

# Simulation and Experiment Study on the Bridge Deflection Measuring Method Based on Secant Inclination

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## Abstract

Deflection is the most direct indicator that reflects the bearing capacity of the bridge and the overall stiffness. There are many ways to measure the deflection of Bridges, and the inclination angle method is the most commonly used indirect method, but the existing theory of inclination angle method is relatively complicated. Based on the facts of the bridge small inclination, this article proposes the method of obtaining the bridge deflection by the inclination of the secant line constructed from the adjacent measurement points. Firstly, according to the bending deformation curve of general simply supported beam, the deflection calculation formula of each measuring point is derived based on the assumption of small deformation and the inclination Angle of measuring point. Secondly, a large commercial finite element software ANSYS 10.0 is used to carry out numerical simulation on the simply-supported beam under concentrated load in mid-span, and the deflection results of the numerical simulation are compared and verified with the theoretical results of the proposed method. Finally, the measured deflection results of the simply-supported beam model under mid-span load are compared with the theoretical results of the proposed method. The verification results show that if the actual model is consistent with the theoretical model, the proposed method has good accuracy.

## Keywords

Bridge Deflection, Measurement, Inclination, Experiment Of Deflection Measurement

## 1. Introduction

Bridge is an important part of the highway and railway. The safety of the bridge

is related to the normal performance of road transportation functions and the safety of the lives and property of the people. Therefore, the safety and health monitoring of the bridge are particularly important [1]. Deflection is an important indicator that reflects the overall stiffness of the bridge and the bearing capacity. It is an important parameter reflecting the health status of the bridge. Therefore, measurement of bridge deflection is very important to evaluate the bearing capacity and safety of the bridge [2].

The methods of bridge deflection measurement can be divided into two categories: direct and indirect method. There are many direct measurement methods, including displacement, full station instrument, GPS, foundation radar measurement system (such as IBIS-S, FastGB SAR) [3], connecting pipeline, level instrument [4], photoelectric imaging, laser, ground microwave interference [5], machine vision [6], theodolite, measuring robot [7], GNSS (Global Navigation Satellite System) [8], digital image correlation (DIC) [9], etc. However, direct measurement methods generally require a fixed reference point, and some methods or operations are difficult, some methods are not accurate enough, some methods or prices are expensive, so people have developed indirect measurement methods.

The parameters of indirect measurement methods are generally strain [5], acceleration [10] [11], and inclination [2] [6]. The deflection is generally obtained from strain by modal vibration method, that is, the corresponding strain modular vibration type is obtained from the measured dynamic strain, and the displacement modular vibration type is then obtained from the relationship between the strain and displacement. Then the coordinates of displacement mode are calculated. At this point, the dynamic deflection of bridge can be obtained from the displacement modular vibration and displacement modal coordinates. The quadratic integration of acceleration with respect to time can also give the bridge deflection. These two methods above are generally used for measurement of dynamic deflection. The strain modular method requires fairly many measurement points, and there are some difficulties in identifying the modular vibration [6]. The modular vibration type is inaccurate [12] or the number of mode identification is insufficient, which will affect the deflection measuring accuracy [13]. The quadratic integration method of acceleration will produce errors, and it is difficult to determine the initial conditions [14]. Bridge deflection can also be obtained from inclination angle by curve fitting and the numerical integration of inclination with respect to coordinates, or by mode superposition method of structural dynamics. In recent years, the inclination method has been increasingly applied to the deflection measurement of the bridge [2].

However, whether curve fitting method or mode superposition method is used to obtain deflection from inclination Angle, these methods are still complicated. In this paper, based on the fact that the inclination Angle of Bridges is generally small, a method to obtain the deflection of Bridges by the inclination Angle and simple trigonometric function is proposed and verified by numerical

simulation and test. The experimental results show that the proposed method has good accuracy as long as the actual model agrees with the theoretical model.

## 2. Review of Bridge Deflection Measurement by Inclination Angle

According to the literature investigated by the author, at present, there are mainly integration method, summation method, fitting method and reconstruction method to measure bridge deflection through inclination Angle.

### 2.1. Integral Method

The integration method [15] is a relatively traditional and simple method, because the first derivative of the deflection  $y$  with respect to its position coordinate  $x$  is equal to the tangent of the inclination Angle  $\theta$ , namely:

$$\frac{dy}{dx} = \tan \theta \quad (1)$$

Assuming that the deformation is small, there approximately is:

$$\tan \theta = \theta \quad (2)$$

Therefore, as long as the inclination curve is fitted from the measured inclination Angle and the inclination Angle is integrated, the deflection of the bridge can be obtained, namely:

$$y = \int_0^x \theta dx \quad (3)$$

### 2.2. Summation Method

The summation method [16] is actually a discrete form of the continuous integral method. Change the length  $dx$  of the Infinitesimal element in Equation (3) of the above integration method into the distance  $\Delta x_i$  of the adjacent measuring points, and change the inclination  $\theta$  at any position  $x$  into the inclination  $\theta_i$  of secant line of the adjacent measuring points, then (3) becomes the following discrete summation form [17]:

$$y_j = \sum_{i=1}^j \theta_i \Delta x_i \quad (4)$$

### 2.3. Fitting Method

Although the integration method is simple and direct, its integration results are easily disturbed by test datum. Therefore, in order to ensure the accuracy, a large number of measuring points need to be arranged [18]. Therefore, people seek other inclination angle method. According to the results of literature survey, fitting method is the most popular inclination method for bridge deflection measurement. In 2002, Yang Xueshan *et al.* [19] proposed a fitting method to measure bridge deflection by inclination Angle. The fitting method uses a linear combination of a set of basis functions to fit the curve of the bridge, and its basic

form is shown as follows:

$$y(x) = A(x) \sum_{j=1}^n X_j g_j(x) \quad (5)$$

where,  $A(x)$  represents the boundary value condition,  $X_j$  is the combination coefficient, and  $g_j(x)$  is the basis function.

The determination of combination coefficient and basis function is the key of fitting method.

The combination coefficient is basically determined by the following two methods [20].

1) Solve linear equation set directly

When the measurement points are set such that the number of equations  $m$  is equal to the number of unknowns  $n$ , then the set of equations has a unique solution, and the combination coefficient  $X_j$  can be obtained by directly solving the set of equations.

2) The least square method

When the measurement points are set so that the number of equations  $m$  is greater than the number of unknowns  $n$  and the rank of the coefficient matrix of the set of equations is different from that of the augmented matrix, the combined coefficient  $X_j$  can be solved by the least square method.

Because of the measurement error, more measuring points are generally arranged, and the least square method is used to determine the combination coefficient.

The basis functions come in a variety of forms. In early stage, it was common to use power function [21] or displacement mode (or shape function) [22] determined by unit characteristic load as basis function. In order to improve the accuracy and stability [23] and reduce the number of measuring points [20], spline curve was proposed as the basis function. In order to adjust the shape of the deflection curve and further improve the fitting accuracy, people later adopted rational spline curve as the basis function [23]. In order to avoid singularity of coefficient matrix and get rid of restrictions on the placement of measurement points, Liu Yan put forward an improved spline fitting method with no measurement point restrictions based on the comprehensive application of traditional cubic spline curves, least-squares method, and integral method [24]. In order to adapt to the complex bridge with varying stiffness [25], naturally meet the structural boundary conditions and make use of the orthogonal characteristics of the mass matrix of the vibration mode, people use the natural vibration mode of the bridge as the basis function to fit the deflection curve [20]. However, the mode shape function needs to be calculated and determined by establishing a finite element model, which will be quite different from the actual model [25]. In order to solve this problem, Yao Jingchuan *et al.* proposed the fitting method of simplified mode shape function [26] [27] as the basis function.

## 2.4. Reconstruction Method

Reconstruction method is a method to determine the deflection of a measuring

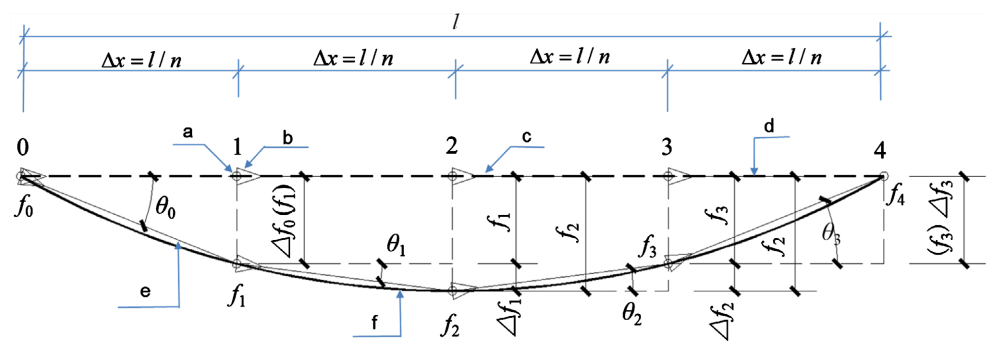
point by determining the relationship between the deflection of a measuring point and the inclination of a certain point [18]. Because these two quantities have different relationships with different structures and loads, the relationship between these two quantities is relatively easy to determine for simple structures and loads, but it is difficult to determine for complex cases.

### 3. Principle of Secant Inclination Deflection Measurement

The principle of measuring the bridge deflection by the secant inclination method is shown in Figure 1 [17]. The horizontal dashed line in the figure indicates the position of the bridge before deformation, and the arc-shaped solid line indicates the position after deformation,  $i$  (in this case  $i = 0, 1, 2, 3, 4$ ) is the position of the measuring point, where  $i_0$  and  $i_n$  (in this example  $i_0 = 0, i_n = 4$ ) are also the position of the support,  $f_i$  is the deflection corresponding to the  $i$ -th measuring point, and  $\Delta f_i$  is the  $i + 1$ -th measuring point Relative to the deflection increment of the  $i$ -th measuring point,  $l$  is the span of the bridge,  $n$  is the number of equal divisions of the measured points of the bridge, and  $\Delta x$  is the horizontal distance between two adjacent measuring points before deformation, namely:

$$\Delta x = l/n \tag{1}$$

In actual measurement, a linear chord can be erected between adjacent measuring points (*i.e.* each internode). One end of the chord is restrained by a hinge support, and the other end is restrained by a roller support to ensure that the chord can be free to expand and contract as the bridge deformed, without bending deformation to affect the accuracy of angle measurement. The inclinometer is fixed on the chord. Before the bridge is deformed, or when the chord is in a horizontal state, the inclinometer's reading is 0. After the bridge is deformed, the inclinometer's reading  $\theta_i$  is the inclination angle of the connecting line between the  $i + 1$ th and the  $i$ th measuring points, namely the inclination angle of the secant



Notice:

- 1)  $n=4$  in this paper;
- 2) Inclinometers installed on elastic chord are for measuring inclination of the chord;
- 3) Notation:  
 a—anchor point; b—inclinometer; c—beam position before deformation;  
 d—chord position before deformation; e— chord position after deformation;  
 f— beam position after deformation.

Figure 1. Principle of deflection measurement by secant inclination method.

line of the deflection curve.

It is easy to know from **Figure 1** that the deflection increment of the measuring point of  $i + 1^{\text{th}}$  relative to the  $i^{\text{th}}$  is:

$$\Delta f_i = \Delta x \cdot \tan \theta_i \quad (2)$$

The deflection corresponding to the  $i^{\text{th}}$  measuring point is:

$$f_i = f_{i-1} + \Delta f_{i-1} \quad (3)$$

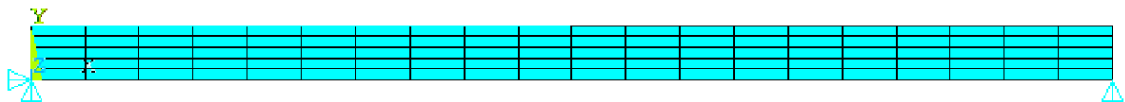
The deflection  $f_0$  of the initial measurement point at the left support can be directly measured by a displacement meter erected on the pier (the pier can be approximately regarded as a static reference point). After  $f_0$  is measured, the measured inclination angle  $\theta_i$  of each chord, as well as formula (2) and formula (3) can be used to obtain the deflection of each measuring point.

It can be seen from **Figure 1** that the cross-section of the bridge will rotate after the deformation, so that the measuring point is not right under the point where before the bridge deforms, resulting in an error in the deflection calculated according to the above formula. A test of this error will be made below by using numerical simulation method.

#### 4. Verification of the Measured Deflection Value under Static Load

In this section, a large-scale commercial finite element software, ANSYS 10.0, will be used to numerically simulate a simply supported steel beam under static load to verify the accuracy and reliability of the proposed secant inclination deflection measurement theory. The span of the simply supported beam is  $l = 10$  m, the width of the rectangular section is  $b = 0.2$  m, the section height is  $h = 0.5$  m, the steel's elastic modulus is  $E = 2 \times 10^{11}$  N/m<sup>2</sup>, its Poisson's ratio is  $\mu = 0.3$ , and its density is  $\rho = 7800$  kg/m<sup>3</sup>. The plane 42 plane stress element is used in simulation model, and the finite element model is shown in **Figure 2**.

Here, three load cases are studied, namely the deflection of the concentrated load acting on the 1/2 span (that is, the middle span) and the 1/4 span, and the uniform load. The deflections at the 1/2 span and 1/4 span are investigated. Set the concentrated load  $P = 10,000,000$  N and the uniform load  $q = 1,000,000$  N/m. In the numerical simulation, the secant inclination is calculated with the coordinates between the two measuring points after the bridge is deformed. The analysis results are shown in **Table 1**. It can be seen from **Table 1** that the maximum error is only 0.91%, which is less than 1%. Because the actual deflection-span ratio limit 1/600 [7] is much smaller than the ratio 1/16.7 of this example, the actual error should be much smaller than the error of this example.



**Figure 2.** ANSYS finite element model.

**Table 1.** Numerical verification of deflection calculation under static load.

	Deflection of mid-span	Deflection of 1/4 span
Deflection from proposed theory (m):	-0.481	-0.387
Deflection from ANSYS (m):	-0.478	-0.386
Error	0.628%	0.259%

## 5. Experimental Study on Deflection Measurement by Inclination Method

In order to verify the feasibility of the proposed deflection measurement method and to compare with the numerical simulation results, an experimental study on the single-span simply supported beam is conducted in this paper.

### 5.1. Test Device

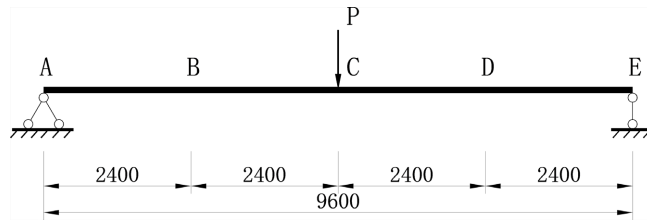
In this test, a simply supported steel beam is used to simulate the bridge. The section size of the steel beam is 6 mm × 20 mm, and the beam span is 9600 mm. There are a total of five measuring points A, B, C, D and E, and the distance between the measuring points is 2400 mm, as shown in **Figure 3**. Among them, A, B, D and E are arranged to measure the inclination Angle, B, C and D are arranged to measure the deflection, and C is arranged to load. Inclinator is HUOTO brand DC111 model digital display high-precision electronic inclinometer box, its resolution is 0.05°, accuracy is ±0.02°, repeatability is 0.1°. The load is  $P = 43.9\text{N}$ . The test site device for measuring the Angle of point A and deflection of point B is shown in **Figure 4**.

### 5.2. Test Results

By substituting 240 mm distance between measuring points in the test into  $\Delta x$  in formula (2), and the measured inclination Angle of each measuring point into  $\theta_i$  in formula (2), the theoretical increment  $\Delta f_i$  of deflection of each measuring point can be obtained. By substituting the result of formula (2) into formula (3), the obtained result is used as the theoretical value of deflection of each measuring point. The comparison results of theoretical and measured deflection of each measuring point are shown in **Table 2**. From the comparison results, it can be seen that there is a certain error between the theoretical value of deflection and the measured value at point B at 1/4 span and point C in the middle span. Among them, the error at point B at 1/4 span is small, which is close to the error value in **Table 1** of Section 3. The error at point C in the middle span is big, which is much bigger than the error value in **Table 1** in Section 3.

### 5.3. Analysis and Discussion

According to the test results in **Table 2** of Section 5.2, it can be seen that there are the following characteristics:



**Figure 3.** Schematic diagram of the experiment.



**Figure 4.** Experimental setup. (1-simply supported steel beam, 2-load, 3-dial indicator, 4-support, 5-base, 6-inclinometer).

**Table 2.** Comparison of bridge deflection test and theoretical results (Measuring point spacing 240mm, supposing secant Angle equal to tangent Angle).

load (N)	parameters	left support(point A)			Left quarter span (Point B)				mid-span (Point C)					
		readings		measurement	readings		theory	error (%)	readings		measurement	theory	error (%)	
		before loaded	after loaded		before loaded	after loaded			before loaded	after loaded				
	angle (°)	0.000	2.350	2.350	0.000	1.800	1.800							
	deflection (mm)				1.590	11.430	9.840	9.849	0.093	1.740	15.630	13.890	17.391	25.209
43.9	parameters	Right support (point E)			Right quarter span (Point D)				mid-span (Point C)					
		readings		measurement	readings		measurement	theory	error (%)	readings		measurement	theory	error (%)
		before loaded	after loaded		before loaded	after loaded				before loaded	after loaded			
	angle (°)	0.050	2.350	2.300	0.050	1.900	1.850							
	deflection (mm)				1.180	10.970	9.790	9.639	-1.538	1.690	15.590	13.900	17.391	25.118

**1) The test data of symmetric points are not completely symmetric**

In the test beam, the point A of left support and the point E of right support is symmetrical, and the point B at left 1/4 span and the point D at the right 1/4 is



also symmetrical, but no matter the measured values of Angle or the deflection at point B and point D are not completely symmetrical, which suggests that there may exist supporting conditions that are not absolutely symmetrical, support and geometry of steel beam that is not absolutely smooth, quality and geometry of materials is not absolutely even, loading position for each time is not absolutely the same, and the disturbance that comes from different loading position. All the factors mentioned above could affect the measurement result and lead to the deviation of the measurement data.

### 2) There exist errors in the measurement itself

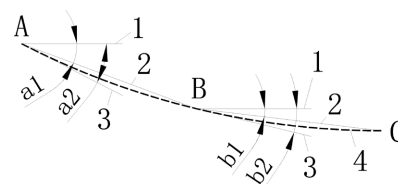
According to the measurement datum in **Table 2**, the deflection readings obtained from two measurements of the same point C are not exactly the same, which indicates that there may be errors in the measurement itself (of course, there may also be deviations caused by the disturbance of the test operation to the conditions of two tests). In addition, because the inclination box has a certain size, the measured inclination Angle is not strictly speaking the Angle of the measuring point, but only the Angle of the two ends of the inclination box near the measuring point.

### 3) The mid-span error is larger than that at 1/4 span

According to the comparison results in **Table 2**, the theoretical deflection values of point B and point D at the 1/4 span are very close to the measured values with very small errors, while there is a certain gap between the theoretical deflection values of point C at the middle span and the measured values with large errors, which is mainly caused by the error in using the tangent inclination approximation to replace the secant inclination Angle.

Due to the limitations of the test model and the size of the inclinometer, it is difficult to measure the inclination Angle of the line between two neighbor measurement points (secant inclination Angle) in the model test, so it is approximately replaced by the tangent inclination Angle of the measurement point, which is generally larger than the secant inclination Angle, as shown in **Figure 5**, resulting in a larger theoretical value.

As can be seen from **Figure 5**, for the measurement point A of the support, the tangent inclination  $a_1$  is close to the secant inclination  $a_2$ . Therefore, the tangent inclination  $a_1$  is approximately used to replace the secant inclination  $a_2$  to calculate the deflection of the measurement point B, and the error will be relatively



**Figure 5.** Inclination Angle of tangent line and secant line. (A, B, C- measuring points; 1- horizontal line; 2- Line between two neighbor measuring points, namely secant line; 3- tangent of measuring point; 4- beam line after deformation;  $a_1$  and  $a_2$ - Secant angle and tangent angle of point A;  $b_1$ ,  $b_2$ - Secant angle and tangent angle of point B).

small. However, for measuring point B at 1/4 span, there is a large difference between tangent Angle b1 and secant Angle b2. Therefore, the tangent Angle b1 is set to approximate the secant Angle b2 to calculate the deflection of measuring point C, and the error will be relatively large. According to visual estimation, the secant inclination Angle of point B at 1/4 span is about 1/2 of the tangent inclination Angle. Therefore, when the secant inclination Angle of point B is approximately set to 1/2 of its tangent inclination Angle, that is,  $b_1 = b_2/2$ , the theoretical deflection of mid-span measurement point C calculated by this method has a great reduction in its error, as shown in **Table 3**.

## 6. Conclusions and Future Work

The following conclusions can be drawn from the above research:

- 1) As long as the actual model is consistent with the theoretical model, the secant line inclination method proposed in this paper has high accuracy.
- 2) Bearing conditions, the flatness of supports and components, the uniformity of materials, and the measurement error may be the factors that affect the accuracy of the secant line inclination method of bridge deflection measurement. The influence of these factors should be considered and evaluated when the method is applied to the actual bridge deflection measurement.
- 3) The measurement operation can be simplified by approximating the inclination angle of measurement point to replace the inclination angle of the line between two neighbor measurement points (secant inclination Angle), which makes the measurement of bridge deflection easier. However, the influence of this approximate operation on the measurement accuracy needs to be considered and evaluated.

**Table 3.** Discussion of bridge deflection test results (Measuring point spacing 240 mm, secant Angle of point B is approximately taken as half of its tangent Angle).

load (N)	left support(point A)			Left quarter span (Point B)				mid-span (Point C)						
	readings		measurement	readings		measurement	theory	error (%)	readings		measurement	theory	error (%)	
	before loaded	after loaded		before loaded	after loaded				before loaded	after loaded				
	angle (°)	0.000	2.350	2.350	0.000	1.800	1.800							
	deflection (mm)				1.590	11.430	9.840	9.849	0.093	1.740	15.630	13.890	13.619	-1.948
43.9	Right support(point E)			Right quarter span (Point D)				mid-span (Point C)						
	readings		measurement	readings		measurement	theory	error (%)	readings		measurement	theory	error (%)	
	before loaded	after loaded		before loaded	after loaded				before loaded	after loaded				
	angle (°)	0.050	2.350	2.300	0.050	1.900	1.850							
	deflection (mm)				1.180	10.970	9.790	9.639	-1.538	1.690	15.590	13.900	13.514	-2.774

4) In this paper, numerical simulation and experimental verification are only carried out for simply-supported beams with mid-span concentrated loads. Future work can carry out verification research under other load forms (such as uniform load, moving load, and even random load).

5) This paper only studies the case of single-span simply supported Bridges, and future work can study other bridge forms, such as multi-span continuous Bridges, arch Bridges, cable-stayed Bridges and even suspension Bridges.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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