \tag{1.8}$$

$$div\vec{B} = 0. \tag{1.9}$$

When we spoke about the continuity equation (1.1), we have got the acquaintance with the operation div. The operation *rot* (a rotor, literally, a whirlwind, a rotation) is included into the equation system (1.6)-(1.9). In the coordinate notation, for instance, the equation (1.6) will be written as: $\partial H_z/\partial y - \partial D_y/\partial z = j_x$, $\partial H_x/\partial y - \partial D_z/\partial z = j_y$ etc. We hope that the reader is well acquainted with the concept of the ordinary derivative du/dx; in equations (1.6)-(1.9) there are the *partial derivatives* of the field components by coordinates. Since the field values are functions of several variables $(x, y, z, t \equiv \vec{r}, t)$, the derivative on the one from them is a partial derivative. Understanding will come to the reader: one should have a will.

We directly see from Maxwell equations (1.6)-(1.9) that *outside currents* $\vec{j}_{\text{out}}$ and *outside charges* ρ_{out} are the *sources*, which excite the electromagnetic field $\vec{E}, \vec{H}$. At the same time, the field $\vec{E}, \vec{H}$ induces currents and charges $\vec{j}, \rho$ on the conducting bodies or in dielectric or other media. For instance, the electric field $\vec{E}$ stimulates the conduction current $\vec{j}_{\text{con}} = \sigma\vec{E}$, according to the *Ohm law* (1.3), in each point of the conducting body, and the total current J will be defined according to the "integrated" *Ohm law* (1.4). In accordance with the famous *Maxwell hypothesis*, the conduction current in the circuit break (for example, between capacitor plates; Figure 1.2) should be supplemented by the *displacement current* $\vec{j}_{dis}$, so that the net current included in the equation (1.6) is a sum of the conduction current and the bias current: $\vec{j} = \sigma\vec{E} + \vec{j}_{\text{dis}}$.

The statement that *field* $\vec{H}, \vec{E}$ *variations in time* (the *operation* $\partial/\partial t$ in time) *correspond* to *any field* $\vec{E}, \vec{H}$ *variations in space* (the *operation rot* in space coordinates) is another important consequence of Maxwell equations.

The equation system does not include the values related to the media characteristics, in which the EMF $\vec{E}, \vec{H}$ exists and electromagnetic waves propagate.

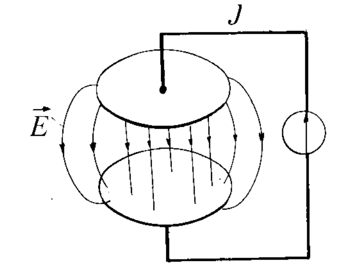

Figure 1.2. To explanation of the bias current effect in a capacitor.

That is why, the main equation system of electrodynamics – the Maxwell equation system (1.6)-(1.9) should be supplemented by some conditions.

First of all, these are *constitutive equations*, i.e. equations connecting the field intensities $\vec{E}, \vec{H}$ and their induction parameters $\vec{D}$, $\vec{B}$ through the media parameters ε, μ, which are the *dielectric and magnetic permeability* of the media. In the general case, ε, μ are the complex variables: $\varepsilon = \varepsilon' + i\varepsilon''$, $\mu = \mu' + i\mu''$. The vector $\vec{D}$ is sometimes called as a *vector of electric displacement.* The parameter ε as a relative dielectric permeability is connected with $\varepsilon_a = \varepsilon \varepsilon_0$ - the absolute permeability measured in *farad per meter* $([F/m])$ (Michael Faraday: 1791-1867); ε_0 is the permeability of the vacuum. In practice one can frequently use the relative values of ε, μ.

The concept of an *index of refraction* of the medium $n^2 = \varepsilon \mu$ is used as well. The sense of signs in front of imaginary parts of the permeability and the index of refraction we shall discuss later.

Constitutive equations in the simplest case (ε, μ, σ are scalar parameters) will take the following view:

$$\vec{D} = \varepsilon \vec{E}, \quad \vec{B} = \mu \vec{H}, \tag{1.10}$$

In equation (1.10) the quantity of σ means the specific (differential) conductance in the given space point. The coefficient of proportionality μ between vectors of the magnetic field $\vec{B}$ and $\vec{H}$ is called the *absolute magnetic permeability* of the medium and is measured in Henry per meter $([Hn/m])$. Constitutive equations (1.10) in vacuum will be written as: $\vec{D} = \varepsilon_0 \vec{E}$, $\vec{B} = \mu_0 \vec{H}$, where $\varepsilon_0 = (1/36\pi)\cdot 10^{-9}$ (F/m), $\mu_0 = 4\pi \cdot 10^{-7}$ (Hn/m).

The equation (1.6) is usually called as the *first Maxwell equation*, and the equation (1.7) – the second one. The equation (1.7) is the differential form of the *Faradey law of magnetic induction*. Equation (1.6) is the differential form of the generalized *total current law* (*Ampere law*). We shall be acquainted with the more general integrated formulation of Maxwell equations a little later (see Section 1.4), and now we shall write, for instance, the total current law in the integrated form: $\oint\limits_L \vec{H}\, d\vec{l} = \int\limits_S \vec{j}\, d\vec{s} = J$. The total current law connects the magnetic field circulation along the contour L and the total current $\vec{j} = \sigma \vec{E} + \vec{j}_{dis}$, which is covered by the contour. In the concept of the total current we include, as we already noted a little earlier, the *outside current* $\vec{j}_{\text{out}}$, which is the field source and considered to be given, however, it itself is not a results of the EMF under consideration. In turn, the outside current $\vec{j}_{\text{out}}$ is the consequence of the *electromotive forces* $\vec{E}_{\text{out}}(\vec{r},t)$ action, i.e. forces of the non-electromagnetic origin. Such forces are processes of chemical, bio-electromagnetic, space, diffusion and etc. characters. Thus, for the outside current the differential form of the *Ohm law* is correct: $\vec{j}_{\text{out}} = \sigma \vec{E}_{\text{out}}$. The term $\vec{j}_{\text{dis}} = \partial \vec{D}/\partial t$, which is called as the *displacement current* representing of the famous *Maxwell hypothesis* about this current existence supplementing in the space the *conduction current* $\vec{j}$, is included also into the total current $\vec{j}$. Thus, the lines of the total current are closed in the space either to themselves or to sources (electric charges), as it is shown in Figure 1.2 on an example of a capacitor. The equation for the conduction current (1.3) is the differential form of the *Ohm law*.

It is important that in the Maxwell equation system the medium parameters (ε, μ, σ) are not included and since these equations are suitable for any media, then they must be complemented by the constitutive equations (1.10) (they are sometimes called as *equations of states* because they characterize the medium). A little lately we shall give the summary of the main classes of media, with which in practice of antenna systems and radio wave propagation we need to deal.

1.3. ENERGY OF AN ELECTROMAGNETIC FIELD. THE UMOV-POYNTING VECTOR. THE EQUATION OF THE ENERGY BALANCE (THE UMOV THEOREM)

The system of main equations of electrodynamics (1.6)-(1.9) is the *total* and *sufficient* system to provide an analysis and calculations of the specific problems in the truly boundless area of human activity including the problems of antenna-feeder devices and radio wave propagation. However, this system is the system of *equations in partial derivatives*, and therefore, it allows the boundless variety of solutions. That is why, at examination of practical or simulation problems, the main equation system should be supplemented by a number of conditions, among which there are the necessary consideration of the field behavior character near boundaries of media with different parameters (*boundary conditions*), the field behavior on the infinite separation from the source (*conditions of radiation*, the *principle of extinguishing*, the *principle of the limited amplitude* etc.), and, at last, the field behavior in the area of the knee of the formative surface (the non-analytical boundary representation) of the object under consideration, near sharp bends etc. (*conditions on edges*). At solution of non-stationary problems, when the field quantity dependence in time is manifested (instead of harmonic one), there are necessary the *initial conditions* as well, i.e. field values in the some initial time moment (for example, at $t = 0$). Mentioned conditions are necessary for the correct problem statement satisfying the requirements of the *unicity theorem* of the Maxwell equation solution.

The action of the electromagnetic field becomes apparent, as we already mentioned, on the basis of its influence upon a moving charge (1.1) as well as according to its impact on any body, which is situated in the area occupied by the field. This action is manifested owing to the electromagnetic energy W distributed in some area V, limited by the surface S (Figure 1.3; the surface S can be both real (realizable) and imaginary (virtual):

$$W = (1/2)\int_V \left(\varepsilon_{\text{a}}\,\vec{E}^2 + \mu_{\text{a}}\,\vec{H}^2\right)dV. \tag{1.11}$$

Thus, the electromagnetic energy W is distributed in space with the bulk density

$$w = (1/2)\,\varepsilon_{\text{a}} E^2 + (1/2)\mu_{\text{a}} H^2. \tag{1.12}$$

From (1.11) it directly follows that EMF energy W represents a sum of the electric W^{el} and magnetic W^{m} field energy, so

$$W^{\text{el}} = (1/2)\int_V \varepsilon_{\text{a}}\vec{E}^2 dV = (1/2)\int_V \vec{E}\,\vec{D}\,dV, \tag{1.13}$$

$$W^{\text{m}} = (1/2)\int_V \mu_{\text{a}}\vec{H}^2\,dV = (1/2)\int_V \vec{H}\,\vec{B}\,dV. \tag{1.14}$$

The EMF energy W (1.11) in the area V can change in time owing to, at least, two processes. First of all, it can be transformed into another energy type of the “non-electromagnetic” character, for instance, into thermal energy, for heating the body with conductivity σ, chemical, bio-physical energy and many others. The variation of energy amount in the volume V can be also at the expense of its going away from the volume through some holes S_Σ or through the surface S itself by virtue of complete or partial transparency (for example, owing to mentioned imagination) or due to some features of the field itself (see below). It is evident that energy may not only go away from the volume V but to enter it from outside through the same surface S or though some of its part (for instance, S_Σ).

The first of the mentioned processes of EMF energy variation in the volume W (transformation to another energy types) is described as a power delivered (deliverable) by the field in the time unit as

$$P = \int_V \vec{j}\,\vec{E}\,dV, \tag{1.15}$$

The integration element in (1.15) defines the *bulk power density* p, which is $p = \vec{j}\,\vec{E}$.

Energy going away (emitted) from the volume V in time unit (or coming into it from outside) is defined as follows:

$$P_\Sigma = \oint_S \vec{S}\,d\vec{s}, \tag{1.16}$$

where $\vec{S}$ is the *Umov-Poynting vector* (N.A.Umov: 1846-1915; Poynting John Henry: 1852-1914) representing the *energy flow density*.

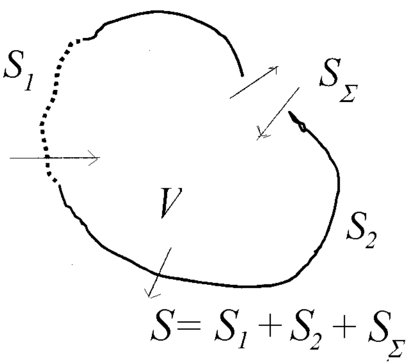

Figure 1.3. The scheme of the arbitrary volume V, closed by some combined surface $S = S_1 + S_2 + S_\Sigma$. Surfaces S_1, S_2 differ by their properties, and the surface S_Σ represents a hole through which energy is going away from the volume V or enters into it.

The Umov-Poynting vector $\vec{U}$ is defined as the cross product of vectors of electric $\vec{E}$ and magnetic $\vec{H}$ field intensities in each point of the surface S:

$$\vec{U} = \left[\vec{E}, \vec{H}\right]. \tag{1.17}$$

Energy quantities W, P, P_Σ are related each other by the "integrated" relation – the *equation of power balance*:

$$dW/dt + P + P_\Sigma = 0. \tag{1.18}$$

In essence, this is a *law of energy conservation* for EMF: each variation of energy W in some volume V is corresponded by the variation owing to losses P or by