Open Journal of Fluid Dynamics, 2023, 13, 262-273
https://www.scirp.org/journal/ojfd
ISSN Online: 2165-3860
ISSN Print: 2165-3852

# Vortex-Induced Vibrations of Two Cylinders in Tandem Arrangement at Low Reynolds Number

**Joseph Sulemani Mloy**

College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, China
Email: jomloyjj@gmail.com

**How to cite this paper:** Mloy, J.S. (2023) Vortex-Induced Vibrations of Two Cylinders in Tandem Arrangement at Low Reynolds Number. *Open Journal of Fluid Dynamics*, **13**, 262-273.
https://doi.org/10.4236/ojfd.2023.135020

**Received:** November 2, 2023
**Accepted:** December 25, 2023
**Published:** December 28, 2023



## Abstract

The objective of this study is to apply numerical methods to investigate the effects of the spacing on the vortex shedding of two elastically mounted cylinders in tandem arrangement. 2-D computational simulations are carried out at low Reynolds number of 100. The study utilized a commercial software ANSYS FLUENT to carry out the computational simulations. First, a number of test cases, including flows past one and two cylinders with predetermined motions, are simulated to evaluate the solver's accuracy. The vortex shedding and hydrodynamic forces from the current findings and those from literature show good agreement, which supports the accuracy of the current solver. Multiple simulations were the performed for flow around two elastically mounted cylinders in tandem arrangement. The subsequent relative flow fields demonstrated that for a certain range of spacing, vortex shedding was completely eliminated while it remained completely unaffected or partially reduced for other ranges of spacing. This suggests that the spacing between the two cylinders can be utilized as a passive method of suppressing vortex shedding.

## Keywords

Vortex Shedding, Elastically Mounted Cylinders, Tandem, Reynolds Number

## 1. Introduction

Vortex induced vibrations of cylindrical structures have been a focal point of study for many researchers in computational fluid dynamics (CFD). Researchers have studied this area thoroughly and extensively publishing a good amount of literature. Fluid flow around a circulation cylinder is a classical problem in the study of fluid structure interactions as circular cylinders have a simple section firm and a distinctively located separation point. More so, aerodynamic optimization of circular cylinders to manipulate their vortex shedding has a wide range of applications in the world of engineering. Vortex shedding is applicable in submarine pipes, marine risers, offshore platforms, heat exchangers, transmission cables and high-rise buildings. When fluid flow occurs around such bluff structures, there are vortexes in their wake which in turn produce aerodynamic forces. When the frequency of the aerodynamic forces approaches the natural frequency of the bluff structures, vortex induced vibrations tend to occur. Vortex induced vibrations have been identified to be the main cause of fatigue in such structures. It is therefore important to develop functional engineering solutions to suppress vortex induced vibrations in these structures to reduce fatigue and in turn make these structures safer as well as extend their service life.

Williamson C.H.K. 1996 [1] concluded that flow past a stationary cylinder becomes unstable when the Reynolds number exceeds 47 and vortex shedding starts to occur. The cylinder is then subjected to unsteady aerodynamic forces which in turn cause vortex induced vibrations. S.P. Singh and S. Mittal, 2005 [2] conducted a study on flow past and elastically mounted cylinder at low Reynolds number with the focus being on observing modes of vortex induced oscillations. The study investigated the effect of Reynolds number ($50 \leq Re \leq 500$) on VIV which was found to have a significant influence. Lyu Z, Zhang W. W. and Kou J Q, 2023 [3] conducted vortex induced vibrations experiments at sub critical Reynolds numbers that affirmed that vortex shedding occurs at Reynold numbers as low as 23.

According to Saman Rashidi, Javad Abolfazli Esfahani and Masoud Hayatdavoodi, 2016 [4] manipulation of vortex shedding can be identified by two main methods. The first identifies vortex suppression by the type of external source or modification of the circular cylinders geometry to control the vortex shedding. The second method identifies vortex control by the part of the flow, whether it is the boundary or the wake that is modified to control the vortex shedding. There are numerous attempts by researchers to classify vortex control mechanism by researchers but generally they're classified into active control methods (energy consuming) and passive control methods. S. R. Bukka, A. R. Magee and R. K. Jaiman, 2020 [5] conducted a stability analysis of passive suppression devices for the vortex-induced vibration (VIV) in the laminar flow condition. Tao, S., Tang, A., Xin, D., Liu, K., and Zhang, H., 2016 [6] applied the suppression method of vortex-induced vibration that occurs on a circular cylinder fitted with vortex generators, based on the wind tunnel experiment. Chirathalattu, A. T., Santhosh, B., Bose, C., Philip, R., and Balaram, B., 2023 [7] investigated the suppression mechanism of instabilities induced by fluid-structure interactions (FSI) using passive vibration absorption devices, such as nonlinear energy sink (NES).

Y. Wang, C. Shu, C. J. Teo and J. Wu, 2015 [8] studied the vortex induced vibrations of an elastically mounted cylinder at Re 100 and monitored the displacement at varying natural frequencies varying from 4 to 8. This paper will apply similar physical parameters to two elastically mounted cylinders in tandem arrangement. The objective is to investigate the influence of the positioning of the downstream cylinder on the vortex shedding of the upstream cylinder as well as its own vortex shedding. To achieve the objective, multiple simulations are performed with the rear cylinder being placed at different lengths from the upstream cylinder in each simulation. The obtained results are compared to observe key differences in the vortex shedding patterns as well as hydrodynamic forces. Results can also be compared to those of a single elastically mounted cylinder to give the study additional context on the influence having the two elastically mounted cylinders in tandem arrangement.

# 2. Numerical Simulation Model

## 2.1. Governing Equations of Fluid Mechanics

### 2.1.1. Conservation of Mass Equation (Continuity Equation)

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

where $V$ is velocity and $\nabla$ the gradient operator.

### 2.1.2. Conservation of Momentum Equation

$$\rho \frac{D\boldsymbol{V}}{Dt} = -\nabla p + \rho \boldsymbol{g} + \mu \nabla^2 \boldsymbol{V} \tag{2}$$

where; $\rho \frac{D\boldsymbol{V}}{Dt}$ is Momentum convection, $\nabla p$ is Surface force, $\rho \boldsymbol{g}$ is Mass force and $\mu \nabla^2 \boldsymbol{V}$ is Viscous force.

## 2.2. Governing Equations of Structural Dynamics (Vibration of the Cylinder)

The circular cylinder of diameter D, is elastically mounted in a uniform free stream with velocity $U_\infty$. As a result of the unsteady hydrodynamic forces, the cylinder is subjected to two DOF vibrations in the OXY plane. The in-line and traverse oscillations are governed by the following equations.

$$\ddot{X} + 4\pi F_n \zeta \dot{X} + (2\pi F_n)^2 X = \frac{2C_d}{\pi m^*} \tag{3}$$

$$\ddot{Y} + 4\pi F_n \zeta \dot{Y} + (2\pi F_n)^2 Y = \frac{2C_d}{\pi m^*} \tag{4}$$

$\ddot{X}$, $\dot{X}$ and $X$ denote the in-line acceleration, velocity and displacement respectively; while $\ddot{Y}$, $\dot{Y}$ and $Y$ represent the cross-flow acceleration, velocity and displacement. $\zeta$ denotes the structural damping, $F_n$ denotes the reduced natural frequency which is described by the equation $F_n = f_n D / U_\infty$. $m^*$ represents the cylinder's normalized mass which is given by the equation $m^* = 4m / (\pi \rho D^2)$. The reduced velocity $U^*$ is defined by $U^* = U_\infty / (f_n D) = 1/F_n$. In order to maximize the amplitude of the oscillations, the damping ratio is taken as zero. The vibrations governed by these equations are dynamically modeled in ANSYS FLUENT using a user defined function that discretizes the equations using the 4-stage-Runge-Kutta scheme.

### 2.3. Geometrical Model and Boundary Equations

A rectangular domain is used in the set up with the inlet being 20D away from the origin (center of the upstream cylinder), the outlet is 30D away from the origin and the upper and lower symmetry boundaries are 20D away from the origin as displayed Figure 1. The computational domain is discretized by an unstructured mesh that is significantly refined around the circular cylinders Figure 2.

The mesh is developed using ICEM first as a structured mesh then converted to a triangular mesh to facilitate utilization of the remeshing function in ANSYS Fluent during the simulation. The free stream flows from left to right with the left side being the velocity inlet the upper and lower sides are set asymmetry boundaries and the right side is set as the velocity outlet with a relative pressure of zero.

### 2.4. Numerical Procedure

The flow is characterized by the equation;

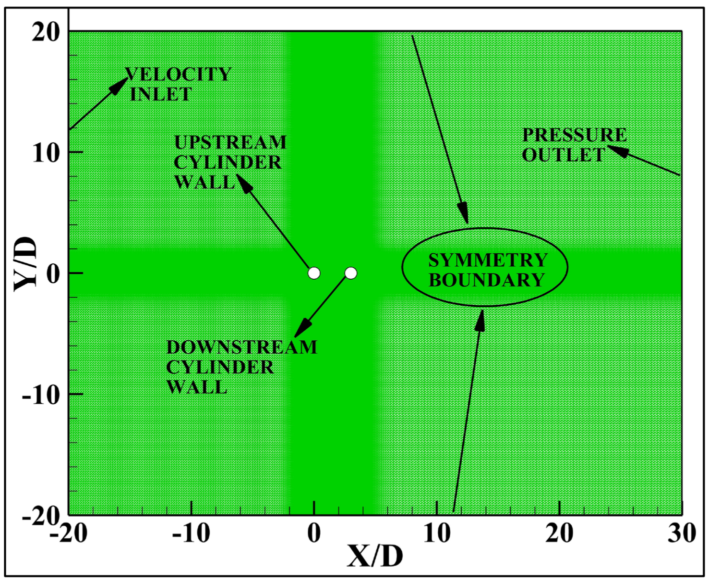


Figure 1. Computational domain.

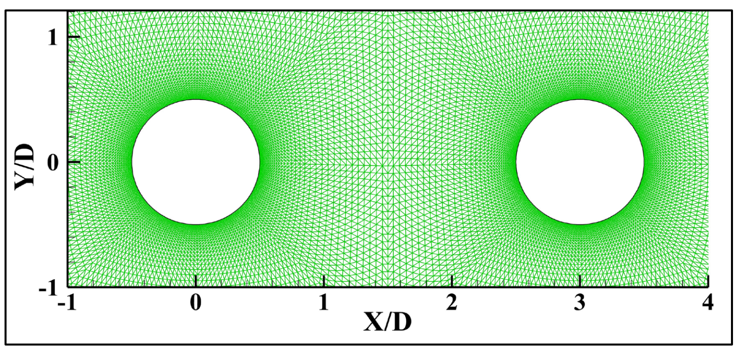


Figure 2. Geometry of the mesh around the cylinder wall.

$$Re = \frac{\rho u_\infty D}{\mu} \tag{5}$$

For this case study we will set $\rho = 1.0$, $u_\infty = 0.1$ and $D = 1.0$ Free stream properties are applied on the inlet and natural boundary conditions are applied on the upper and lower symmetry boundary and the pressure outlet. The viscous model is set as Laminar. The SIMPLE algorithm is employed in coupling of pressure and velocity fields. The least square based scheme is used to discretize the gradient and the Standard and second order upwind solution schemes are used to compute the pressure and momentum respectively.

To replicate the motion of the elastically mounted cylinder ANSYS Fluent User-Defined-Functions (UDF), which are C and C++ based codes for server interaction are used. This paper utilized the DEFINE_CG_MOTION (DMG) macro to implement motion of the boundary mesh. The UDF used to model the traverse and inline vibration uses and a hook function DEFINE_EXECUTE_AT_END.

## 2.5. Numerical Procedure

In order to ensure that the simulation yielded accurate results a grid independence study is conducted on the mesh to ensure its quality. The mesh convergence test was conducted for 7 cases of the elastically mounted with a reduced velocity $U^* = 5$. Mesh properties in the mesh are altered in the 7 different cases notably the first layer height of the inflation of the mesh surrounding the cylinder wall. As the results in **Figure 3** shows doubling the first layer height and increasing the total number of cells significantly seems to have negligible influence on the results of the simulations from the fourth mesh. Mesh **M5** is selected to perform the rest of the simulations as it is deemed sufficiently accurate.

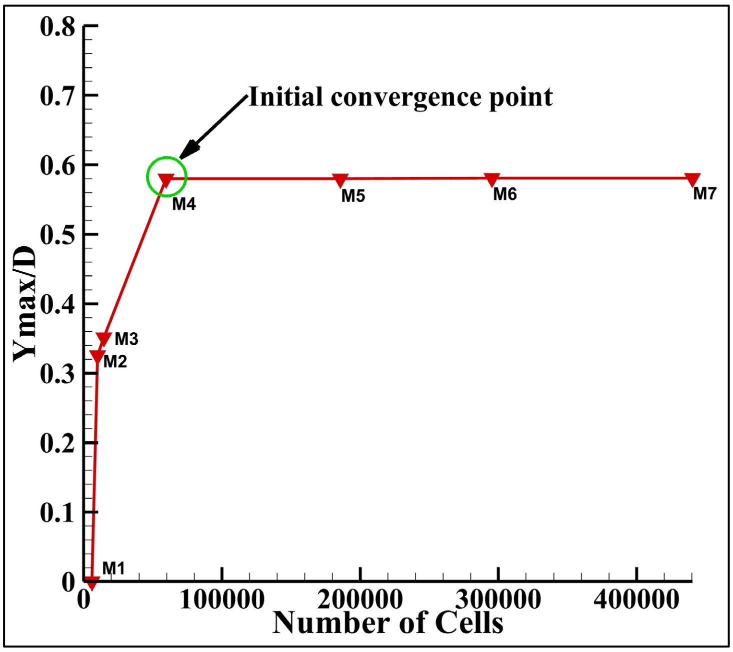


**Figure 3.** Mesh convergence results.

# 3. Results and Discussions

## 3.1. Preamble

In the results presented in this paper certain parameters have been specified. A nondimensional time step $T$ to represent time. $T$ is obtained by the formula;

$$T = \frac{U_{\infty}\Delta_t}{D} \tag{6}$$

## 3.2. Validation

In order to validate the current solver a simulation of a standard case of flow past a stationary cylinder unsteady ($Re$ = 100) case. Unsteady periodic flow occurs. A quantitative analysis and study of the lift and drag coefficients and the Strouhal number is made in **Table 1**. Again, the data is in line with available literature.

While it is not the focus of this study, the simulation of the two-dimensional flow past a fixed cylinder at different Reynolds numbers is included here as a basis of comparison with other numerical and experimental results for a complete validation of the current solver. After going a step further and conducting more simulations at $Re$ = 110, 126, 160, 185 and 200, a quick comparison of the Strouhal numbers was made in accordance with E. Guilmineau and P. Queutey, 2001 [12]. The data obtained had excellent agreements with prior studies as shown in **Figure 4**.

## 3.3. Flow over an Elastically Mounted Cylinder

The reduced velocity ($U^*$) is altered from 4 to 8 in order to examine the vortex shedding. The simulation data of the maximum $Y$ displacement ($Y_{max}$) of an elastically mounted cylinder at different reduced velocities is plotted in comparison to data from Y. Wang, C. Shu, C. J. Teo, and J. Wu, 2015 [8] obtained from an immersed boundary-lattice Boltzmann flux solver and, S.P. Singh and S. Mittal, 2005 [2] obtained by finite element method. Good agreements are achieved

**Table 1.** Comparison of dynamic parameters for unsteady flow past a stationary cylinder.

| References | $C_L$ | $C_D$ | $S_T$ |
|---|---|---|---|
| (Braza, Chassaing, Minh, H.H., and M., P., 1986) [9] | ±0.30 | 1.28 ± 0.02 | 0.16 |
| (Benson, M.G., Bellamy-Knights, P.G, Gerrard, and Gladwell, 1989) [10] | ±0.28 | 1.325 ± 0.008 | 0.164 |
| (Ding, H., Shu, C., Yeo, K.S., and Xu, D, 2004) [11] | ±0.38 | 1.46 ± 0.01 | 0.17 |
| (Y. Wang, C. Shu, C.J. Teo and J.Wu, 2015) [8] | ±0.37 | 1.334 ± 0.012 | 0.163 |
| Present | ±0.325 | 1.351 ± 0.009 | 0.1639 |

which verify the process to be accurate up to this point **Figure 5**. The highest displacement for all the cases is considered to be when $U^* = 5$. From the point where the reduced velocity equals 5 to where it equals to 8 the maximum displacement reduces steadily.

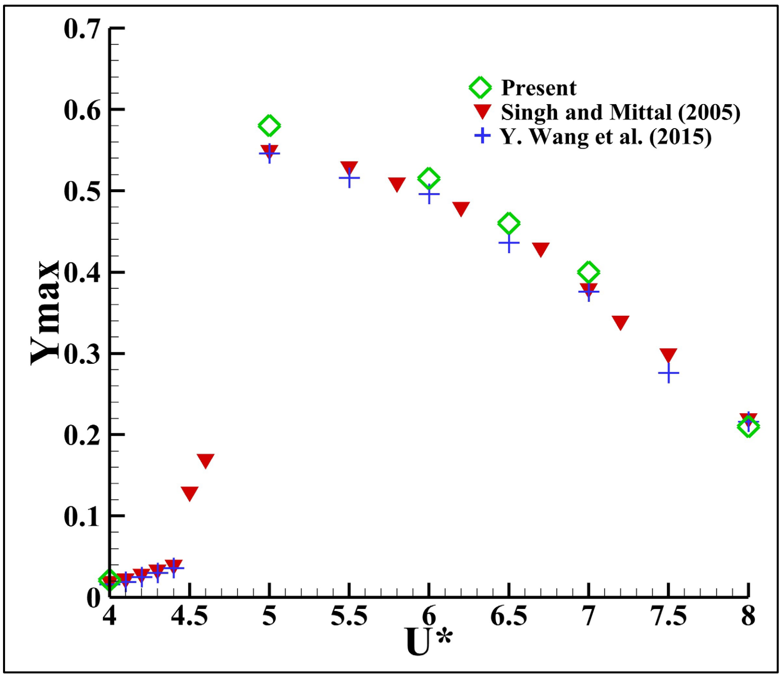


**Figure 4.** Comparison of Strouhal number versus Reynolds number for transient flow over a cylinder.

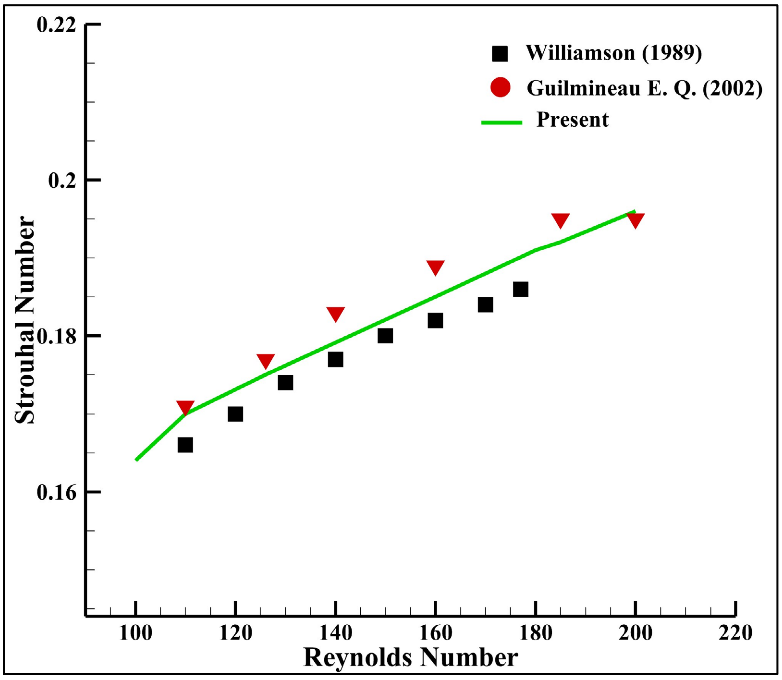


**Figure 5.** Comparison of the maximum crossflow displacement to reduced velocity.

For all the simulations taken into consideration in this study, the peak oscillation amplitude $Y_{max}$ is $0.58D$ at the reduced velocity $U^* = 5$. drag coefficients for different reduced velocities. In each case the lift and drag forces settle to a regular sinusoidal function after the onset of wake instability leads to vortex shedding.

## 4. Flow over Two Elastically Mounted Cylinders in Tandem Arrangement

### 4.1. Results

Figure 6 shows the streamline contours of the flow over the two cylinders at different spacing. Figure 7 shows the instantaneous vorticity contours and it can be observed that the so-called 2S mode, in which two single vortices are released from the cylinder's sides once every period. This phenomenon was also presented by Mittal, 2005 [2].

#### 4.1.1. *L/D* = 1.5

By looking at the streamlines at this spacing it can be observed that the downstream cylinder is within the recirculation bubble of the upstream cylinder. In Table 2 it can be observed that despite having negligible displacement the cylinders are still subjected to relatively significant lift and drag forces. At this spacing the effect of vortex shedding is significantly reduced but still observable. The vorticity contours also suggest vortex shedding is occurring.

#### 4.1.2. *L/D* = 3.0

At this spacing the downstream cylinder is observed to be perfectly immersed into the recirculation bubble of the upstream cylinder. The displacement and lift forces of both cylinders are effectively cut down to zero. Only drag forces are observed as per Table 2.

When you look at the vortex shedding it can be observed right from the onset at the upstream vortex shedding has been trimmed out completely.

**Table 2.** Comparison of dynamic parameters for flow past two elastically mounted circular cylinders in tandem arrangement.

| $L/D$ | $C_l$ | | $C_d$ | | $Y_{max}$ | |
|---|---|---|---|---|---|---|
| | Upstream cylinder | Downstream cylinder | Upstream cylinder | Downstream cylinder | Upstream cylinder | Downstream cylinder |
| 1.5 | ±0.1313 | ±0.3912 | 1.18 | −0.0799 | 0 | 0 |
| 3.0 | 0 | 0 | 1.1246 | −0.0507 | 0 | 0 |
| 4.5 | 0 | 0 | 1.0922 | 0.2010 | 0 | 0 |
| 6.0 | ±1.601 | ±1.8959 | 1.4128 ± 0.2306 | 0.7045 ± 0.5698 | 0.3743 | 0.3011 |
| 10.0 | ±1.4822 | ±1.9944 | 1.4245 ± 0.2371 | 0.8791 ± 0.5430 | 0.3405 | 0.4071 |

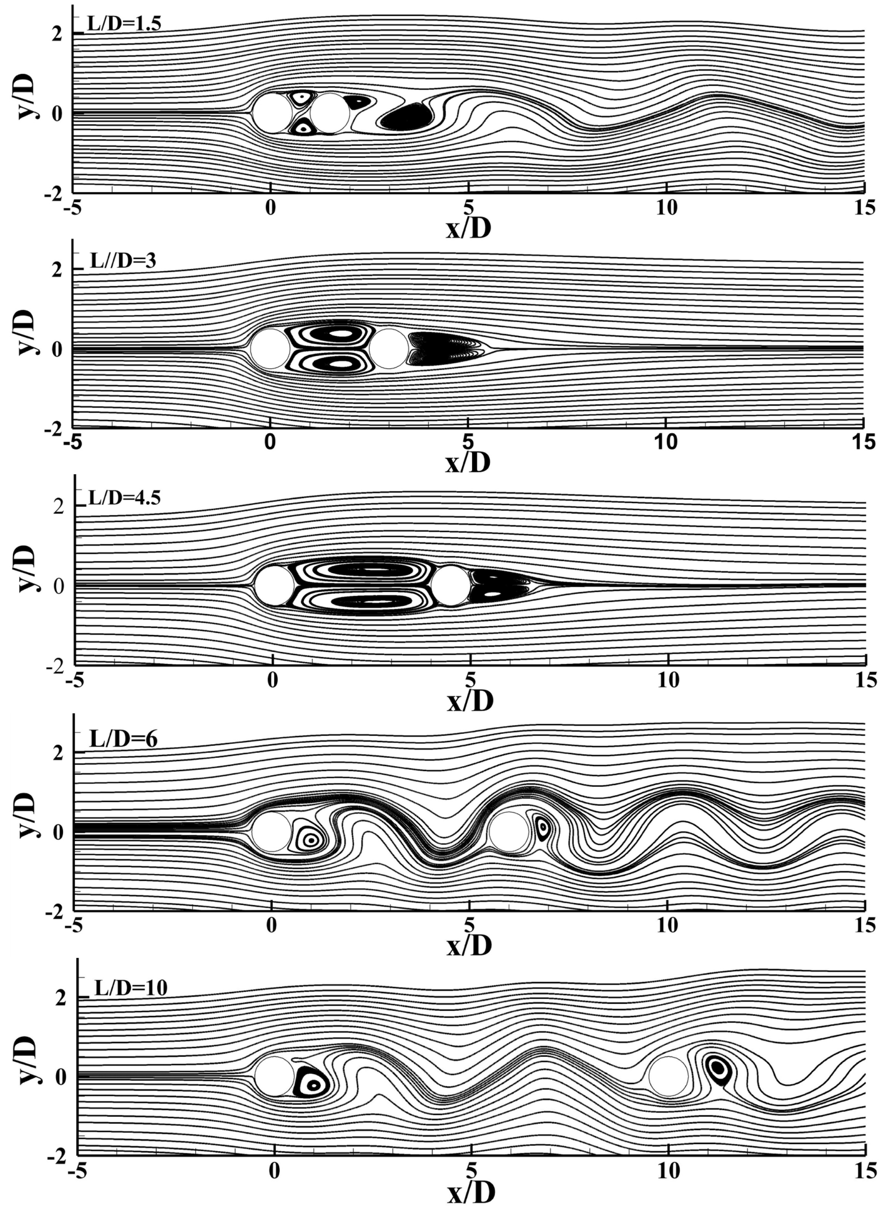


Figure 6. Streamline contours for flow past two elastically mounted cylinders in tandem arrangement.

#### 4.1.3. *L/D* = 4.5

The observations at this spacing are pretty much similar to *L/D* = 3.0. By observing the streamlines, the downstream cylinder is still in the zone of the recirculation bubble of the upstream cylinder. The vortex shedding is canceled out for both the upstream and downstream cylinders. There are no lift forces or displacement for both cylinders as well. Only drag is observed on the cylinders. At this spacing the vortex shedding has also been controlled effectively and cannot be observed completely.

#### 4.1.4. *L/D* = 6.0

Y. Wang *et al.* [8] observed that at the reduced velocity $U^* = 5$ there are two

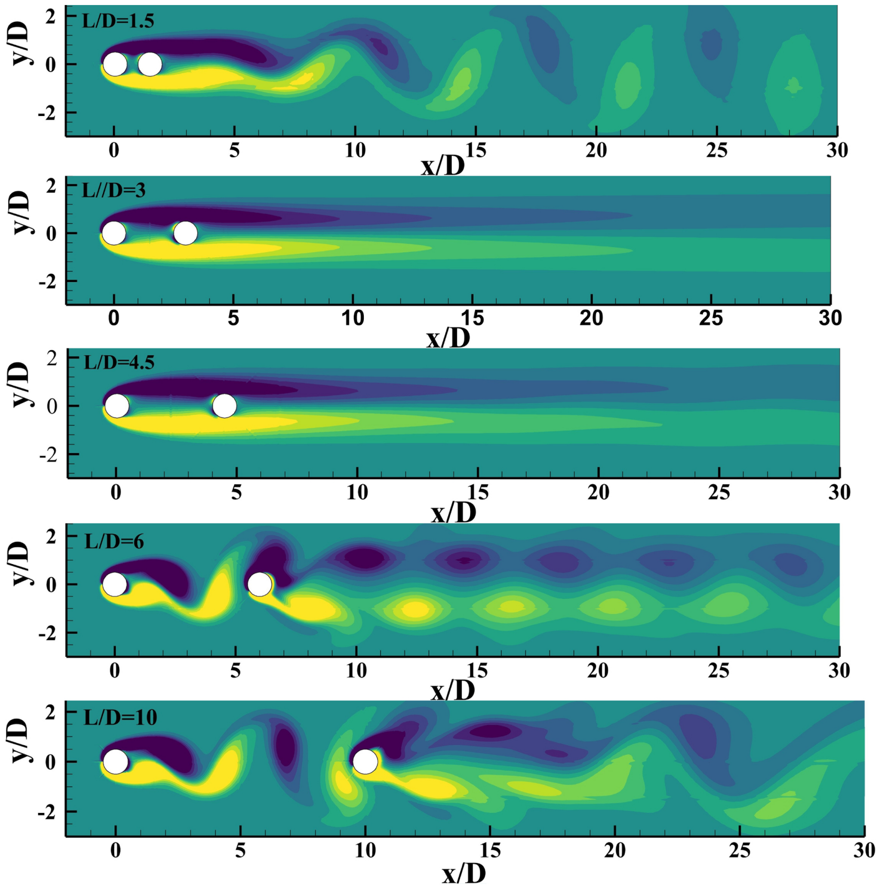


**Figure 7.** Instantaneous vorticity contours for flow past two elastically mounted cylinders in tandem arrangement.

arrays of vortices moving downstream without converging into a single vortex street. In the current series of simulations this phenomenon is only observed when the spacing between the two cylinders $L/D = 6$. Two vortex form behind the downstream cylinder and flow downstream without converging This suggests that in this current set up the spacing has minimal effects on the vortex shedding.

#### 4.1.5. $L/D$ = 10.0

At this spacing the gap between the upstream and downstream cylinders has no effect on the vortex shedding of both cylinders. By observing the vortex sheet, it can be concluded that there is a decent amount of vortex shedding on both cylinders individually. When the forces are studied its can be seen that both cylinders have Lift forces and vertical displacements similar to those of a single elastically mounted cylinder.

## 5. Conclusions

The wake characteristics of elastically mounted circular cylinders in tandem arrangement haven't been explored or documented in the literature. This study mainly focused on these characteristics and if the spacing between the two tandem cylinders could be manipulated to inhibit vortex shedding. For the range $L/D$ = 1.5, 3 and 4.5; no vertical displacement was observed for both the upstream and downstream cylinders. For $L/D$ = 6 and $L/D$ = 10 displacement was observed as can be seen in **Table 2**. From **Table 2** it is also important to note that taking the drag coefficients of $L/D$ = 10 as the reference values; the drag coefficients of the upstream cylinders for $L/D$ = 6, $L/D$ = 4.5, $L/D$ = 3.0 and $L/D$ = 1.5 are lower by 7%, 23%, 21% and 16% respectively (The reduction is in comparison to the drag coefficients of $L/D$ = 10.0). Drag coefficients of the downstream cylinders for $L/D$ = 6, $L/D$ = 4.5, $L/D$ = 3.0 and $L/D$ = 1.5 are lower by 20%, 77%, 105% and 109% respectively. This shows that there is a significant reduction in the drag when $L/D$ is reduced to be within the range of the recirculation bubble. The vorticity contours in **Figure 7** show that; there is no vortex shedding for $L/D$ = 3.0 and $L/D$ = 4.5, light vortex shedding for $L/D$ = 1.5 and observable vortex shedding for $L/D$ = 6.0 and $L/D$ = 10.

From the results obtained from this study it's clearly evident that for a case of two circular cylinders in tandem arrangement the spacing between them can be exploited as mechanism of passive control of vortex induced vibrations. In instances where the downstream cylinder was placed within the recirculation bubble of the upstream cylinder, the vortex shedding of both cylinders was effectively reduced if not completely eliminated. When the downstream cylinder is placed out of range of the recirculation bubble, the vortex shedding is still observed. This suggests that the downstream cylinder possesses the capability to inhibit vortex shedding if strategically positioned. I think that this concept has great potential and can actually be applied in real life Fluid-structure interactions (FSI) where there are two identical cylinders in tandem arrangement.

## Acknowledgements

We thank the editor and the anonymous referees for their comments and suggestions to enrich the manuscript.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

## References

[1] Williamson, C.H.K. (1996) Vortex Dynamics in the Cylinder Wake. *Annual Review of Fluid Mechanics*, **28**, 477-539. https://doi.org/10.1146/annurev.fl.28.010196.002401

[2] Singh, S.P. and Mittal, S. (2005) Vortex-Induced Oscillations at Low Reynolds Numbers: Hysteresis and Vortex-Shedding Modes. *Journal of Fluids and Structures*, **20**, 1085-1104. https://doi.org/10.1016/j.jfluidstructs.2005.05.011

[3] Lyu, Z., Zhang, W.W. and Kou, J.Q. (2023) Experiments on Vortex-Induced Vibration of a Cylinder at Subcritical Reynolds Numbers. *Acta Aerodynamica Sinica*, **41**, 101-107.

[4] Rashidi, S., Esfahani, J.A. and Hayatdavoodi, M. (2016) Vortex Shedding Suppression and Wake Control: A Review. *Ocean Engineering*, **126**, 57-80. https://doi.org/10.1016/j.oceaneng.2016.08.031

[5] Bukka, S.R., Magee, A.R. and Jaiman, R.K. (2020) Stability Analysis of Passive Suppression for Vortex-Induced Vibration. *Journal of Fluid Mechanics*, **886**, A12. https://doi.org/10.1017/jfm.2019.1026

[6] Tao, S., Tang, A., Xin, D., Liu, K. and Zhang, H. (2016) Vortex-Induced Vibration Suppression of a Circular Cylinder with Vortex Generators. *Shock and Vibration*, **2016**, Article ID 5298687. https://doi.org/10.1155/2016/5298687

[7] Chirathalattu, A.T., Santhosh, B., Bose, C., Philip, R. and Balaram, B. (2023) Passive Suppression of Vortex-Induced Vibrations Using a Nonlinear Energy Sink—Numerical and Analytical Perspective. *Mechanical Systems and Signal Processing*, **182**, 109556. https://doi.org/10.1016/j.ymssp.2022.109556

[8] Wang, Y., Shu, C., Teo, C.J. and Wu, J. (2015) An Immersed Boundary-Lattice Boltzmann Flux Solver and Its Applications to Fluid-Structure Interaction Problems. *Journal of Fluids and Structures*, **54**, 440-465. https://doi.org/10.1016/j.jfluidstructs.2014.12.003

[9] Braza, M., Chassaing, P. and Ha Minh, H. (1986) Numerical Study and Physical Analysis of the Pressure and Velocity Fields in the near Wake of a Circular Cylinder. *Journal of Fluid Mechanics*, **165**, 79-130. https://doi.org/10.1017/S0022112086003014

[10] Benson, M.G., Bellamy-Knights, P.G., Gerrard, J.H. and Gladwell, I. (1989) A Viscous Splitting Algorithm Applied to Low Reynolds Number Flows Round a Circular Cylinder. *Journal of Fluids and Structures*, **3**, 439-479. https://doi.org/10.1016/S0889-9746(89)80026-X

[11] Ding, H., Shu, C., Yeo, K.S. and Xu, D. (2004) Simulation of Incompressible Viscous Flows Past a Circular Cylinder by Hybrid FD Scheme and Meshless Least Square Based Finite Difference Method. *Computer Methods in Applied Mechanics and Engineering*, **193**, 727-744. https://doi.org/10.1016/j.cma.2003.11.002

[12] Guilmineau, E. and Queutey, P. (2001) A Numerical Simulation of Vortex Shedding from an Oscillating Circular Cylinder. *Journal of Fluids and Structures*, **16**, 773-794. https://doi.org/10.1006/jfls.2002.0449