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A Study of Two Dimensional Unsteady MHD Free Convection Flow over a Vertical Plate

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Abstract

In this paper, unsteady free convection heat transfer flow over a vertical plate in the presence of a magnetic field is discussed in detail. The dimensionless partial differential equations of continuity, momentum along energy are analyzed with suitable transformations. For numerical calculation, an implicit finite difference method is applied to solve a set of nonlinear dimensionless partial differential equations. Dimensionless velocity and temperature profile are also investigated due to the effects of assumed parameters in the concerned problem. An explicit finite difference technique is used to compute velocity and temperature profiles. The stability conditions are also examined.

Keywords

Free Convection, Heat Transfer Flow, Magnetic Field, Explicit Finite Difference Method

1. Introduction

Unsteady MHD free convection flow is created a significant effect on heat transfer over a vertical plate. It has great interest on both theoretical and practical points of view because of its enormous applications in several engineering and geophysical field. In 1952, Ostrach [1] obtained a similar solution along with a vertical plate for free convection. In 1958, Stewartson [2] observed the free convection from a horizontal plate. In 1961, Sparrow and Cuss [3] discussed the effect of the magnetic field on free convection heat transfer on the isothermal vertical plate. In 1969, Roten and Claassen [4] studied natural convection above the unconfined horizontal plate. In 1972, Soundalgekar [5] studied the viscous dissipation effects on unsteady free convection flow past an infinite, vertical porous plate with constant suction. In 1977, Soundalgekar and Wavre [6] investigated unsteady free convection flow past an infinite vertical plate with variable suction and **mass transfer**. In 1984, Hussain and Begum [7] observed the outcome of mass transfer and free convection past a vertical plate. In 1984, Afzal and Hussain [8] discoursed the mixed convection over a horizontal plate. In 1985, Tokis [9] considered a class of exact solutions of the unsteady magnetohydrodynamic free convection flows. In 1990, Huang and Chen [10] presented local similarity solutions at free convective heat transfer from a vertical plate to non-Newtonian power-layer fluids. In 1992, Hossain [11] observed the viscous and joule heating effects on MHD free convection flow with variable plate temperature. In 1992, Sattar and Alam [12] studied unsteady hydromagnetic free convection flow with hall current and mass transfer along with an accelerated porous plate. Here temperature and concentration profiles are time-dependent. In 1997, Crepeau and Clarksean [13] attained a similar solution to natural convection with internal heat generation, which decays exponentially. In 1999, Cheng [14] analyzed the consequence of a magnetic field on heat and mass transfer with natural convection along a vertical surface in porous media an integral approach. In 1999, Pop and Postelnicu [15] perceived similar solutions of free convection boundary layers over vertical and horizontal surface porous media with internal heat generation. In 2009, Cao and Baker [16] presented the slip effects on mixed convection flow and heat transfer from a vertical plate. In 2005, Chamaka and Al-Mudhaf [17] studied unsteady heat and mass transfer from rotating vertical cone with magnetic field and heat generation or absorption effect which develop our studies nearly to this field. In 2006, Alam *et al.* [18] investigated the numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. In 2007, Aydin and Kaya [19] scrutinized the mixed convection flow of a viscous dissipating fluid about a vertical plate. In 2008, Alam *et al.* [20] explored the effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation. In 2011, Bhattacharyya [21] [22] *et al.* discussed the MHD boundary layer ship flow over a flat plate and porous plate embedded in a porous medium. In 2011, Singh and Kumar [23] calculated the fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous considered plate in the slip-flow regime. In 2013, Sarkar [24] studied the Hall effects on unsteady MHD free convective flow past an accelerated moving vertical plate with viscous and Joule dissipations. In 2013, Narahari and Debnath [25] investigated the unsteady MHD-free convection flow past an accelerated vertical plate with constant heat flux and heat source. In 2013, Alam *et al.* [26] premeditated the effect of heat and mass transfer in MHD free convection flow over an inclined plate with hall current. In 2017, Pandya *et al.* [27] observed the combined effects of Soret-Dufour, radiation, and chemical reaction on unsteady MHD flow of dusty fluid over inclined porous plate embedded in a porous medium.

The principal objective of this paper is to study the MHD free convection flow over a vertical plate. Because in recent years, free convection flow in the presence of the magnetic is attracted the attention of several researchers due to various applications in science and technology. MHD is used in a wide range of applications in engineering science, for example, MHD power generators, MHD bearings, MHD pumps etc. Also, it is used in solar physics, geophysics, meteorology, aeronautical plasma flows, electronics and chemical engineering. Therefore, it is necessary to investigate, in detail, the distributions of velocity and temperature for the flow of a viscous incompressible electrically conducting fluid along with a vertical plate across the boundary layer in the presence of a magnetic field. In this regard, the explicit finite difference techniques are used for solving non-linear partial differential equations.

2. Mathematical Model and Governing Equations

Let us consider an electrically conducting incompressible viscous fluid of unsteady MHD free convection flow along with the vertical plate. In the Cartesian coordinate system, X-axis is along with the plate in the direction of the flow, and the Y-axis is normal to it. T_w and T_∞ is the temperature of the plate and outside of the plate respectively. A uniform magnetic field $\boldsymbol{B} = (0, B_0, 0)$ is enacted normal to the plate, and the magnetic field is anticipated to be negligible while B_0 is constant which are illustrates in Figure 1.

The equations for the flow and temperature unsteady heat and mass transfer over a vertical plate in the presence of a magnetic field are given below with boundary conditions.

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

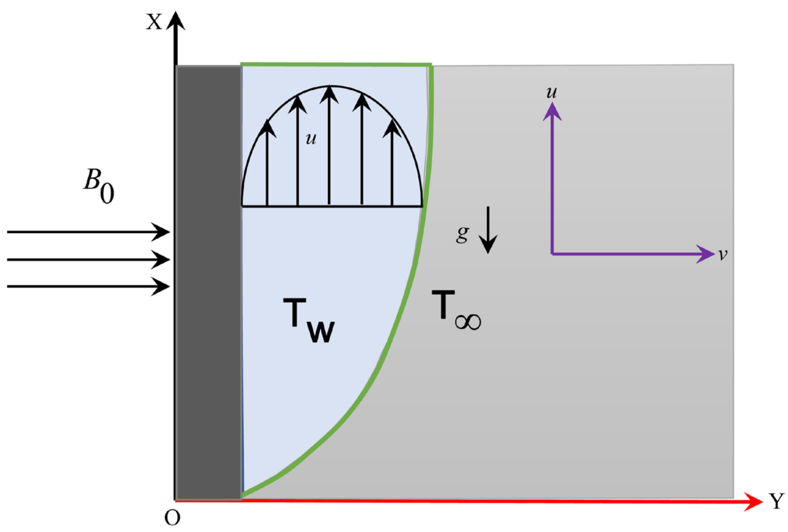


Figure 1. Physical configuration and coordinates system.

Momentum equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\cdot\frac{\partial^2 T}{\partial y^2}+\frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The appropriate boundary condition for velocity and temperature are given by

$$\begin{aligned}&u=U_0, v=0,\ T=T_w,\ \text{at}\ y=0\\&u\to 0,\ T\to T_\infty,\ \text{as}\ y\to\infty\end{aligned} \tag{4}$$

where v is the kinematic viscosity, g is the acceleration due to gravity, β is the co-efficient of volumetric expansion, T is the temperature of the fluid inside the thermal boundary layer, T_∞ is the temperature in the free stream, σ is the electric conductivity, B_0 is the magnetic field, ρ is the fluid density, κ is the kinematic viscosity, C_p is the specific heat with constant pressure, U_0 is the uniform velocity of the fluid, T_w is the temperature of the plate and remaining symbols have their usual meaning.

3. Mathematical Formulation

Applying the following usual transformations, the system of partial differential equations with boundary conditions transformed into a non-dimensional system.

$$u=U_0U, v=VU_0, Y=\frac{yU_0}{v}, X=\frac{xU_0}{v}, \eta=\frac{tU_0^2}{v}, T=T_\infty+\left(T_w-T_\infty\right)\overline{T}.$$

To apply the above transformation in Equations (1)-(3), and with corresponding boundary conditions (4), by simplification obtain the following non-linear differential equations in terms of dimensionless variables such as

Continuity equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

Momentum equation

$$\frac{\partial U}{\partial \eta}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+Gr\overline{T}-MU \tag{6}$$

Energy equation

$$\frac{\partial \overline{T}}{\partial \eta}+U\frac{\partial \overline{T}}{\partial X}+V\frac{\partial \overline{T}}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 \overline{T}}{\partial Y^2}+Ec\left(\frac{\partial U}{\partial Y}\right)^2 \tag{7}$$

with boundary conditions

$$\begin{aligned}&U=1,\ V=0,\ \overline{T}=1,\ \text{at}\ Y=0\\&U=0,\ \overline{T}=0,\ \text{as}\ Y\to\infty\end{aligned} \tag{8}$$

where,

Magnetic parameter, $M = \dfrac{\sigma \nu B_0^2}{\rho U_0^2}$

Grashof number, $Gr = \dfrac{\nu g \beta \left(T_w - T_\infty\right)}{U_0^3}$

Prandtl number, $Pr = \dfrac{\nu}{\alpha}$

Eckert number, $Ec = \dfrac{U_0^2}{C_p \left(T_w - T_\infty\right)}$

4. Numerical Solution

A set of nonlinear partial differential dimensionless governing equations is solved numerically with related boundary conditions along with an explicit finite difference method which is tentatively stable. The region of the flow is divided into a grid or mesh of lines parallel to X- and Y-axes, where X-axis indicates the plate and Y-axis is normal to the flow of the fluid which is demonstrates in Figure 2. This study measures the height of the plate $X_{\max}$ (=100), *i.e.*, X varies from 0 to 100 and assumed $Y_{\max}$ (=25) as taken to $Y \to \infty$, *i.e.*, Y varies from 0 to 25.

$m = 250$ and $n = 250$ are taken to grid spacing in the X and Y directions correspondingly and as follows $\Delta x = 0.4\left(0 \le x \le 100\right)$ and $\Delta Y = 0.1\left(0 \le Y \le 25\right)$

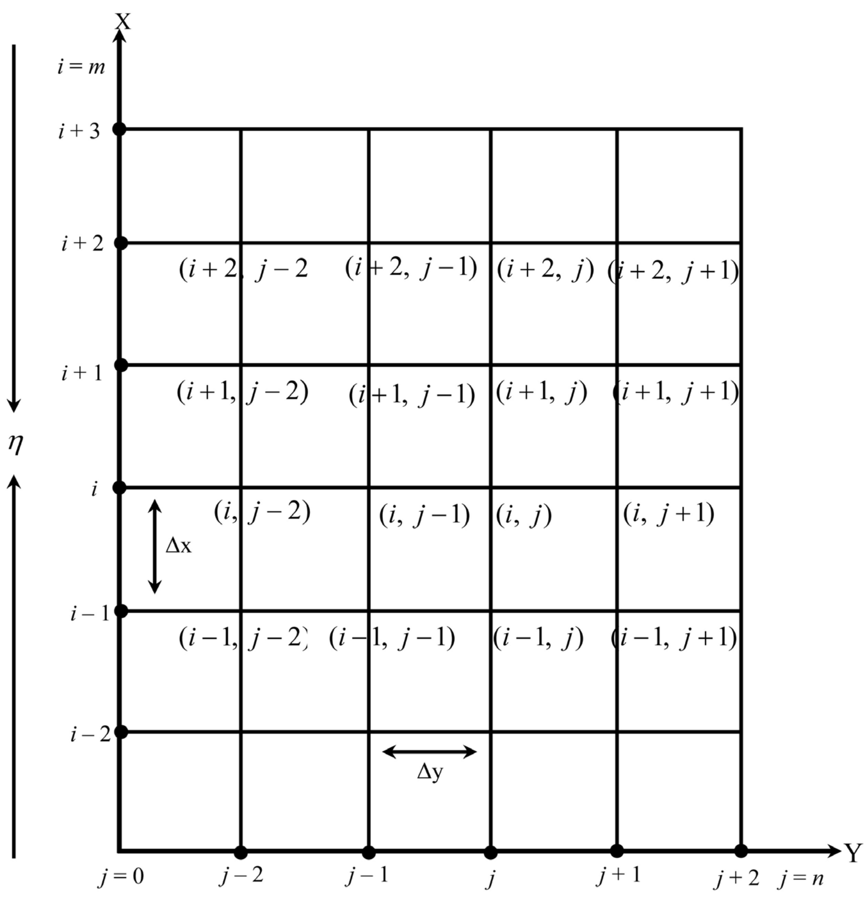


Figure 2. Explicit finite difference system.

with the smaller time step $\Delta\eta = 0.005$. Let $U', \bar{T}'$ denote the values of $U, \bar{T}$ at the end of a time-step respectively.

Applying the explicit finite difference method into the partial Equations (5)-(7) with boundary conditions (8) we get,

$$(5) \Rightarrow \frac{U_{i,j} - U_{i,j-1}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \tag{9}$$

$$\begin{aligned}
&(6) \Rightarrow \frac{U'_{i,j} - U_{i,j}}{\Delta\eta} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \\
&= \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\bar{T}_{i,j} - MU_{i,j}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow U'_{i,j} = U_{i,j} + \Delta\eta\Bigg(& -U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} - V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} \\
& + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\bar{T}_{i,j} - MU_{i,j}\Bigg)
\end{aligned} \tag{10}$$

$$\begin{aligned}
&(7) \Rightarrow \frac{\bar{T}'_{i,j} - \bar{T}_{i,j}}{\Delta\eta} + U_{i,j}\frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta Y} \\
&= \frac{1}{Pr}\frac{\bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \bar{T}'_{i,j} = \bar{T}_{i,j} + \Delta\eta\Bigg(& -U_{I,J}\frac{\bar{T}_{i,j} - \bar{T}_{i-1,j}}{\Delta X} - V_{i,j}\frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{\Delta Y} \\
& + \frac{1}{Pr}\frac{\bar{T}_{i,j+1} - 2\bar{T}_{i,j} + \bar{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2\Bigg)
\end{aligned} \tag{11}$$

The initial and boundary conditions with the finite difference methods are as follows:

$$\begin{gathered}
U_{i,0}^n = 1,\ V_{i,0}^n = 0,\ \bar{T}_{i,0}^n = 1 \\
U_{i,L}^n = 0, \bar{T}_{i,L}^n = 0
\end{gathered} \tag{12}$$

where $L \to \infty$.

Here, the subscripts i and j indicate the grid points with X and Y coordinates correspondingly and $\bar{T}$ is the temperature.

5. Stability and Convergence Analysis

Stability analysis is a better tool for attaining the projection of the compatibility of a real-world mathematical problem. Before doing the real computations, it's always helpful. It also represents numerical solutions that are trusted and reliable. The general terms of the Fourier expansion for U, θ, φ at time arbitrary say $\eta = 0$ is $e^{i\bar{\alpha}\bar{x}}$ and $e^{i\bar{\beta}\bar{y}}$ apart from a constant, where $i = \sqrt{-1}$.

Then

$$\begin{gathered}
U : \Psi(\eta)e^{i\bar{\alpha}\bar{x}} \cdot e^{i\bar{\beta}\bar{y}} \\
\bar{T} : \theta(\eta)e^{i\bar{\alpha}\bar{x}} \cdot e^{i\bar{\beta}\bar{y}}
\end{gathered} \tag{13}$$

And after the time step, these terms will become

$$\begin{aligned} &U:\Psi'(\eta)\mathrm{e}^{i\bar{\alpha}\bar{x}}\cdot\mathrm{e}^{i\bar{\beta}\bar{y}} \\ &\bar{T}:\theta'(\eta)\mathrm{e}^{i\bar{\alpha}\bar{x}}\cdot\mathrm{e}^{i\bar{\beta}\bar{y}} \end{aligned} \tag{14}$$

Applying Equation (13) and Equation (14) into Equation (10) & Equation (11), the following equations are found by simplification

$$\frac{\Psi'-\Psi}{\Delta\eta}+U\frac{\Psi\left(1-\mathrm{e}^{-i\bar{\alpha}\Delta X}\right)}{\Delta X}+V\frac{\Psi\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)}{\Delta Y}$$

$$=\frac{2\Psi(\Delta Y\cdot\cos\beta-1)}{(\Delta Y)^2}+Gr\theta'-M\Psi$$

$$\Rightarrow\Psi'-\Psi+\frac{\Delta\eta}{\Delta X}U\Psi\left(1-\mathrm{e}^{-i\bar{\alpha}\Delta X}\right)+\frac{\Delta\eta}{\Delta Y}V\Psi\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)$$

$$=2\frac{\Delta\eta}{(\Delta Y)^2}\Psi(\Delta Y\cos\beta-1)+Gr\Delta\eta\theta'-M\Psi$$

$$\Rightarrow\Psi'=\Psi\left\{1-\frac{\Delta\eta}{\Delta X}U\left(1-\mathrm{e}^{-\bar{\alpha}i\Delta X}\right)-\frac{\Delta\eta}{\Delta Y}V\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)\right.$$

$$\left.+\frac{2\Delta\eta}{(\Delta Y)^2}(\Delta Y\cos\beta-1)-M\Delta\eta\right\}+Gr\Delta\eta\theta'$$

$$\Psi'=A\Psi+B\theta' \tag{15}$$

where

$$A=1-\frac{\Delta\eta}{\Delta x}U\left(1-\mathrm{e}^{-i\bar{\alpha}\Delta X}\right)-\frac{\Delta\eta}{\Delta Y}V\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)+\frac{2\Delta\eta}{(\Delta Y)^2}(\cos\beta\cdot\Delta Y-1)-M\Delta\eta$$

$$B=Gr\Delta\eta$$

and

$$\frac{\theta'(\eta)-\theta(\eta)}{\Delta\eta}+U\theta(\eta)\frac{1-\mathrm{e}^{-i\bar{\alpha}\Delta X}}{\Delta X}+V\theta(\eta)\frac{\mathrm{e}^{-i\bar{\beta}\Delta Y}-1}{\Delta Y}$$

$$=\frac{1}{Pr}\frac{2\theta(\eta)(\cos\beta\cdot\Delta Y-1)}{(\Delta Y)^2}+Ec\left(\frac{U\Psi(\eta)\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)}{(\Delta Y)^2}\right)$$

$$\Rightarrow\theta'(\eta)=\theta(\eta)\left\{1-\frac{\Delta\eta}{\Delta x}U\left(1-\mathrm{e}^{-i\bar{\alpha}\Delta X}\right)-\frac{\Delta\eta}{\Delta Y}V\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)\right.$$

$$\left.+\frac{1}{Pr}\frac{2\Delta\eta}{(\Delta Y)^2}(\cos\beta\cdot\Delta Y-1)+EcU\frac{\Delta\eta}{(\Delta Y)^2}\Psi(\eta)\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)\right\}$$

$$\Rightarrow\theta'(\eta)=G\theta+H\Psi \tag{16}$$

where, $G=1-\frac{\Delta\eta}{\Delta X}U\left(1-\mathrm{e}^{-i\bar{\alpha}\Delta X}\right)-\frac{\Delta\eta}{\Delta Y}V\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)+\frac{1}{Pr}\frac{2\Delta\eta(\cos\beta\cdot\Delta Y-1)}{(\Delta Y)^2}$ and $H=EcU\frac{\Delta\eta}{(\Delta Y)^2}\left(\mathrm{e}^{i\bar{\beta}\Delta Y}-1\right)$

Equation (15), Equation (16) can be written as:

$$\Psi' = A\Psi + B\left(G\theta + H\Psi\right) = \left(A + H\right)\Psi + BG\theta$$

$$\Rightarrow \Psi' = A_1\Psi + B_1\theta \tag{17}$$

where,

$$A_1 = A + H$$

$$B_1 = BG$$

and

$$\theta' = G\theta + H\Psi \tag{18}$$

Equation (17), Equation (18) can be expressed in matrix form.

$$\begin{pmatrix} \psi' \\ \theta' \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ H & G \end{pmatrix} \begin{pmatrix} \psi \\ \theta \end{pmatrix}$$

$$i.e.\ \ \eta' = T\eta$$

where, $T = \begin{pmatrix} A_1 & B_1 \\ H & G \end{pmatrix}$ and $\eta = \begin{pmatrix} \psi \\ \theta \end{pmatrix}$

As eigenvalues of the amplification matrix T is crucial for attaining the stability condition, as a result, let

$$B_1 \to 0,\ \ H \to 0.$$

Hence, matrix T is as follows:

$$T = \begin{pmatrix} A_1 & 0 \\ 0 & G \end{pmatrix}$$

Thus, the eigenvalues of T are

$$\lambda_1 = A_1,\ \ \lambda_2 = G.$$

Here, values of λ_1, λ_2 must not surpass in modulus.

Therefore, the stability conditions are as follows:

$$\left|A_1\right| \le 1,\ \ \left|G\right| \le 1.$$

Let,

$$a = \frac{u\Delta\eta}{\Delta X}, b = \frac{\Delta\eta}{\Delta Y}V, c = \frac{\Delta\eta}{\left(\Delta Y\right)^2}$$

Hence,

$$A = 1 - a\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - b\left(e^{i\bar{\beta}\Delta Y} - 1\right) + 2c\left(\cos\beta\Delta Y - 1\right) - M\Delta\eta$$

$$H = EcUc\left(C^{i\bar{\beta}\Delta Y} - 1\right)$$

The co-efficient of a, b, c is real and non-negative. Therefore, the maximum modulus of A_1, G arise when $\bar{\alpha}\Delta X = m\pi$ and $\bar{\beta}\Delta Y = n\pi$ where m and n are integers and hence A_1, G are real. The values of $\left|A_1\right|, \left|G\right|$ are greatest when m and n are odd integers, which are

$$A_1 = A + H = 1 - 2a - 2b - 4c - 2cEc.$$

To satisfy $\left|A_1\right| \le 1, \left|G\right| \le 1$ the most negative permissible values are,

$$A_1 = -1, \quad G = -1.$$

Thus, the stability conditions of the problem are given below.

$$1 - 2a - 2b - 4c - 2cEc \le -1$$
$$2(a + b + 2c + cEc) \le 2$$
$$a + b + 2c + cEc \le 1$$
$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{2\Delta\eta}{(\Delta Y)^2} + \frac{\Delta\eta}{(\Delta Y)^2} Ec \le 1$$

Analogously, the 2nd condition is as follows

$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{1}{Pr}\frac{2\Delta\eta}{(\Delta Y)^2}$$

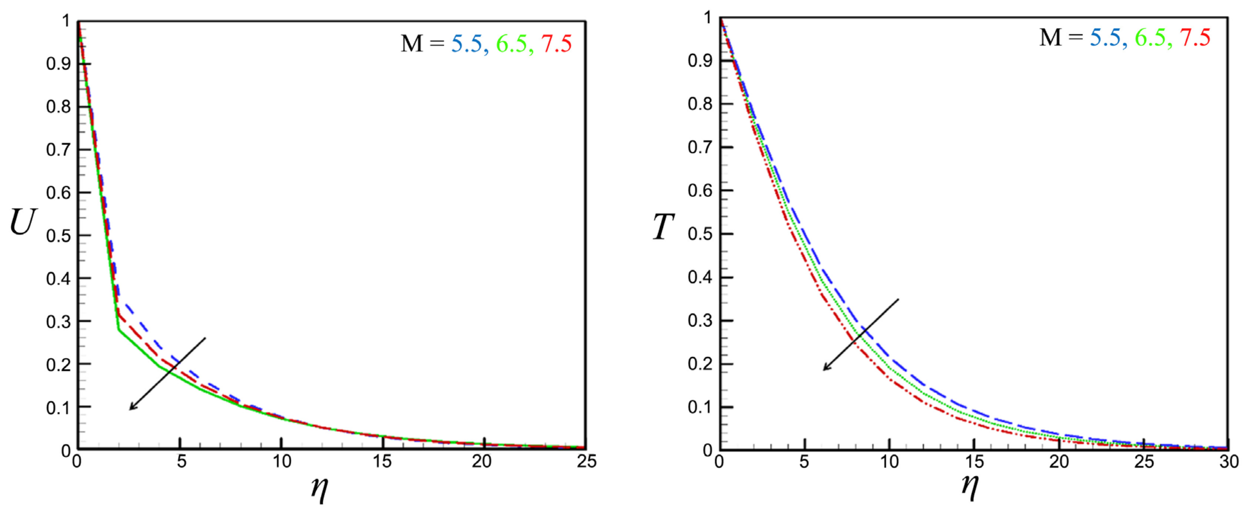
and convergence criteria of the method is $Pr \ge 1$.

6. Results and Discussion

To investigate the physical significance of the concerned problem, the several dimensionless parameters values such as magnetic Parameter *M*, Grashof number *Gr*, Prandtl number *Pr*, Eckert number *Ec* are executed with numerical computations. To illustrate the computed result, the velocity and temperature profile are plotted, the physical explanation is explained here. The velocity and temperature distributions are as follows:

6.1. Effect of Magnetic Parameter (*M*)

Figure 3(a), Figure 3(b) illustrates the variation of velocity and temperature profiles of the flow field for different values of the magnetic parameter *M* keeping other parameters as constant. Figure 3(a) displays that for rising values of magnetic parameter *M*, the velocity of the flow field falls-down. It's the fact that the application of a transverse magnetic field to an electrically conducting fluid gives to escalate a body force accredited as Lorentz force. It reduces the speed of



(a) (b)

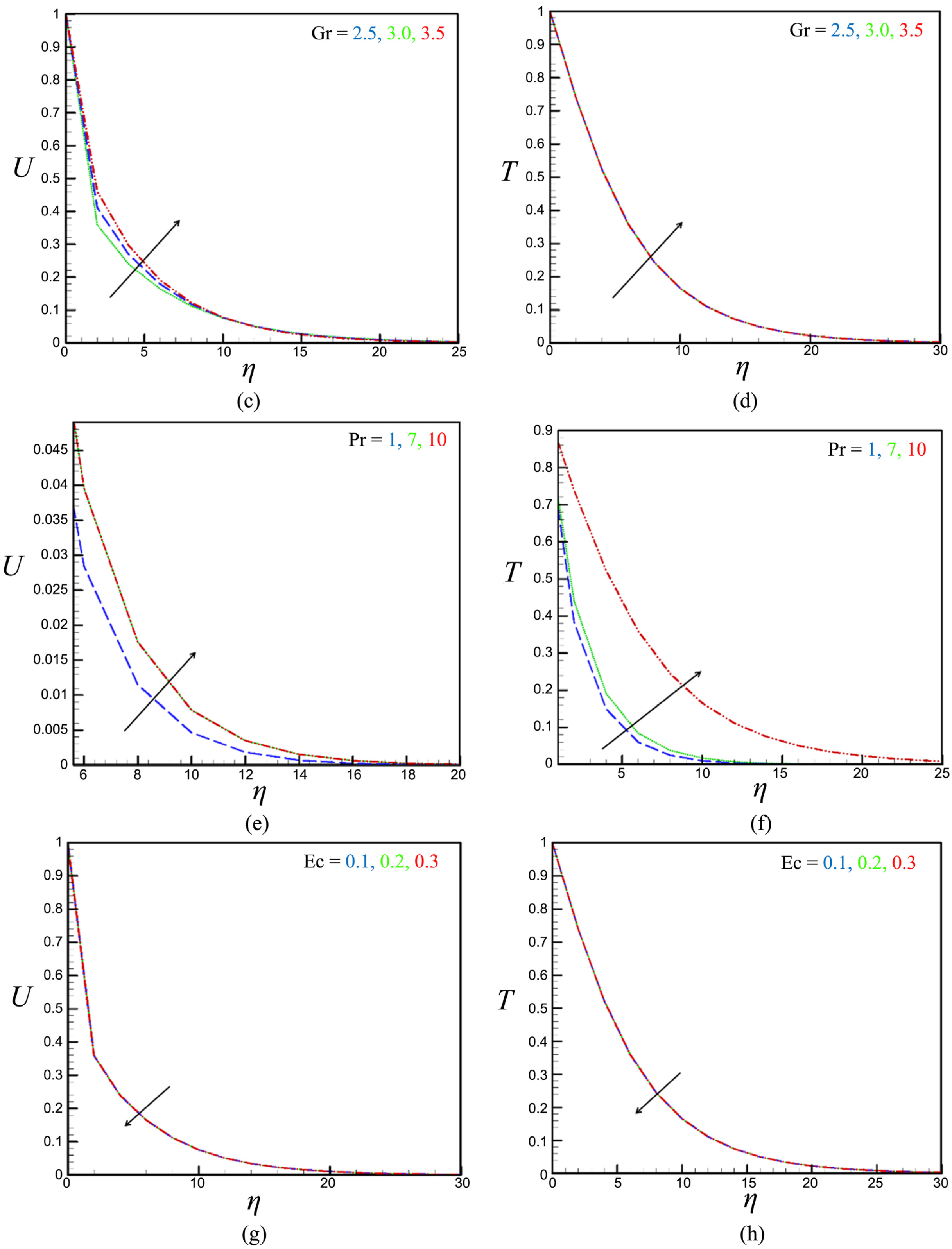


Figure 3. (a-b) Effect of Magnetic parameter (M) on velocity and temperature profile where $Gr = 2.5$, $Pr = 1$, $Ec = 0.1$; (c-d) Effect of Grashof number (Gr) on velocity and temperature profile where $M = 5.5$, $Pr = 1.0$, $Ec = 0.1$; (e-f) Effect of Prandtl number (Pr) on velocity and temperature profile where $Gr = 2.5$, $M = 5.5$, $Ec = 0.1$; (g-h) Effect of Eckert number (Ec) on velocity and temperature profile where $Gr = 2.5$, $M = 5.5$, $Pr = 1.0$.

the motion of the fluid in the boundary layer. Figure 3(b) reveals that the temperature profiles are lessening due to the increasing values of *M*. Since higher values of magnetic field parameters generate lower heat transfer, the temperature gradient decreases for reducing the rate of heat convection in the flow.

6.2. Effect of Grashof Number (*Gr*)

Analogously, Figure 3(c) and Figure 3(d) demonstrate the effect of *Gr* on the velocity and temperature profiles while other parameters are constant. Figure 3(c) demonstrates that the velocity profile is amplified for increasing values of *Gr*. It means to raise the velocity due to the thermal buoyancy force which generates the pressure gradient. The increasing values of *Gr* grow the thermal buoyancy effect. Thus, the rate of heat transfer increases, which plots in Figure 3(d).

6.3. Effect of Prandtl Number (*Pr*)

From Figure 3(e), it is lucid that the velocity profiles are enhanced for increasing values of *Pr*. The thermal boundary layer thickness decreases for increasing values of *Pr*, in general lower average temperature within the boundary layer. Velocity and temperature in the boundary layer fall very quickly for large values of the Prandtl number.

Figure 3(f) reveals that the temperature profile rises for the rising values of *Pr*. It is fact that the smaller values of *Pr* are comparable to increase in the thermal conductivity of the fluid. Furthermore, heat is diffused away from the heated surface more rapidly for higher values of *Pr*.

6.4. Effect of Eckert Number (*Ec*)

Figure 3(g) exemplifies that the velocity profile is declined owing to rises up *Ec*. Because of transferring heat to the kinetic energy of the flow, the driving force is reduced.

As frictional heating and heat energy are stored in liquid, the temperature profile is declined for rising values of *Ec*, which illustrate in Figure 3(h).

7. Conclusions

This thesis work analyzes the governing equations for two dimensional unsteady MHD free convection flow over a vertical plate. The governing partial differential equations and analytical explanation, graphical presentations are displayed depending on some different parameters such as Magnetic Parameter *M*, Grashof number *Gr*, Prandtl number *Pr* and Eckert number *Ec*. The effects of the mentioned parameters on the velocity and the temperature profile are analyzed significantly on the flow and heat transfer. The numerical investigations of current work are drawn in the following conclusion.

- The velocity profile falls with increasing values of *M*, *Ec* and opposite scenario for *Gr*, *Pr*.

- The temperature profile is declined with the risen up values of M, Ec and contrary consequence for Gr, Pr.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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