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Measuring the Adequacy of Loss Distribution for the Ghanaian Auto Insurance Risk Exposure through Maximum Likelihood Estimation

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Abstract

Loss distribution plays an influential role in evaluating risks from policyholders' claims. Nevertheless, the auto insurance market in Ghana pays little attention to policyholders' claims distribution, resulting in the market's inefficiency. This study investigates the type of loss distribution function that best approximates policyholders' claims in Ghana. We applied the Kullback-Leibler divergence, Kolmogorov Smirnov, Anderson-Darling statistical tests and maximum likelihood estimation (MLE) to estimate policyholders' claims. The results suggest that Ghana's auto policyholder's claims are better approximated using the lognormal probability distribution. Through the lognormal distribution, the industry can adequately evaluate policyholders' claims to minimize potential loss. Additionally, this distribution could enable the market reach decisions on premiums and expected profits theoretically.

Keywords

Leptokurtic, Loss Distribution, Policyholders Claims, Auto Insurance, Lognormal

1. Introduction

The significant contribution of auto insurance markets in every country's economic growth and development cannot be underrated (Azaare & Wu, 2020; Azaare et al., 2021). Hence, safeguarding the market to understand insurers' exposed risk probability distribution is essential (Azaare & Wu, 2020; Stojakovic & Jeremic, 2016). Insurers' risk exposure may be defined as their susceptibility to potential losses. Every insurer has a duty to price premiums profitably, and this can be guided if the insurer has a clue on which probability law better approximates the risk posed by the policyholders (Ibiwoye et al., 2011; Packová & Brebera, 2015; Walhin & Paris, 1997). This, according to researchers (Brouhns et al., 2003; Azaare & Wu, 2020; Pinquet, 2000; Spindler et al., 2014) guide the insurer in making proper evaluations and predictions to avoid or minimize the potential losses. From the above, it's evident that there have been many uncertainties on the part of insurance companies concerning their risk exposure. The uncertainties have an adverse effect on these insurers and the economy they operate as a whole. The interest of loss distributions by authors in the area of insurance and finance can be associated with their ability in predicting and pricing of premiums, in most instances relying on historical claims from policyholders (Ahmad et al., 2020; Bali & Theodossiou, 2008; Brouhns et al., 2003; Azaare & Wu, 2020; Pinquet, 2000; Spindler et al., 2014; Tremblay, 1992).

Unlike developed countries where auto insurers have robust pricing systems that capture policyholders' claims, the Ghana National Insurance Commission (NIC) uses a pricing system that pays little attention to the importance of claims histories (Azaare & Wu, 2020). However, drawing from the above literature, the danger posed by policyholders' claims can never be undermined. Notwithstanding, the claims ratios from most market players in Ghana according to the NIC annual reports are below the internationally accepted standards. The claim ratio is calculated as the net claims incurred divided by the Net Earned Premiums. It is an influential ratio indicating the strength an insurer exercise in paying claims and to some extent, how well policyholders are treated. Through this ratio, policyholders can measure how much they receive in return for each Ghana cedi premium paid to their insurers. According to the NIC 2018 annual report, the claim ratio overall market for the past years has been low as already posited compared to the internationally acceptable benchmark, which falls between 40% and 60%. In 2018, for instance, the market average increased to 42% from 37% in 2017. Though the market average performance falls within the internationally accepted standard, the figures being recorded by the company under consideration have shown some slight deviations. This company in 2017 recorded a ratio of 43%, which increased to 64% in 2018. If care is not taken, this figure could increase further or may not fall within the market average since historic records from **Figure 1** show some remarkable variations. In 2016 for example, this company recorded a ratio of 46%, which is a consecutive decline from the two previous years of 55% and 64% according to the NIC.

Aside the from market claims ratio, another influential indicator of probability is the total expense ratio (Management Expense + Commission expense). This ratio is determined as a percentage of the Net Earned Premium with an internationally accepted ratio usually less than 40%. As this ratio becomes larger, it implies that the company is inefficiently discharging its duties, and this is more likely to impact its prompt payment of claims to policyholders. In the year under which the sample is considered for this research (2018), the market mean expense ratio was 99%, whereas that of the insurer under consideration was 100%. The market average for this ratio has, over the years, not been so good. This is because almost all the key players in the market have been performing below the international standard which, indicates that the market in general is not that efficient to guarantee policyholders' claims payment. Therefore, we argue that for an insurer to be financially solvent and avoid eroding policyholders trust, the claims from policyholders' must not be taken for granted. Hence, it's imperative to properly evaluate and predict policyholder's claims distribution to help offset the market's inefficiencies. Therefore, to guide the insurer in making proper evaluations and predictions to avoid or minimize potential losses that could end up eroding trust and to attain financial solvency, this study seeks to investigate the type of loss distribution function that best approximate the policyholders' claims in Ghana using real data from a major insurance company.

Many researchers using a generalized linear model (GLM), apply the method of maximum likelihood estimation (MLE), method of moments and Bayesian estimation in their quest to have optimal pricing models for auto insurance based on policyholder's claims histories and a priori rating variables (Bolancé et al., 2007; Azaare & Wu, 2020). As postulated by Bali and Theodossiou (2008) and Fang (2003), the basic problem requires selection of loss models for severity and frequencies of claim. Probability distribution functions are forward-looking because they are founded on actual data (Sarpong, 2019), and to estimate them perfectly, one does not necessarily need long historical time series data. Moreover, probability distribution functions have the merit of being fairly free of mathematical priors, have the capacity of adapting instantaneously to any change in data and also dealing accurately with any intrinsic risks in the data (Bali & Theodossiou, 2008; Sarpong, 2019).

In the past decades, the applications of loss distribution in finance and insurance have been very predominant. It is known that most insurance and financial assets return are positive (Klugman, Panjer, & Willmot, 2012), have heavier tails and unimodal hump-shaped with much higher kurtosis that requires the application of distributions such as the exponential, gamma, Weibull, lognormal, the inverse Gaussian rather than the standard normal distribution (Ahmad et al., 2020; Cooray & Ananda, 2005; Godin, Mayoral, & Morales, 2012; Lane, 2000). Using the normal distribution, "an insurance risk pricing model was developed based on a measure of risk distortion" (Wang, 2000). This model was later modified by Godin et al. (2012) using the normal inverse Gaussian to accommodate heavier skewed tails data. Packová and Brebera (2015), through actuarial modeling found that the claims size of third-party liability insurance better fits the

Pareto distribution than its contenders like the Weibull and the Gamma after performing a series of statistical tests. Sarpong (2019) demonstrates that the lognormal distribution optimally models the seasonal volatilities existing between the American dollar and the Ghana cedi. "A semi-parametric approach based on the generalized method of moments (GMM) to tackle the specification situation concerning claim frequency distributions has been proposed" (Fang, 2003). Extensive research by Bali and Theodossiou (2008) "estimate the conditional and unconditional value at risk (VaR) thresholds, evaluate the performance of three extreme value distributions, generalized Pareto distribution (GPD), generalized extreme value distribution (GEV), and Box-Cox-GEV, and four skewed fat-tailed distributions, skewed generalized error distribution (SGED), skewed generalized *t* (SGT), the exponential generalized beta of the second kind (EGB2), and inverse hyperbolic sign (IHS)".

According to Egan (2011), several probability distributions were fitted to the daily percentage returns regarding Standard and Poor's 500 portfolios, and the optimum selection was found to be the student t-distribution. An empirical study comparing compound distributions of stock returns shows that monthly data returns were accurately approximated by normal distribution (Akgiray & Booth, 1987). According to Tucker and Pond (1988), "the distribution of exchange returns satisfies normality because of their long-tailed and leptokurtic behavior". In providing a clear description to insurance data, Eling (2012) utilized two available data sets in insurance and demonstrated competition between the skew-normal and the skew-student t distribution compared with other distributions. Bolancé et al. (2007) shows evidence of the bivariate claims model by applying the skew-normal and log-skew-normal distributions using Spanish auto insurance datasets. As a parametric alternative in modeling heavy-tailed data, Ahn et al. (2012) applied the log-phase type distribution. In fitting auto insurance claims, performance comparison was made using popular insurance data by Kazemi and Noorizadeh (2015).

From the research work done on finance and insurance data, it has been noticed that though their distributions are mostly not normal and asymmetric the quest of approximating such data with appropriate probability function has always been successful. Therefore, to guide the insurer in making proper evaluations and predictions to avoid or minimize potential losses that could end up eroding trust and to attain financial solvency, this study seeks to investigate the type of loss distribution function that best approximate the policyholders' claims in Ghana using real data from a major insurance company.

2. Methods

2.1. Data Collection/Source of Data

Information on risk exposure and premiums for (n = 23,434) vehicle insurance policyholders was obtained throughout 2018 from a leading Ghanaian insurance company (Azaare et al., 2021). Due to existing market competition, this leading insurer opted to be unanimous. Out of the total sample, 3,733 (15.9%) drivers had reported claims with an average claim of 9,547.60 Cedi with minimum and maximum being respectively 25.00 and 746,016.00 Ghana cedi. The policyholders' claim is the variable of interest in this research since we aimed to investigate the appropriate probability function that best approximates it. This variable has an average driver age of 49 years, whiles the minimum and the maximum ages stood at 21 and 76 years, respectively.

2.2. Loss Distributions

Here, we reviewed some loss distribution functions with the sole aim of testing the one that best fits the data on policyholders' claims. The data's density plots under consideration shown in **Figure 2** and **Figure 4** exhibited the distribution's features; hence, the justification of their selection for investigation. Also, the selection process was influenced by the Cullen and Frey plot in **Figure 3** obtained through the Bootstrapping method. This graph produced positive values of both skewness and kurtosis, indicating a heavier tail distribution. Therefore, the most likely distributions with the above features to fit the data are; lognormal, gamma, exponential and Weibull. We provide the Mathematical functions that make this possible.

2.2.1. The Lognormal Distribution Function

A random positive variable N is claimed to be log normally distributed because $x = \ln(N)$ being a random variable is normally distributed. With parameters μ and σ, the outcome probability distribution function indicating the normal distribution of the lognormal random variable $\ln(N)$ equal

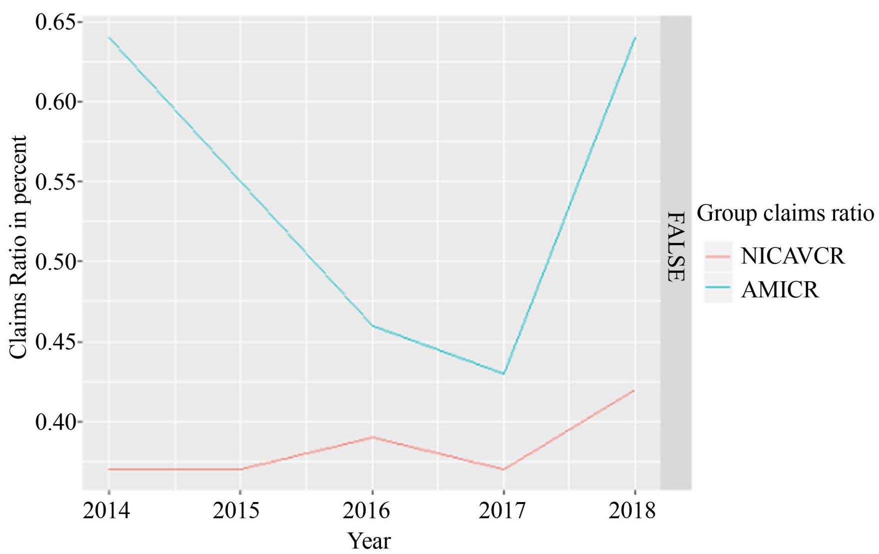

Figure 1. Comparing the annual NIC average claims ratio against a major insurance company (2014-2018). *National insurance average claims ratio (NICAVCR). *A major insurance company claims ratio (AMICR).

$$f(N;\mu,\sigma)=\begin{cases}\sqrt{2\pi}\sigma N\sqrt{2\pi}\sigma N e^{-\frac{\ln(N-\mu)^2}{2\sigma^2}}, & N\le 0\\ 0, & N<0\end{cases}$$

The lognormal distribution has its expectation and variance respectively as

$$E(N)=e^{\mu+\sigma^2 2/2},\quad Var(N)=e^{2(\mu+\sigma^2)}-e^{2\mu+\sigma^2}$$

2.2.2. The Gamma Distribution Function

A random variable N being continuous is said to follow the Gamma distribution function having parameters $\alpha>0;\beta>0$ if has a PDF given as

$$f(N)=\begin{cases}\frac{1}{\alpha}\beta^{\alpha}N^{\alpha-1}e^{-\beta N}, & N>0\\ 0, & \text{otherwise}\end{cases}$$

This distribution function has expectation and variance respectfully as $E(N)=\beta$ and $Var(N)=\beta$.

2.2.3. The Exponential Distribution Function

With location and scale parameter respectively as $\lambda>0;\alpha>0$, a random variable N follows an exponential distribution function if $f(N)=\frac{1}{\alpha}e^{-\left(\frac{N-\lambda}{\alpha}\right)}$, $N\ge\lambda;\alpha>0$. This distribution function has its expectation and variance respectfully as $E(N)=\alpha$ and $Var(N)=\alpha$.

2.2.4. The Weibull Distribution Function

A random variable N follows the Weibull distribution with respectively the scale, the shape parameters $\eta>0;\beta>0$, if the probability distribution function of N having threshold parameter λ is

$$f(N;\eta,\beta)=\frac{\beta}{\eta^{\beta}}(N-\lambda)^{\beta-1}e\left[-\left(\frac{N-\lambda}{\eta}\right)^{\beta}\right], N\ge\lambda,\eta,\beta>0,N$$

The Weibull distribution has its expectation and variance, respectively as;

$$E(N)=\eta\Gamma\left(1+\frac{1}{\beta}\right)+\lambda, Var(N)=\eta^2\left(\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right).$$

2.3. Quantification of Information Lost through Kullback-Leibler Divergence and Model Selection Information Criteria

To ascertain the quantum of information lost in our data, we evaluate the entropy of the probability distribution. The probability distributions' entropy of the data is

$$E=-\sum_{i=0}^{N}p(x_i)\log p(x_i)$$

The Kullback-Leibler divergence (D_{KL}) is obtained by modifying this equation to give the actual information missing for approximating one probability distribution by another. The D_{KL} is;

$$D_{KL}(p \parallel q) = \sum_{i=0}^{N} p(x_i) \log \frac{p(x_i)}{q(x_i)}.$$

where the real and the fitted probability distributions are $p(x_i)$ and $q(x_i)$ respectively. It's always desirable for the D_{KL} value of a particular probability distribution to be smaller. The smaller the value, the less information lost when such probability distribution is used to approximate the data.

In the model selection process, relative values of different statistical distributions are compared using information criteria for an observed data. The information criteria employed to compare the best distribution for the data under consideration are the Bayesian Information Criterion (BIC) and the Akaike Information Criterion (AIC). These criteria though helps in obtaining the optimum selection, adjust the fit of the model and factors parsimoniously by taking fundamental interest considering the different parameters involved. Hence, with a preference for the smaller values, the best distribution fit was obtained through these statistics based on the log-likelihood function calculated at the MLE. Readers may see for example, (Ahmad et al., 2020; Gómez-Déniz & Calderín-Ojeda, 2018; Azaare & Wu, 2020), for details.

2.4. Maximum Likelihood Estimation

Let's assume that a random variable $N = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$ be a vector of n independent observations drawn from a population with PDF $x = \ln(N)$, where $\theta = [\theta_1, \theta_2, \cdots, \theta_q]^{\mathrm{T}}$ denotes a vector for q unknown parameters. The likelihood function $L(\theta; N)$ is defined as $L(\theta; N) = \prod_{i=0}^{n} g(x_i; \theta)$. The value of θ for the maximum likelihood estimate $\hat{\theta} = \hat{\theta}(N)$ is the one that maximizes $L(\theta; N)$.

3. Results and Discussions

The purpose of this paper was to investigate the PDF that adequately approximates the risk exposure of auto insurers. In doing so, we first produced a depicting plot of the data's non-parametric density. As observed in **Figure 2**, the data is positively skewed since the graph tails towards the right direction. We employed the bootstrapping approach in obtaining the Cullen and Frey plot. In **Figure 3**, it's confirmed that there are many possible loss distributions to approximate the data. The summary statistics from this graph produced a skewness and kurtosis of 14.81 and 373.88, respectively. This indicates leptokurtic data since from the statistics, it is noticed that the data has a heavier right tail compared to the left, and hence, the possible loss distribution is positively skewed. The distributions in **Figure 3** includes: Normal, Exponential, Logistic, Beta, Lognormal, Gamma and Weibull. However, the only distributions that will be looked at to ascertain which one the data follow are the Lognormal, Gamma, Exponential and the Weibull since they are the positively skewed distributions as

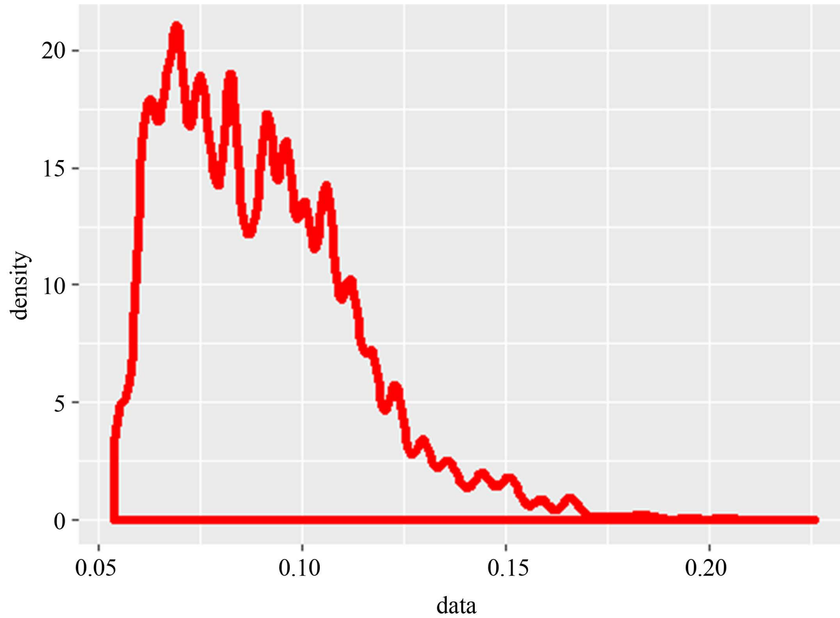

Figure 2. Non-parametric density plot of policyholders claims in Ghanaian major insurance company.

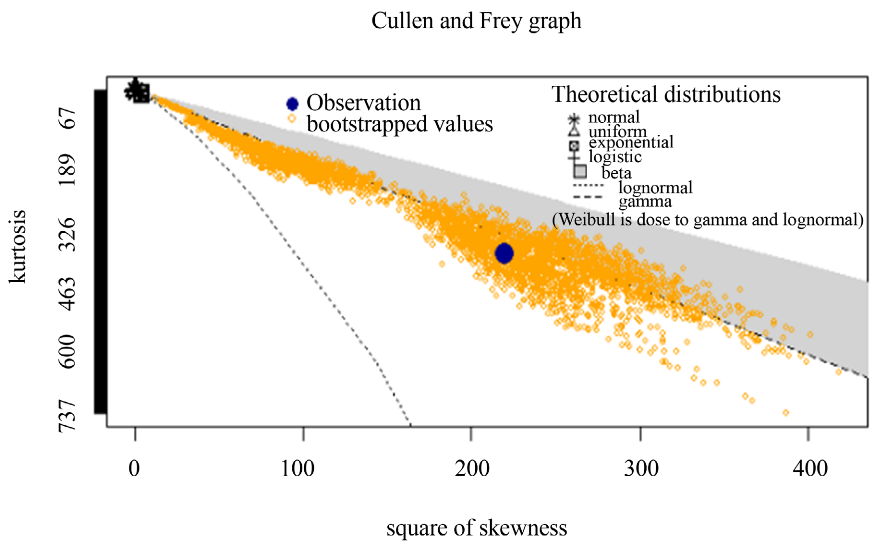

Figure 3. Plot of Skewness and Kurtosis for policyholders claims.

shown in Figure 4. The theoretical QQ and PP plots for these positively skewed distributions are shown in Figure 5. From this figure, it is clearly shown which distribution is more likely to approximate the data. These plots as indicated is not favoring the exponential distribution as it deviates out of the distribution functions hypothesized. Therefore, the most expected PDF could be Lognormal, Gamma or Weibull as confirmed by the graph shown in Figure 6.

The optimal distribution has the smallest AIC and BIC values but with the highest log-likelihood statistic. It is observed from Table 1 that the lognormal distribution received the smallest AIC and BIC values with a high log-likelihood

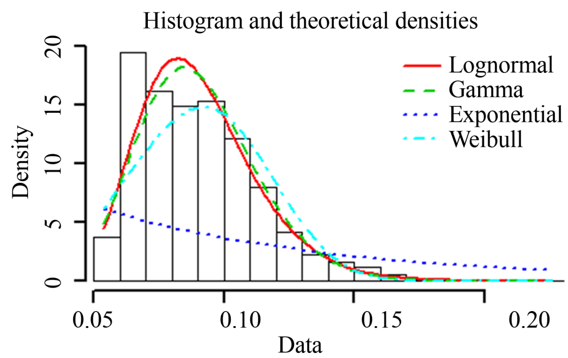

Figure 4. A histogram and theoretical densities plot for policyholders claims.

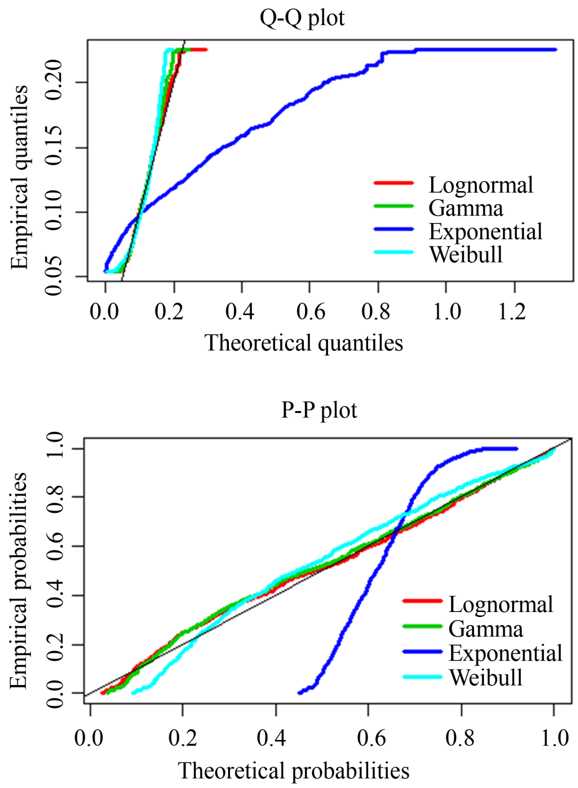

Figure 5. QQ and PP-plots for policyholders claims.

Table 1. The Goodness-of-fit Criteria, Log likelihood and Kullback-Leibler divergence Statistics for insurers risk exposure.

Loss distribution	Log likelihood	AIC	BIC	Kullback-Leibler divergence
Gamma	−7,530.687	15,065.37	15,077.82	−2.30
Exponential	−11,463.57	22,929.14	22,935.37	−1.50
Lognormal	−7,322.54	14,649.09	14,661.54	−2.50
Weibull	−7,456.07	14,916.13	14,928.58	−2.47

Note: Smaller statistics are preferred for Akaike's information criteria (AIC), Bayesian information criteria (BIC) and Kullback-Leibler divergence. Bigger statistic is preferred for the Log likelihood estimates.

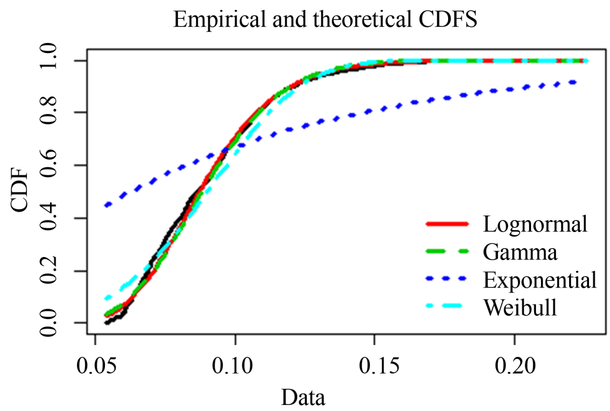

Figure 6. A plot of empirical and theoretical cumulative distributions for policyholders claims.

statistic, and hence the data is more expected to be fitted by the lognormal distribution. To be sure that the data is lognormal, we relied on goodness-of-fit information. The results obtained from the analysis using the Kolmogorov Smirnov and Anderson-Darling statistics show that the data is better approximated with the lognormal distribution. Therefore, it is ascertained that policyholders' claims is better approximated using the lognormal distribution with $\mu = -2.439$ and $\sigma = 0.248$. As far as these information criteria and test statistics are concerned with insurance and financial loss distributions, our findings are in line with several studies (Ahn et al., 2012; Azaare & Wu, 2020; Gómez-Déniz & Calderín-Ojeda, 2018; Dutta and Jason, 2011).

To further confirm that the insurers' risk exposure is approximated by the lognormal distribution, data based on this distribution was simulated. The simulated data has $\mu = -2.440$ and $\sigma = 0.250$. We then compared the simulated data from the lognormal distribution with our actual data (insurance claims). It's observed from **Figure 7** and **Figure 8** that, the plots for both the original and the simulated risk exposures have similar characteristics. Also shown in **Figure 9** is the empirical density plot of the simulated data that can be compared with the original plot in **Figure 2**. Thus, based on the graphs in **Figures 7-9**, we noticed that these sets of data are very comparable and having almost all the points fallen alone the fitted curves from the theoretical empirical and cumulative distributions of the QQ and the PP plots. Therefore, the optimal distribution for the data is lognormal. To determine which of these distributions fits the data with less information, the Kullback-Leibler divergence test was performed. From **Table 1**, the various probability distribution functions and their associated Kullback-Leibler divergence values are shown. Our observation here is that smaller amount of information is lost by approximating the data using the lognormal distribution.

Finally, it was very needful to ascertain whether these two datasets have the same probability distribution. Therefore, with continuity correction, Wilcoxon-signed rank test was performed. It was observed from the test result that there was a *p*-value of 0.102. Hence, at 5% significant level, the two datasets are identical. Therefore, we can conclude that auto insurance policyholders claim in Ghana is approximated by the lognormal probability distribution.

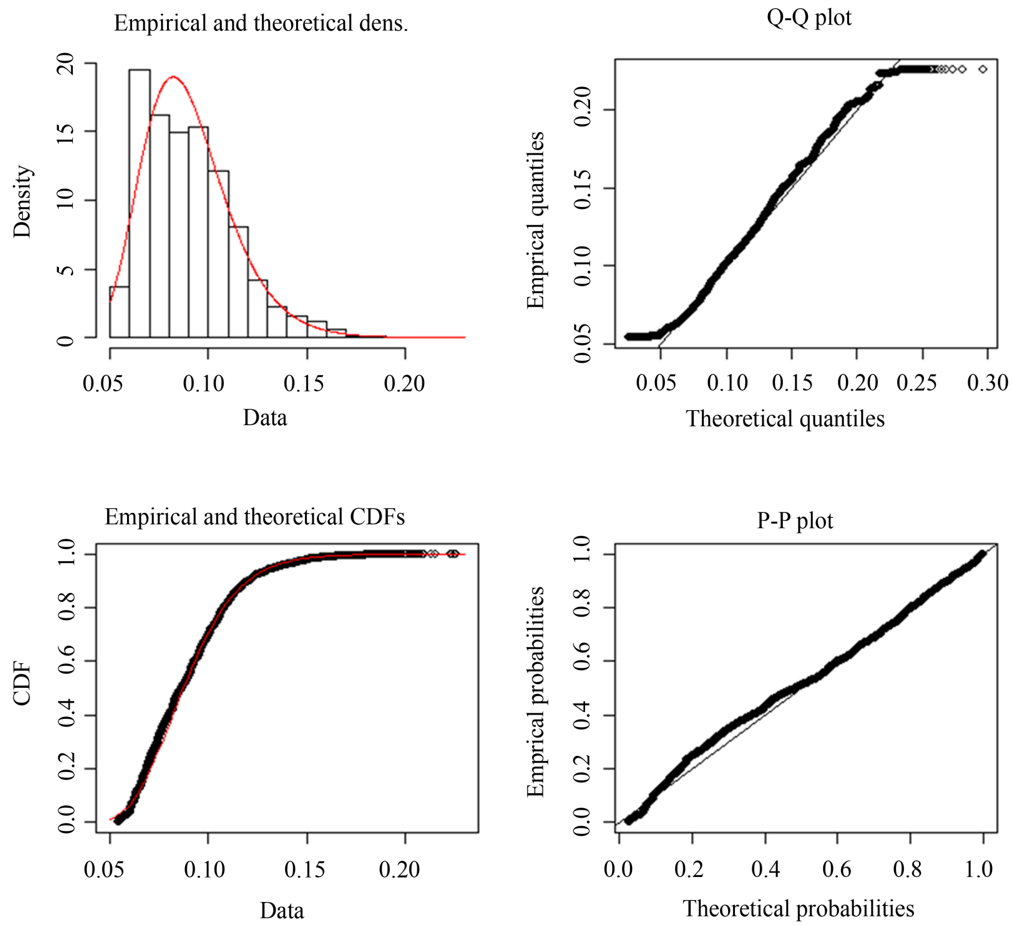

Figure 7. Plot of the original data of policyholders' claims.

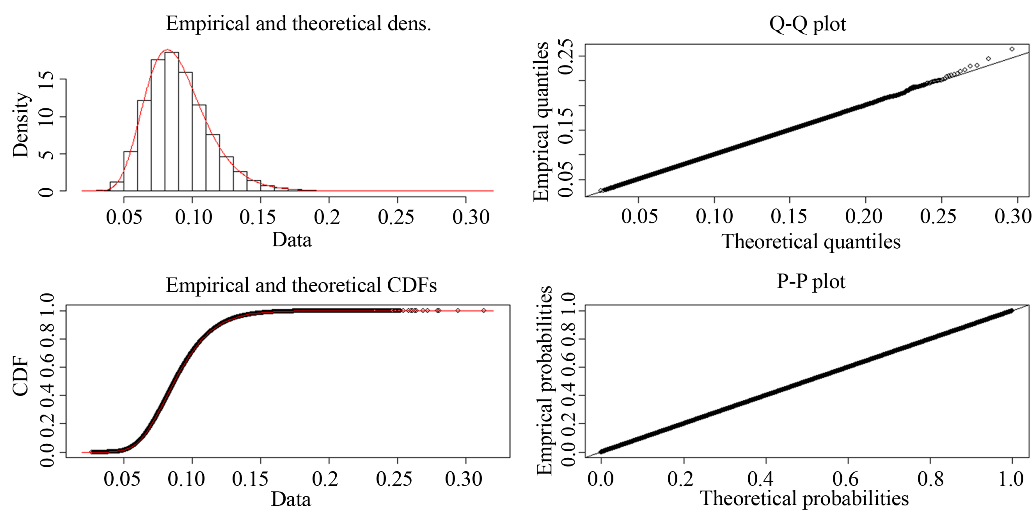

Figure 8. Plot of the simulated data of policyholders' claims.

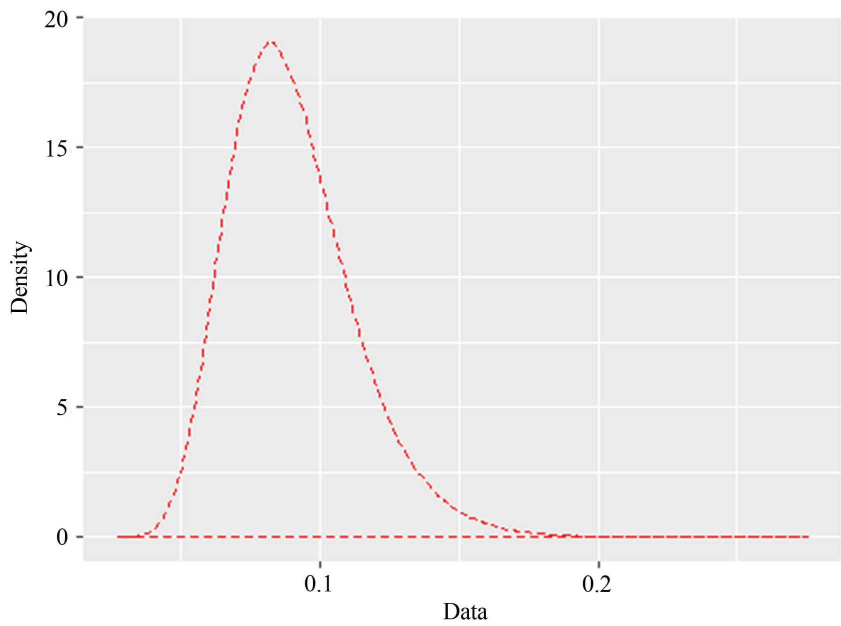

Figure 9. A graph of the distribution of simulated claims of policyholders.

4. Conclusion and Practical Implications

Though insurers have a duty to manage policyholder's risk, nonetheless, they are profit-making companies. Therefore, their survival in every economy depends on how they can properly evaluate their risk exposures to attain financial solvency. Hence, predicting policyholders' claims probability distribution function would maximize profit. This paper has adequately established that the risk policyholders posed to insurers follows lognormal probability distribution. Using the lognormal probability distribution to evaluate the potential losses or policyholders' risk will end up maximizing insurers' profit. From the AIC, BIC, log-likelihood statistics, and the Wilcoxon signed-rank test, it was observed that this distribution properly approximates both the simulated and the original data. It was also observed from the Kullback-Leibler divergence statistics that the amount of information loss using the lognormal distribution to fit the data is less compared to the contending distributions. Thus, the lognormal is the best to approximate the auto insurance policyholders' claims.

Managerially, this paper is expected to help insurers properly evaluate and manage policyholder's risk to profit from it. Evaluating and predicting insurers' risk exposures would translate into attaining financial solvency through maximization of profit. The lognormal distribution provides insurers useful and tractable mathematical features of the risk posed by policyholders. Aside from the useful and tractable features with the lognormal distribution, it also provides information to insurers to reach decisions on premiums loadings, expected profits, and necessary reserves needed to ensure profit margins and the effects of deductibles and reinsurance. The skewed nature of the lognormal distribution would provide insurance companies with the best alternative in modeling their risk exposures. Hence, it is recommended that auto insurers risk (policyholders' claims) in Ghana and other developing economies are predicted using the lognormal distribution. Furthermore, while the lognormal distribution is statistically proven to provide a good fit for our data, we suggest that future research could also look into other long tailed distributions and categorize policyholders' claims based on the insurance type.

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Conflicts of Interest

Authors declare no conflict of interest.

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