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Modelling and Simulating Dynamics Efficiency of Rural-Community Banks (RCBs) in Ghana

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Abstract

The present work seeks to develop a model for measuring efficiency of RCBs in Ghana by means of financial key performance indicators pairing macroeconomic indicators. A stochastic differential equation model for predicting the efficiency of RCBs in the future is developed and simulated using gaussian jumps to evaluate the models' performance in unpredicted situations with four distinct phases of efficiency. Unique solution Exit multiple 4-dimensional stochastic differential equations and Macroeconomic model is proven to be the best-fitting model for the data with the lowest information criterion.

Keywords

Efficiency, Rural and Community Banks, Stochastic Differential Equations, Stochastic Jumps

1. Introduction

Banks serve as financial intermediaries that accept deposits from customers and use them in the form of loan facilities to deficit spending units in the economy [1] In Ghana, the business of banking remains one of the most lucrative industries despite increasing competition and in the attempt to remain competitive; banks are exposed to several factors which can affect their profitability [2]. One of the key players in the Ghanaian banking industry is the Rural and Community Banks (RCBs) to fast track the development of rural areas of Ghana. They are regulated by the Bank of Ghana and thereby form part of the regulated financial sector in Ghana.

Performance efficiency is an important factor in the banking sector as it has a direct relationship with stability and growth. It can be hard to choose which measures to focus on, for example profitability, solvency, and liquidity [3]. The question, therefore, is to what role these sound financial indicators and economic indicators play in performance and predictions of future performance of rural banks [4].

The work of [5]-[10] is some of the models that have undoubtedly broadened perspectives on differentials in performance. The issue of managing and measuring bank performance remains on the agenda and draws great attention from scholars and non-academic researchers [11]. Performance of banks can be expressed in terms of competition, concentration, efficiency, productivity, and profitability [12]. There is still no consistent viewpoint about what performance measures better reflect a Bank's current position and its potential for growth [6].

RCBs over the years have experienced tremendous growth in its operation from a total manual method of operation to computerization in recent times. It is believed that this revolution has also led to some challenges in efficiency and productivity [13]. [13] mentioned that not all RCBs are solvent, however; in 2008 seven RCBs were insolvent, and the continued operation of poorly performing RCBs is one of the key issues facing the banking industry in Ghana. Bank of Ghana report (2012) examine the operational performances of 135 RCBs; 85 RCBs are "satisfactory", 19 RCBs were rated as "mediocre", and the rest of 31 RCBs were considered "distressed" RCBs that needed close monitoring and nurturing to avoid being closed. [14] examines that there is a positive relationship between non-performing loans and rural banks' profitability revealing that there are higher loan losses, but banks still earn profit which significantly impacts future performance. [8] indicate performance of RCBs in Ghana is positively influenced by deposit and liquidity. [9] stated that negative trend in banks financial performance in Ghana pass on their inefficiencies to their customers by raising their lending rates and lowering their deposit rates, despite it being cited as illiquid and stresses on Non-Performing Loan (NPL) as key problem affecting banks' performance. [5] reveals that an improvement in the funding risk of a rural bank in a particular period, signifies a drop in its profitability in the future. [15] [16] investigate that with varying degrees while continuous profitability performance of RCBs can curtail shortfall in funding risk and enhancement of RCBs stability.

According to ARB Apex report (2019); RCBs in the Strong and Satisfactory category decreased from 21 to 9 and 76 to 67 respectively. Fair and Marginal category increased from 20 to 43, and 11 to 12. 9 RCBs not rated due to operational difficulties and 4 RCBs were in distressed position, which show a decline in RCB's performance overtime. RCBs industry continued to record weak assets quality, non-performing loans (NPL) increased to 13.73% as 102 banks recorded NPL above 5% in fourth quarter 2019 compared with 84 in third quarter 2019. The industry year-on-year profit before tax is also declined by 82.11%.

According to [10], the collapse of most of these institutions can be seen as a result of them not being able to evaluate and predict their financial standings in the years ahead. Several studies like [5] [6] [7] cited that performance efficiency of RCBs in Ghana is dipping by profitability indicators, they fail to extend their investigations to identify the factors that predict efficiency. A common observation made from these existing studies is that accounting-based financial ratios approach is consistently used to measure and rank RCB's performance. The accounting ratios approach to performance measurement focuses only on either the input or output side of the financial intermediation process. It does not consider the input-output combinations in the intermediation process. Therefore, it is not able to fully identify inefficiencies and changes in efficiency. This research seeks to develop a model for measuring efficiency of RCBs in Ghana by means of financial key performance indicators pairing macroeconomic indicators, stochastic differential equation model formulated was simulated with jumps incorporation to predict and evaluate the performance of various models.

2. Methodology and Model Development

Developing a prediction methodology for rural bank financial position in Ghana is the primary goal of this research. According to the literature, there are three methods for determining a bank's efficiency: financial ratios; parametric techniques; and non-parametric approaches. Using financial ratios to evaluate a decision-making unit's efficiency has the drawback of creating a false sense of priority for various sorts of input and output. Simulated Data from R was used for model simulation, with the results divided into four categories: profitability, liquidity, solvency, and macroeconomic indicators, to meet the study's goals. For this investigation, repeated responses over time are evaluated and assessed using Stochastic Differential Equations algorithm.

2.1. Model Formulation

The Efficiency equation can be derived using conditional stationary SDE's. Let $X^j(t)$ denote the (random) performance of a Bank at time $t \geq 0$. Here, we focus on a multivariate mixed effect parameters SDE model of bank profitability $P^j(t)$, liquidity $L^j(t)$, solvency $S^j(t)$ and Macroeconomic variables $M^j(t)$ dynamics. Formulate a state's efficiency equation using ODE, we let:

$X^j(t)$ = Efficiency of a Bank at time $t \geq 0$;

$P^j(t)$ = Profitability of a Bank at time t;

$L^j(t)$ = Liquidity of a Bank at time t;

$S^j(t)$ = Solvency of a Bank at time t;

$M^j(t)$ = Macroeconomic situation at time t;

j = Efficiency at a particular state.

Thus, the state's parameter can be defined as:

$$X^j(t) = \left[P^j(t), L^j(t), S^j(t), M^j(t)\right]^{\mathrm{T}} \tag{1}$$

2.2. Model Assumptions

1) The efficiency process is Multivariate discrete-time stochastic process with a countable state space Markov process.

2) Efficiency consists of 4 time-dependent subpopulations

$P^j(t), L^j(t), S^j(t), M^j(t)$

3) The transition of $P^j \to X^j(t):\alpha$, $L^j \to X^j(t):\beta$, $S^j \to X^j(t):\gamma$, $M^j \to X^j(t):\delta$

4) Let Efficiency rate in P^j, L^j, S^j and M^j are; $r_1(t), r_2(t), r_3(t)$, and $r_4(t)$

5) Noise in the efficiency rate is deemed to capture the environmental effects such as accuracy of banks growth.

2.3. Model Process

From the states Equation (1), the derivative of Equation (1) can be written as.

$$\mathrm{d}X^j(t) = \left[\mathrm{d}P^j(t), \mathrm{d}L^j(t), \mathrm{d}S^j(t), \mathrm{d}M^j(t)\right]^{\mathrm{T}} \tag{2}$$

Using the flow chart and model assumptions above, ODE can be formulated below.

$$\frac{\mathrm{d}X^j(t)}{\mathrm{d}t} = \begin{cases} (\alpha + r_1(t))P^j(t) \\ (\beta + r_2(t))L^j(t) \\ (\gamma + r_3(t))S^j(t) \\ (\delta + r_4(t))M^j(t) \end{cases} \tag{3}$$

But $\dfrac{\mathrm{d}X^j(t)}{\mathrm{d}t} = \left[\dfrac{\mathrm{d}P^j(t)}{\mathrm{d}t}, \dfrac{\mathrm{d}L^j(t)}{\mathrm{d}t}, \dfrac{\mathrm{d}S^j(t)}{\mathrm{d}t}, \dfrac{\mathrm{d}M^j(t)}{\mathrm{d}t}\right]^{\mathrm{T}}$.

2.4. Formulating of Differential Equation (ODE's)

$$\begin{cases} \text{Profitabilty}: \dfrac{\mathrm{d}P^j(t)}{\mathrm{d}t} = (\alpha + r_1(t))P^j(t) \\ \text{Liquidity}: \dfrac{\mathrm{d}L^j(t)}{\mathrm{d}t} = (\beta + r_2(t))L^j(t) \\ \text{Solvency}: \dfrac{\mathrm{d}S^j(t)}{\mathrm{d}t} = (\gamma + r_3(t))S^j(t) \\ \text{Macroeconomic}: \dfrac{\mathrm{d}M^j(t)}{\mathrm{d}t} = (\delta + r_4(t))M^j(t) \end{cases} \tag{4}$$

2.5. Incorporating White Noise (Brownian Motion, $r_i(t)\mathrm{d}t = \sigma_i \mathrm{d}B^t(t)$) into ODE Due to Randomness in the Efficiency Rate We Obtain

$$\begin{cases} \dfrac{\mathrm{d}P^j(t)}{\mathrm{d}t} = \alpha P^j(t) + \sigma_1 \dfrac{\mathrm{d}B^1(t)}{\mathrm{d}t} P^j(t) \\ \dfrac{\mathrm{d}L^j(t)}{\mathrm{d}t} = \beta L^j(t) + \sigma_2 \dfrac{\mathrm{d}B^2(t)}{\mathrm{d}t} L^j(t) \\ \dfrac{\mathrm{d}S^j(t)}{\mathrm{d}t} = \gamma S^j(t) + \sigma_3 \dfrac{\mathrm{d}B^3(t)}{\mathrm{d}t} S^j(t) \\ \dfrac{\mathrm{d}M^j(t)}{\mathrm{d}t} = \delta M^j(t) + \sigma_4 \dfrac{\mathrm{d}B^4(t)}{\mathrm{d}t} M^j(t) \end{cases} \tag{5}$$

2.6. Simplifying the Equation (5) of the System into SDE Form by Multiplying dt

$$\begin{cases} \mathrm{d}P^j(t) = \alpha P^j(t)\mathrm{d}t + \sigma_1 P^j(t)\mathrm{d}B^1(t) \\ \mathrm{d}L^j(t) = \beta L^j(t)\mathrm{d}t + \sigma_2 L^j(t)\mathrm{d}B^2(t) \\ \mathrm{d}S^j(t) = \gamma S^j(t)\mathrm{d}t + \sigma_3 S^j(t)\mathrm{d}B^3(t) \\ \mathrm{d}M^j(t) = \delta M^j(t)\mathrm{d}t + \sigma_4 M^j(t)\mathrm{d}B^4(t) \end{cases} \tag{6}$$

2.7. The Stochastic Differential Equations Can Be Written in Matrix Form As

$$\mathrm{d}X(t) = AX(t)\mathrm{d}t - \sigma X(t)\mathrm{d}B(t)^{\mathrm{T}} \tag{7}$$

This Equation (7) is the matrix form of SDE's using Brownian motion for efficiency of RCB's.

$\mathrm{d}X(t) = AX(t)\mathrm{d}t - \sigma X(t)\mathrm{d}B(t)^{\mathrm{T}}$ can be $\mathrm{d}X(t) = \mu\mathrm{d}t + \sigma\mathrm{d}W(t)$, For certain constants $\mu > 0$ and σ, called respectively the drift and the diffusion efficiency of a Bank. To assess whether formulated model have a solution, we conduct positivity test.

3. Positivity Test (Zero Solutions)

Consider a multiple 4-dimensional stochastic differential equation (SDE)

$$\begin{cases} \mathrm{d}P^j(t) = \alpha P^j(t)\mathrm{d}t + \sigma_1 P^j(t)\mathrm{d}B^1(t) \\ \mathrm{d}L^j(t) = \beta L^j(t)\mathrm{d}t + \sigma_2 L^j(t)\mathrm{d}B^2(t) \\ \mathrm{d}S^j(t) = \gamma S^j(t)\mathrm{d}t + \sigma_3 S^j(t)\mathrm{d}B^3(t) \\ \mathrm{d}M^j(t) = \delta M^j(t)\mathrm{d}t + \sigma_4 M^j(t)\mathrm{d}B^4(t) \end{cases} \tag{8}$$

where $f: R^4 \times R_+ \to R^4$ and $g: R^4 \times R_+ \to R^{4\times 4}$, and $B(t) = (B_1(t), \cdots, B_m(t))^{\mathrm{T}}$ is a 4-dimensional Brownian motion, with initial $P^j(0) > 0$, $L^j(0) > 0$, $L^j(0) > 0$, $M^j(0) > 0$ the solution is positive for initial values.

Theorem 1: for any given initial value $X(0) = X_0 \in R^4$, the SDE has a unique positive solution denoted by $X(t: X_0)$. Assume furthermore that $f(0,t) = 0$ and $g(0,t) = 0$ for all $t \geq 0$. Hence the SDE admits the trivial solution $X(t:0) \equiv 0$. $\varphi = \{P^j, L^j, S^j, M^j\} \to R^4 : P_0^j \geq 0, L_0^j \geq 0, S_0^j \geq 0, M_0^j \geq 0$ Then the solution $\{P^j, L^j, S^j, M^j\}$ is positive for $t \geq 0$. Hence, by Itô' sformula;

3.1. Profitability Stochastic Differential Equation

$$\mathrm{d}P^j(t) = \alpha P^j(t)\mathrm{d}t + \sigma_1 P^j(t)\mathrm{d}B^1(t) \tag{9}$$

Let; $\alpha P^j(t) = \mu_1$ (drift) and $\sigma_1 P^j(t) = \theta_1$ (diffusion)

$$\mathrm{d}P^j(t) = \mu_1\mathrm{d}t + \theta_{11}\mathrm{d}B^1(t)$$

$$\mathrm{d}\left(\log\left(P^j\right)\right) = \left(\mu_1 - \frac{\theta_1^2}{2}\right)\mathrm{d}t + \theta_1\mathrm{d}B^1(t)$$

$$\int_0^T \mathrm{d}\left(\log\left(P^j\right)\right) = \left(\mu_1 - \frac{\theta_1^2}{2}\right)\int_0^T \mathrm{d}t + \theta_1 \int_0^T \mathrm{d}B^1(t)$$

$$\log P_T^j - \log P_0^j = \left(\mu_1 - \frac{\theta_1^2}{2}\right)T + \theta_1 B^1(T)$$

$$\frac{P^j(t)}{P_0^j} = \mathrm{e}^{\left(\mu_1 - \frac{\theta_1^2}{2}\right)T + \theta_1 B^1(T)}$$

$$\mathrm{d}P^j(t) = P_0^j \mathrm{e}^{\left(\mu_1 - \frac{\theta_1^2}{2}\right)T + \theta_1 B^1(T)} \tag{10}$$

With the solution (10), we can characterize the qualitative behavior of the process at $t \to \infty$: We observe that profitability is always positive, assuming the initial P_0^j is positive. Since $\mathrm{d}P^j(t) = \alpha P^j(t)\mathrm{d}t + \sigma_1 P^j(t)\mathrm{d}B^1(t)$ implies

$$P^j(t) = P_0^j + \int_0^t \alpha(t) P^j(t)\mathrm{d}t + \int_0^t \sigma_1 P^j(t)\mathrm{d}B^1(t)$$

$$\therefore E\left(\int_0^t \sigma_1 P^j(t)\mathrm{d}B^1(t)\right) = 0$$

$$E\left(P^j(t)\right) = P_0^j + \int_0^t \alpha(t) P^j(t)\mathrm{d}t$$

$$E\left(P^j(t)\right) = P_0^j \mathrm{e}^{\alpha(t)t} \quad \text{for } t \geq 0 \tag{11}$$

3.2. Liquidity Stochastic Differential Equation Model

$$\mathrm{d}L^j(t) = \beta L^j(t)\mathrm{d}t + \sigma_2 L^j(t)\mathrm{d}B^2(t) \tag{12}$$

Let; $\beta L^j(t) = \mu_2$ (drift) and $\sigma_2 L^j(t) = \theta_2$ (diffusion),

$$\mathrm{d}L^j(t) = \mu_2 \mathrm{d}t + \theta_2 \mathrm{d}B^2(t)$$

$$\mathrm{d}\left(\log\left(L^j\right)\right) = \left(\mu_2 - \frac{\theta_2^2}{2}\right)\mathrm{d}t + \theta_2 \mathrm{d}B^2(t)$$

$$\int_0^T \mathrm{d}\left(\log\left(L^j\right)\right) = \left(\mu_2 - \frac{\theta_2^2}{2}\right)\int_0^T \mathrm{d}t + \theta_2 \int_0^T \mathrm{d}B^2(t)$$

$$\log L_T^j - \log L_0^j = \left(\mu_2 - \frac{\theta_2^2}{2}\right)T + \theta_2 B^2(T)$$

$$\frac{L^j(t)}{L_0^j} = \mathrm{e}^{\left(\mu_2 - \frac{\theta_2^2}{2}\right)T + \theta_2 B^2(T)}$$

$$\mathrm{d}L^j(t) = L_0^j \mathrm{e}^{\left(\mu_2 - \frac{\theta_2^2}{2}\right)T + \theta_2 B^2(T)} \tag{13}$$

We observe that Liquidity is continuously positive, assuming the initial L_0^j is positive. Since $\mathrm{d}L^j(t) = \beta L^j(t)\mathrm{d}t + \sigma_2 L^j(t)\mathrm{d}B^2(t)$ implies:

$$L^j(t) = L_0^j + \int_0^t \beta L^j(t)\mathrm{d}t + \int_0^t \sigma_2 L^j(t)\mathrm{d}B^2(t)$$

$$\therefore E\left(\int_0^t \sigma_2 L^j(t)\mathrm{d}B^2(t)\right) = 0$$

$$E\left(L^j(t)\right) = L_0^j + \int_0^t \beta L^j(t)\,\mathrm{d}t$$

$$E\left(L^j(t)\right) = L_0^j \mathrm{e}^{\beta t} \quad \text{for } t \ge 0 \tag{14}$$

3.3. Solvency Stochastic Differential Equation Model

$$\mathrm{d}S^j(t) = \gamma S^j(t)\,\mathrm{d}t + \sigma_3 S^j(t)\,\mathrm{d}B^3(t) \tag{15}$$

Let; $\gamma S^j(t) = \mu_3$ (drift) and $\sigma_3 S^j(t) = \theta_3$ (diffusion),

$$\mathrm{d}S^j(t) = \mu_3 \mathrm{d}t + \theta_3 \mathrm{d}B^3(t)$$

$$\mathrm{d}\left(\log\left(S^j\right)\right) = \left(\mu_3 - \frac{\theta_3^2}{2}\right)\mathrm{d}t + \theta_3 \mathrm{d}B^3(t)$$

$$\int_0^T \mathrm{d}\left(\log\left(S^j\right)\right) = \left(\mu_3 - \frac{\theta_3^2}{2}\right)\int_0^T \mathrm{d}t + \theta_3 \int_0^T \mathrm{d}B^3(t)$$

$$\log S_T^j - \log S_0^j = \left(\mu_3 - \frac{\theta_3^2}{2}\right)T + \theta_3 B^3(T)$$

$$\frac{S^j(t)}{S_0^j} = e^{\left(\mu_3 - \frac{\theta_3^2}{2}\right)T + \theta_3 B^3(T)}$$

$$\mathrm{d}S^j(t) = S_0^j \mathrm{e}^{\left(\mu_3 - \frac{\theta_3^2}{2}\right)T + \theta_3 B^3(T)} \tag{16}$$

We observe that Solvency is each time positive, supposing the initial S_0^j is positive. Since $\mathrm{d}S^j(t) = \gamma S^j(t)\,\mathrm{d}t + \sigma_3 S^j(t)\,\mathrm{d}B^3(t)$ implies,

$$S^j(t) = S_0^j + \int_0^t \gamma S^j(t)\,\mathrm{d}t + \int_0^t \sigma_3 S^j(t)\,\mathrm{d}B^3(t)$$

$$\therefore E\left(\int_0^t \sigma_3 S^j(t)\,\mathrm{d}B^3(t)\right) = 0$$

$$E\left(S^j(t)\right) = S_0^j + \int_0^t \gamma S^j(t)\,\mathrm{d}t$$

$$E\left(S^j(t)\right) = S_0^j \mathrm{e}^{\gamma t} \quad \text{for } t \ge 0 \tag{17}$$

3.4. Macroeconomic Stochastic Differential Equation Model

$$\mathrm{d}M^j(t) = \delta M^j(t)\,\mathrm{d}t + \sigma_4 M^j(t)\,\mathrm{d}B^4(t) \tag{18}$$

Let; $\delta M^j(t) = \mu_4$ (drift) and $\sigma_4 M^j(t) = \theta_4$ (diffusion),

$$\mathrm{d}M^j(t) = \mu_4 \mathrm{d}t + \theta_4 \mathrm{d}B^4(t)$$

$$\mathrm{d}\left(\log\left(M^j\right)\right) = \left(\mu_4 - \frac{\theta_4^2}{2}\right)\mathrm{d}t + \theta_4 \mathrm{d}B^4(t)$$

$$\int_0^T \mathrm{d}\left(\log\left(M^j\right)\right) = \left(\mu_4 - \frac{{\theta_4}^2}{2}\right)\int_0^T \mathrm{d}t + \theta_4 \int_0^T \mathrm{d}B^4(t)$$

$$\log M_T^j - \log M_0^j = \left(\mu_4 - \frac{\theta_4^2}{2}\right)T + \theta_4 B^4(T)$$

$$\frac{M^j(t)}{M_0^j} = \mathrm{e}^{\left(\mu_4 - \frac{\theta_4^2}{2}\right)T + \theta_4 B^4(T)}$$

$$\mathrm{d}M^j(t) = M_0^j \mathrm{e}^{\left(\mu_4 - \frac{\theta_4^2}{2}\right)T + \theta_4 B^4(T)} \tag{19}$$

We observe that Macroeconomic is constantly positive, assuming the initial M_0^j is positive. Since $\mathrm{d}M^j(t) = \delta M^j(t)\mathrm{d}t + \sigma_4 M^j(t)\mathrm{d}B^4(t)$ implies,

$$M^j(t) = M_0^j + \int_0^t \delta M^j(t)\mathrm{d}t + \int_0^t \sigma_4 M^j(t)\mathrm{d}B^4(t)$$

$$\therefore E\left(\int_0^t \sigma_4 M^j(t)\mathrm{d}B^4(t)\right) = 0$$

$$E\left(M^j(t)\right) = M_0^j + \int_0^t \delta M^j(t)\mathrm{d}t$$

$$E\left(M^j(t)\right) = M_0^j \mathrm{e}^{\delta t} \quad \text{for } t \geq 0 \tag{20}$$

4. Model Simulation

In **Figure 1**, show simulated profitability SDE model performance of RCB's over the period to assess the trend of company's ability to earn profits from its sales or operations, balance sheet assets, or shareholders' equity. The series however exhibited cyclic movements which are cycles of rising and falling from initial time point zero (0) to one (1), data values that do not repeat at regular intervals. Such oscillatory movements of time series often have the duration of more than a *Business Cycle*. Cyclic variations of prosperity, recession, depression, and recovery are due to a combination of two or smore economic forces and their interactions.

A company's ability to meet its short-term financial obligations can be assessed by looking at the simulated performance of RCB's liquidity model over time, as shown in **Figure 2**. However, simulated values tend to rise over time, but the magnitude of the seasonal change remains the same, indicating an additive seasonal pattern.

As seen in **Figure 3**, simulated solvency SDE model show how the banking industry has maintained its resiliency and robustness with trend pattern. This indicates that the industry's ability to withstand losses has increased because of the reforms and the recapitalization.

A phenomenon known as the business cycle occurs in **Figure 4**, when long-term trends in macroeconomic growth are superimposed on the levels and rates of change of macroeconomic SDE model. These include the levels and rates of change of major macroeconomic variables, which is expected to go through occasional fluctuations grow on expansions and recessions. The Great Depression of the 1930s served as the impetus for the development of most of the modern macroeconomic theory, and the financial crisis that occurred in 2008 serves as an obvious recent example. According to [17] Ghana's inflation show a decreasing pattern and there is non-seasonal and seasonal pattern in the series. [18] highlighting those changes in money supply, changes in Government of Ghana Treasury bill rates as well as changes in exchange rate as determinants of inflation in the short run.

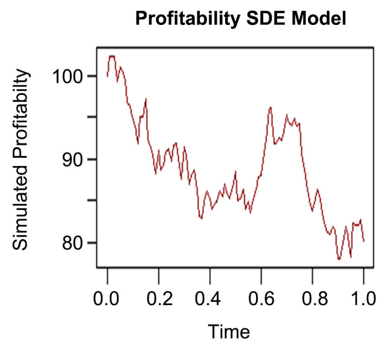

Figure 1. Profitability SDE model.

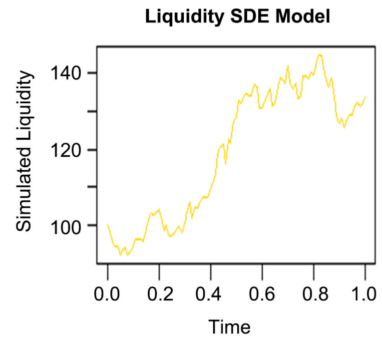

Figure 2. Liquidity SDE model.

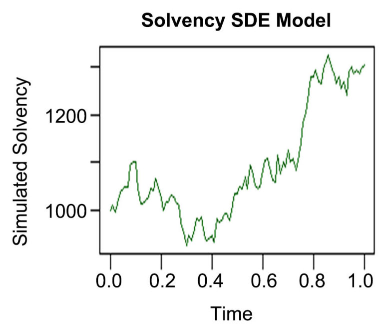

Figure 3. Solvency SDE model.

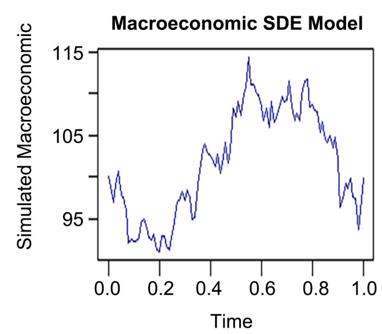

Figure 4. Macroeconomic SDE model.

5. Model Evaluation

The performance of the models is evaluated from the likelihood-ratio tests and information criterions to identify the SDE's resulting in the most significant improvement for measuring RCB's efficiency.

6. Incorporating Jumps in the Stochastic Differential Models

Gaussian jumps, represented as Z_t with time and a Poisson distribution with some intensity λ, are used to simulate the model. The jumps come from $N\left(0,\sigma^2\right)$. There may be unexpected events that have a positive or negative impact on the efficiency of RCBs, hence the most flexible and numerically accessible mathematical framework is SDEs with jumps. These equations can be used to simulate the evolution of financial and other random quantities over time. But wars, central bank actions, and other news can cause the financial instruments to suddenly change. A process with jumps might be used to depict the indicators in this case. Like other Jump processes, Gaussian Jumps follow a Gaussian process in which every finite collection of random variables has a multivariate normal distribution, and every finite linear combination of those variables is normally distributed in discrete and continuous timeframes (Poisson and Markov process) as seen Figures 5-8.

$$\begin{cases} \mathrm{d}P^j(t) = \alpha P^j(t)\mathrm{d}t + \sigma_1 P^j(t)\mathrm{d}B^1(t) + \mathrm{d}Z_t \\ \mathrm{d}L^j(t) = \beta L^j(t)\mathrm{d}t + \sigma_2 L^j(t)\mathrm{d}B^2(t) + \mathrm{d}Z_t \\ \mathrm{d}S^j(t) = \gamma S^j(t)\mathrm{d}t + \sigma_3 S^j(t)\mathrm{d}B^3(t) + \mathrm{d}Z_t \\ \mathrm{d}M^j(t) = \delta M^j(t)\mathrm{d}t + \sigma_4 M^j(t)\mathrm{d}B^4(t) + \mathrm{d}Z_t \end{cases} \tag{24}$$

where $\mathrm{d}Z_t$ is a Gaussian process with

$$Z_t = \sum_{j=1}^{N_t} Y_j, \text{ with } Y_j \sim N\left(0,\sigma^2\right) \tag{25}$$

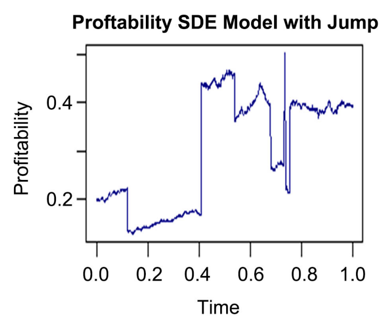

Figure 5. Profitability SDE model.

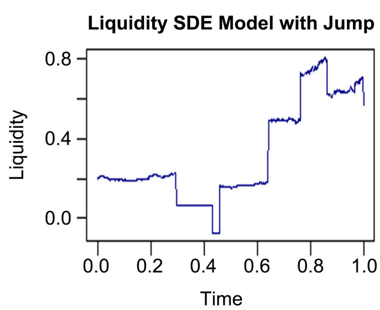

Figure 6. Liquidity SDE model.

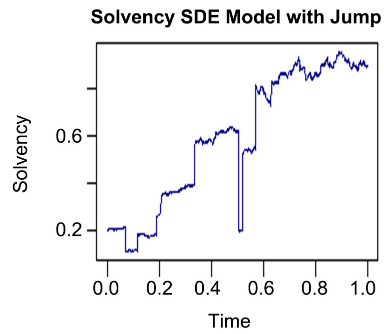

Figure 7. Solvency SDE model.

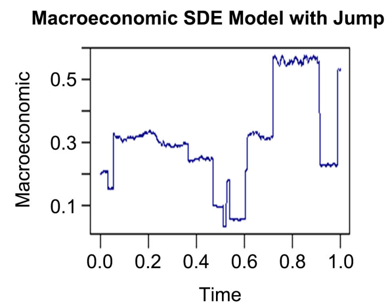

Figure 8. Macroeconomic SDE model.

7. Discussion

In Table 1, Using the log-likelihood value of the SDE models, it is possible to evaluate the model's fit. The better a model matches a dataset, the greater the log-likelihood value. For a particular model, the log-likelihood value can range from zero to infinity. As a result, the Macroeconomic Model's log-likelihood value (−184.1907) is greater than the Solvency Model (−188.3591), the Profitability Model (−199.4506), and the Liquidity Model (−211.3683). To confirm the best-fitting stochastic differential equation models, an information criterion is applied to each model, and the model with the lowest information criterion is the best. Typically, the criteria try to minimize the expected dissimilarity between the chosen model and the true model. In Table 1, the AIC estimates the relative amount of information lost by all the four SDE models, the less information a model loses, the higher the quality of that model. However Macroeconomic model prove to minimize information loss with AIC value (372.3814), BIC (377.5918) and logLik (−184.1907) which offer a better fit.

In Figure 1, the series however exhibited cyclic movements which are cycles of rising and falling from initial time point zero (0) to one (1) for profitability SDE model, as shown in Figure 2. However, simulated values tend to rise over time, but the magnitude of the seasonal change remains the same, indicating an additive seasonal pattern liquidity SDE model. Whiles Figure 3 indicated trend pattern that the industry's ability to withstand losses has increased because of the reforms and the recapitalization. However, a phenomenon known as the business cycle occurs in Figure 4, when long-term trends in macroeconomic growth are covered on the levels and rates of change of macroeconomic SDE model.

Table 1. SDE Model selection.

	Profitability Model	Liquidity Model	Solvency Model	Macroeconomic Model
AIC	402.9012	426.7366	380.7182	372.3814
BIC	408.1115	431.947	385.9286	377.5918
logLik	−199.4506	−211.3683	−188.3591	−184.1907
Std. Error	0.0437	0.0561	0.0425	0.2012

8. Conclusions

Non-financial metrics like loan coverage, productivity, service quality, and management quality have not been included in the current analysis. Based on profitability, liquidity, solvency, and microeconomic variables, the bank's efficiency has been modelled, simulated, and rated. The report focused on these areas because they are of special relevance to investors, prospective investors, employees, management, and the Central Bank. In Ghana, a stochastic differential equation model has been created to measure the efficiency of RCBs.

The model's performance was evaluated using simulated data, and the results demonstrate that it may be utilized to make predictions. Over time, this study advises that these institutions apply real data to simulated stochastic differential models to improve their efficiency. Findings from this study could be useful to develop country rural banking institutions and policymakers, as well as scholars studying banking efficiency.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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