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Modeling Blood Flow in Veins of Uniform Properties (Giraffe Jugular Vein)

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Abstract

This paper models the giraffe's jugular veins as a uniform collapsible tube from a rigid skull. The equations governing one-dimensional steady flow through such a tube for various conditions have been developed. The effects of inertial and inclination angles that have not been discussed previously have been included. It has been shown that different flows for a uniform tube (vein) are possible. However, this flow matches that of a jugular vein which is supercritical, and the steady solution has been given by the balance between the driving forces of gravity and the viscous resistance to the flow at the right atrium of the heart must be sub-critical for a fixed right-atrium pressure which means that an elastic jump is required to return the flow to sub-critical from the supercritical flow upstream this type of relationship gives rise to flow limitation at the same time given any right atrium fixed pressure there exists a maximum flow rate which when exceeded the boundary conditions of the flow do not hold boundary conditions at the right atrium are not satisfied hence making the steady flow impossible this mechanism of flow limitation is slightly different from the other one in that causes airways through forced expiration from the observation made it is clearly shown that there is an intravascular pressure difference with a change in height.

Keywords

Blood Flow, Jugular Vein, Cross-Sectional Area, Supercritical, Subcritical, Jump

1. Introduction and Mathematical Formulation

The study of physiological fluid flow is pursued by many scientists in different fields. The attraction of the subject comes about as a result of the abundance and the diversity raised which differs greatly in many respects from those encountered in engineering fluid flows. Actually, many fluid-conveying vessels (veins and arteries) in an animal's body are highly elastic and undergo deformation substantially in due to the traction (pressure and viscous stress) that the fluid exerts on them. There is also a great deal of interest in the study of flows in deformable elastic tubes in both biomechanical and biological contexts. It also poses a fluid-mechanical challenge of interest and great difficulty on its own. The flow of blood to the brain and the neck in a standing giraffe is of great interest. Experiments and theoretical explanations have been given, but still, the whole issue has not been fully understood. [1], did a study in physiology and biophysics of circulation of fluid in the body system and found out that 'it is no harder in the circulation for the blood to flow uphill than downhill' and that differences in the level of different parts of the vascular bed do not in any way affect the forces for the flow and hence do not affect the circulation directly. In their study, they also found out that the prerequisite for the existence of a vascular siphon is a continuous column of blood in both arterial and venous limbs of the loop [2] [3] [4] while studying Gravitational hemodynamic and Oedema prevention in the giraffe, where the projected pressure range is between −93 and −27 mmHg based just on the current hydrostatic gradient, which is proportional to the total of the gravitational and viscous pressures. In a more recent research, the authors provide more evidence in favor of the idea that the heart only has to overcome the blood vessels' viscous resistance rather than the weight of the blood being pumped to the brain [5], and [6]. This working models the mathematical equations such that (Following [7]) a long straight tube which has the ability to collapse as the fluid flows hence its cross-sectional area is not uniform is considered. For this study, an elastic tube is considered but assumed to be made of uniform material. The flow is considered non-conducting. The tube is assumed to be inclined at an angle ∅ to the horizontal, and the total height moved by the fluid is Lsin∅. The flow is in the x-axis, and at some point, the full-time Navier Stokes equation is considered in which it is assumed that the length of the tube L is much larger than its radius r ($L \gg r$).

2. Basic Equations of Blood Flow

2.1. Continuity Equation

The equation is also called the mass conservation equation. It is derived from the law of conservation of mass, which makes the assumption that mass cannot be generated or destroyed and that during the state flow process, the mass that has been stored in a regulated container remains unchanged. This means that the inflow into the controlled volume is equal to the outflow. The equation is given as;

$$\frac{\partial p}{\partial t} + \frac{\partial \rho u_i}{\partial u_i} = 0 \tag{1}$$

For in-compressible flow

$$\frac{\partial p}{\partial t} = 0 \tag{2}$$

and thus equation the continuity equation reduces to;

$$\frac{\partial \rho u_i}{\partial x_i} = 0 \tag{3}$$

which represents the rate of change of volume of a moving fluid element per unit volume.

2.2. Equation of Momentum Conservation

The principle of conservation of momentum asserts that all external forces acting on the control volume as well as the momentum flux into the control volume are equal. This implies that a closed system's overall momentum is constant. Suggesting that the total of dissipative viscous forces, change in pressure, gravity, and other forces acting on the fluid constitute the change in momentum in a tiny element volume of a fluid. It is possible to present the general equation of momentum in tensor form;

$$\rho\left(\frac{\partial u_i}{\partial t}\right) + U_i\left(\frac{\partial u_i}{\partial x_i}\right) = \rho f_i + \left(\frac{\partial \sigma_{i,j}}{\partial x_i}\right) + F_i \tag{4}$$

where $i = 1, 2, 3$ and $j = 1, 2, 3\ldots$ are summation variables along x, y, z directions respectively. The term ρf_i represents the body forces acting on the fluid and for this study, the forces considered are pressure forces and gravitational forces.

2.3. Poiseuille Equation

This equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(P + \rho g z + \frac{1}{2}\rho u^2\right) = RQ \tag{5}$$

where $u = \frac{Q}{A}$ is the average fluid velocity.

2.4. Tube Law

The law relates the pressure difference between two pints (trans mural pressure) to the cross sectional area of an elastic tube in this case the vein. [8] gives the equation as;

$$p - p_e = \varnothing\left(\frac{A}{A_o}\right) \tag{6}$$

the vein resembles two flattish membrane under tension and the function $\alpha\left(\frac{A}{A_o}\right)$ is defined as;

$$\alpha\left(\frac{A}{A_o}\right)=\begin{cases}K_c\left(1-\frac{A}{A_o}\right) & \text{if } 0<A\le A_o\\ K_E\left(\frac{A}{A_o}-1\right) & \text{if } A>A_o\end{cases} \tag{7}$$

with K_c and K_E are the vein's stiffens when it is distended and collapsed respectively.

3. Some General Governing Equations

The equation of conservation of mass for an incompressible fluid is given by

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{8}$$

where $A(x,t)$ is the cross-sectional area and Q is the volume flow rate, and x is the distance down the vein. If U is the average velocity, then $Q=UA$ and therefore equation 8 becomes,

$$\frac{\partial A}{\partial t}+\frac{\partial (UA)}{\partial x}=0 \tag{9}$$

Since the flow is in the *x*-direction, the momentum equations in the *y* and *z*-directions are ignored. form the fact that $r \ll L$, the equation of conservation of this flow becomes

$$\frac{1}{\rho}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial t}\right)=-\frac{\partial p}{\partial x}+F_x \tag{10}$$

for the purpose of the paper, the forces considered are resistive forces $R(A)$ and gravitational (gz) forces. in the above equation, u is the velocity with which the fluid moves and the viscosity of the fluid is neglected because it's assumed that the radius of the vein is too small compared to the distance through which the blood travels down the vein ($r \ll L$). L is the total length of the vein which is inclined to the horizontal, the above equation can be written as;

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\left(\frac{1}{\rho}\right)\frac{\partial p}{\partial x}+gZ \tag{11}$$

The schematic representation of the tube law in **Figure 2** illustrates the drastic change in the wall distensibility (the inverse stiffness)

$$D=\frac{1}{A}\left(\frac{\partial A}{\partial p}\right)=\frac{1}{A}\left(\frac{\partial P}{\partial A}\right)^{-1} \tag{12}$$

as the tube's cross-section changes to collapse from inflated. In the axisymmetrically inflated state, (i) in **Figure 2**, any deformation of the vein is accompanied by a stretching of the wall. Therefore to induce any change to the vessel's cross-sectional area, large changes in transmural pressure are required. The axisymmetric state of the vessel withstands small compressive loads ($0>P_{atm}>P_b$). However, when the trans-mural pressure falls below P_b, the tube buckles non-axisymmetrically, typically to a two-Iobed state. Once buckled, only the tube's small bending stiffness resists any further collapse. Hence, this means that the vessel undergoes large changes in the cross-sectional area when the transmural pressure changes slightly; see (ii)-(iii) in **Figure 2**. As the compression increases further, the vessel's opposite walls come into contact ($P_{atm} < P_{int}$) and the wall stiffness increases again as the two outer lobes need to be bent strongly in order to further reduce the cross-sectional area, (iv) in **Figure 2** Various curve fits which approximate the $P_{atm} = P(A)$ relationship sketched in **Figure 2** and which incorporate the correct solid-mechanical behavior (see [9]) have been suggested in the literature; see e.g. [8].

4. Bernoulli-Poiseulle

For steady flow in the gradually varying system, inertial forces are accounted by using the "Bernoulli-Poiseulle" equation given as;

$$\frac{\partial}{\partial x}\left(P+\rho gz+\frac{1}{2}\rho u^2\right)=-RQ \tag{13}$$

where $u=\frac{Q}{A}$ is the fluid average velocity. To asses the effect of inertia we need to find the change of $\frac{1}{2}\rho u^2$ as the blood flows from the uncollapsed section as we compare it to the gravitational pressure over a comparable length of the vein. Using the experimental results obtained by Peterson 2009 on jugular venous pooling during lowering of the head, the cross -sectional area of the jugular vein when the head is upright is approximated as (0.14 ± 0.04 cm^2) and (3.19 ± 0.04 cm^2) when lowered. Similarly, jugular flow is 14 ml·s^{-1} which gives the blood velocities of 80 cm·s^{-1} and 4.4 cm·s^{-1} in the collapsed and uncollapsed segments respectively.

This means that a change in $\frac{1}{2}\rho u^2$ is going from the collapsed segment to uncollpsed one. Laboratory experiments have shown that collapse only occurs on small distances. This means that it is possible that inertial is important in the flow of blood in jugular veins of giraffes. [8], gives some theories which are basic to for the steady flow in a tube of uniform properties and non-uniform properties with all possible cases that that can occur. In this chapter we apply this theory to the blood flow in jugular veins of giraffes. The mathematical model used is similar to the siphon mechanism but the inertial force are added.

4.1. Governing Equations and Mathematical Description

A long straight tube is considered through which fluid flows with internal pressure p and height z. the tube is inclined at an angle $\varnothing$ to the horizontal. the tube is assumed to be of uniform properties as in **Figure 1**.

The flow considered will be in the x-direction in which inertial properties/effects will be considered significant in the collapsible section since the collapse will not be gradual. It is noted that collapse of any tube can occur even at a very small distance, especially for thick tubes [10]. this makes inertia an important

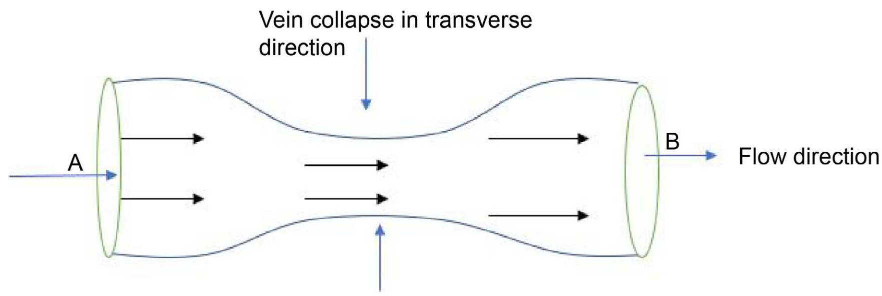


Figure 1. Collapsible vein under consideration in which the ends A and B are fixed.

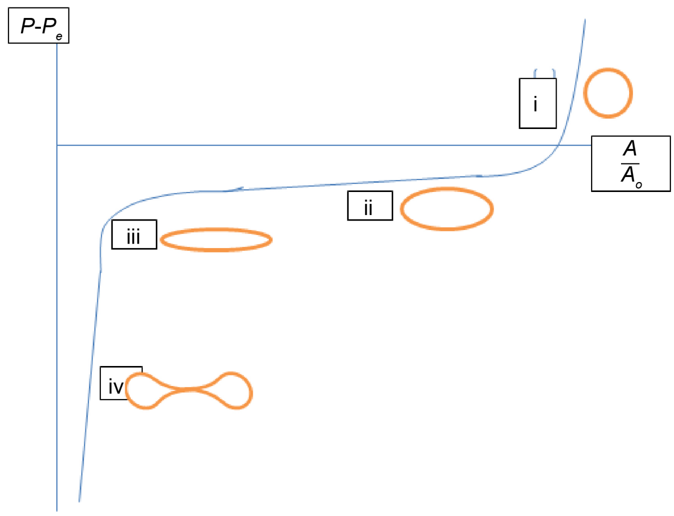


Figure 2. Schematic representation of tube law (The form of pressure area relationship used in is of the form of Ellad 1989).

property as thy evolve self excited oscillations.The sketch diagram for the flow is as shown above.

4.2. Equations Governing the Motion

The equation of conservation of mass is [8]

$$uA = Q \tag{14}$$

where Q is the volumetric flow rate $A(x)$ is the averaged cross-sectional area of the vein and $u(x)$ is the averaged velocity. using the above assumptions, the x-direction momentum equation can be given as the Bernoulli-Poiseulle equation as;

$$u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} - g = \frac{R(A)Q}{\rho} \tag{15}$$

where $R(A)$ is the viscous resistance given by

$$R(A) = \frac{8\mu A^{\frac{1}{2}}}{A^{5/2}} \tag{16}$$

If the tube remains circular as it collapses, the resistance term used is

$R = \dfrac{8\pi\mu}{A^2}$. Equation (16) is used whenever the vein is circular which is not the case in most cases. This means that a quasi steady resistance is developed and used whenever the tube cross sectional area changes in shape as given by [7] as

$$R(A) = \frac{4\mu\pi}{A^2}\left(\delta + \frac{1}{\delta}\right) \tag{17}$$

where δ is the ratio of the minor axis to the major axis of the so formed for the flow. The term $R(A)$ must be chosen such that the resistance increases more rapidly as the area decreases as it would be in a circular tube. The cross-sectional area is related to the internal pressure via a simple tube law

$$p - p_e = K_p F\left(\frac{A}{A_0}\right) \tag{18}$$

where p is the internal pressure, p_e is the external pressure and k_p is the quantity that represents the elastic properties and wall elasticity of the vein [8] and is given by;

$$K_p = \frac{E}{12\sqrt{1-\sigma^2}}\left(\frac{h}{r}\right)^2 \tag{19}$$

E is the Young's Modulus, σ is Poisson ratio and $\dfrac{h}{r}$ is the ratio of wall thickness to the vein radius when it is circular. To increase the effect of the surrounding tissues to the vein in K_p the value of $\dfrac{h}{r}$ is increased. The form of the tube law taken can be described as shown in **Figure 2**.

Positive trans-mural pressure difference.When $p > 0$ the vein is inflated and essentially circular as (i) in the diagram. The pressure difference across the vein is supported by hoop tension and the appropriate stiffness constant is that for the tension in the tube wall, in such state the vein has low compliant hence high transmural pressure is required to cause a given increase in area. If the trans-mural pressure nears zero, the cross-sectional area ceases to be circular and takes an elliptic shape as in (ii) in **Figure 2**. The tube now becomes compliant *i.e* a change in an area requires a curvature of the vein. the compliance of the vein falls again when highly collapsed (iii), making the cross section area to be a dumbell shape.

Negative trans-mural pressure difference. This occurs when $p < 0$, the tube is partially collapsed in the successive shaped shown in **Figure 2**. The pressure difference is primarily supported by the bending stiffness of the vein as in Equation (18)

$$F\left(\frac{A}{A_0}\right) = F(\alpha) = \alpha^n - \alpha^{\frac{-2}{3}} \tag{20}$$

with

$$\alpha = \frac{A}{A_0} \tag{21}$$

where for the veins $n=10$ this for is used to explain the different shapes that the tube exhibits under different conditions. It is to be noted that whenever the transmural pressure is zero, $\alpha \to 0$ and gives $\alpha = 1$. The tube; law used includes modification to include the effect of increased stiffness for $\alpha > 1$. Differentiating Equation (17) and with (20) gives

$$\frac{\partial p}{\partial x} - \frac{\partial p_e}{\partial x} = K_p \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial x} \tag{22}$$

Substituting for $\frac{\partial p}{\partial x}$ in Equation (15) to get;

$$u\frac{\partial u}{\partial x} + \frac{1}{\rho}\left(K_p \frac{\partial F}{\partial \alpha}\frac{\partial \alpha}{\partial x}\right) - g = \frac{R(A)Q}{\rho} \tag{23}$$

upon substitution of Equation (14) into (23) we find

$$\frac{\partial A}{\partial x} = \frac{A}{\rho} \cdot \frac{g\rho - R(A)Q}{c^2 - u^2} \tag{24}$$

where

$$c^2 = \frac{K_p}{\rho}\frac{\alpha \partial F}{\partial \alpha} \tag{25}$$

c^2 is proportional to the gradient of the pressure-area relationship and signifies tube (vein) stiffness at any given value α. It may actually be said to be the speed of propagation of small amplitute pressure wave along the tube. Hence the wavespeed is given by

$$c^2 = \left(\frac{K_p}{\rho}10\alpha^{10} + \frac{3}{2}\alpha^{\frac{-3}{2}}\right)^{\frac{1}{2}} \tag{26}$$

For the purpose our study we consider an inclined vein/tube such that $g = g\sin\phi$, hence Equation (24) becomes;

$$\frac{\partial A}{\partial x} = \frac{A}{\rho} \cdot \frac{g\rho \sin\phi - R(A)Q}{c^2 - u^2} \tag{27}$$

4.3. Non-Dimensiolization

The following parameters for non -dimensionalization are used $\alpha = \frac{A}{A_0}$, $\xi = \frac{x}{L}$, $C = \frac{c}{c_o}$, $U = \frac{u}{c_0}$, $R(\alpha) = \frac{R(A)}{R(A_0 r)}$. Where c_o ($c_o = \frac{K_p}{\rho}$) is the characteristic wave speed and viscous resistance is $\hat{R} = \alpha^{\frac{-5}{2}}$. This variable is only valid for uniform properties otherwise for nonuniform properties will vary with longitudinal distances in which they are defined as $\alpha = \frac{A(\xi)}{A_o(\xi)}$ and

$$c_0^2 = \frac{K_p(\xi = 0)}{\rho} = \frac{K_{po}}{\rho};$$

$$\begin{cases} \frac{\partial A}{\partial x} = \frac{\partial A_0 \alpha}{\partial \xi}\frac{\partial \xi}{\partial x} = \frac{\alpha}{L}\frac{\partial A_o}{\partial \xi} + \frac{A_0}{L}\frac{\partial \alpha}{\partial \xi} \\ \frac{\partial A_0}{\partial x} = \frac{\partial \left(A/\alpha\right)}{\partial \xi}\frac{\partial \xi}{\partial x} = \frac{1}{L\alpha}\frac{\partial A}{\partial \xi} - \frac{A}{\alpha L}\frac{\partial \alpha}{\partial \xi} \\ \frac{\partial u}{\partial x} = \frac{\partial \left(c_0 U\right)}{\partial \xi}\frac{\partial \xi}{\partial x} = \frac{c_0}{L}\frac{\partial U}{\partial \xi} \end{cases} \tag{28}$$

4.4. Supper and Sub-Critical Flow

Substituting Equation (22) to (21) after letting $c^2 = k_p \alpha \frac{\mathrm{d}F}{\mathrm{d}\alpha}$ (where c is the wave speed of blood) we get;

$$\frac{\partial \alpha}{\partial \xi} = \frac{\rho L g \sin\phi - R(A)Q}{c_0^2 \rho \left(C^2 - U^2\right)} \tag{29}$$

[8] suggests that the speed index $S = \frac{U}{C}$ be introduced in the equation above, such that it is given by;

$$\frac{\partial \alpha}{\partial \xi} = \frac{L\left(\rho g \sin\phi - R(A)Q\right)}{c_0^2 \rho C^2 \left(1 - S^2\right)} \tag{30}$$

The behavior of flow characteristics depends on the numerator and sign of the denominator. If $S > 1$ then it is said to be supercritical and it is sub-critical if $S < 1$ otherwise it is critical.

$$\frac{\partial \alpha}{\partial \xi} = \frac{L\left(\rho g \sin\phi - R(A)Q\right)}{c_0^2 \rho C^2 \left(1 - S^2\right)} \tag{31}$$

The speed index S acts as Froude Number in shallow water waves and Mach number in gas dynamics. If the numerator in Equation (31) is zero, then the gravitational and resistive force balance (are equal) *i.e.*

$$Lg\rho \sin\phi = R(A)Q \tag{32}$$

Its clear that there exists a point $\frac{\mathrm{d}\alpha}{\mathrm{d}\xi} = 0$ where there is no change in area and at this point the resistive forces are equal to gravitational forces ($\rho g \sin\phi = R(A)Q$), this paint is called (α_{lim}) [7] and is given by

$$\alpha_{lim} = \left(\frac{8\pi Q}{g \sin\phi \rho A_0^2}\right)^{\frac{2}{5}} \tag{33}$$

Similarly, the denominator approaches zero when the speed index is 1 ($S = 1$), *i.e.* $\alpha = \alpha_1$ such that

$$Q = A_o \alpha_1 c(\alpha_1) \tag{34}$$

The cross-sectional area (α) and flow rate (Q) at which both Equations (32) and (33) are satisfied plays a critical role in the theory ($\alpha_1 = \alpha^*$). The function

$\frac{\mathrm{d}\alpha}{\mathrm{d}\xi}$ in Equation (30) depends only on α and therefore the stability of the stationary points in each region is determined by the sign of the function at different values of α. This gives different cases as shown below.

4.4.1. When There Are No Gravity Forces

If $\rho g \sin\phi = 0$, then the head of the giraffe is in the horizontal position (same level as the heart) and at this point, the gravitational forces have minimal influence on the blood flow, and equation 29 becomes;

$$\frac{\partial\alpha}{\partial\xi} = \frac{-R(A)Q}{c_0^2 \rho C^2 \left(1-S^2\right)} \tag{35}$$

The speed of the waves affects the flow. Given that the wave speed c depends on the stiffness of the tube, it follows that C is substantially larger in regions where the vessel is inflated than in those where it is compressed without the opposing walls coming into contact. Equation (29) predicts an intriguing event known as "choking" (in comparison to a corresponding phenomenon in gas dynamics). Assume that the average fluid velocity (u) at the upstream end of the tube is lower than the wave speed (C). Since Equation (29) assumes that $\mathrm{d}\alpha/\mathrm{d}xi < 0$. Continuity then requires that $\mathrm{d}\xi/\mathrm{d}\tau > 0$, *i.e.* the flow is accelerated in the stream-wise direction. Provided the tube is long enough, a location at which the 'speed index' $S = U/C \to 1$ is approached and thus $\mathrm{d}\alpha/\mathrm{d}\xi \to -\infty$, which violates the long-wavelength assumption in the 1D model. This situation is physically unrealizable, Meaning that steady flows are impossible (according to this simple model) if the flow rate is so large enough that U approaches C anywhere along the tube. Alternatively, if $S > 1$ at any point along the tube, then the downstream conditions variations will not propagate upstream (because small-amplitude waves travel at speeds $U \pm C$); The wave-speed process of flow limiting, which is thought to function in the major airways of the lung, is explained by this (Dawson and Elliot 1977; Elliot). Several authors argued that the supercritical flow $U > C$ that occurs at some point along the tube as fluid flow might coincide with the onset of self-excited oscillations [11]. However, Gavriely *et al.* (1989) showed that while flow-induced oscillations occurred only when the flow was limited in tubes with large wall inertia, the initial flow speed may be as low as S 0.3 (although in the definition of wave speed, they did not account for wall mass). Later experiments by [10] and computations by [12] has also cast doubt over a causal link between flow limitation and self-excited oscillations.

In the framework of the 1D model, the predictions of elastic jumps occur in situations where supercritical (S > 1) flow is generated within the collapsible tube. The change in the cross-sectional area increases the fluid velocity u while the associated reduction in wall stiffness y reduces the wave speed U. The ensuing supercritical flow is still governed by (4.17), which now predicts that $\mathrm{d}\alpha/\mathrm{d}\xi > O$. Hence the cross-sectional area α increases and, provided the tube is long enough, the reduction in fluid velocity U now brings in a situation in which $\mathrm{d}\alpha/\mathrm{d}\xi \to +\infty$ at some point.

As in the case of choking, this violates locally the model's assumptions. However, a transition region (shock-like), in which the flow speed reduces from super to subcritical, develops upstream of the point where $\mathrm{d}\alpha/\mathrm{d}\xi \to +\infty$ would occur. The application of jump conditions (similar to those used in gas dynamics) across the thickness of the elastic jump (Oates 1975, Shapiro 1977, Cowley 1982) establishes how the flow changes as it passes from the supercritical to the subcritical regime. The downstream boundary conditions, which can only affect the subcritical area of the flow downstream of the jump, dictate where the elastic leap will occur. Standing waves are caused by longitudinal bending or tension and are dampened by viscous effects. They can arise either upstream or downstream of elastic jumps (MeClurken *et al.* 1981; Cowley 1983) [10].

4.4.2. Presence of Gravitational Forces

The terms $\rho g \sin\phi$ give the effect of gravitational forces on the flow of blood in the vein. In the presence of gravity, choking can be avoided because smooth transitions from sub- to supercritical flows are possible if the flow becomes critical $U = C$) at a location where $g = Ru$. other physiologically relevant scenarios in which smooth transitions through $u = c$ are possible are listed by [8]. They are (i) axial variations of the vessel's elastic properties (corresponding to flow in tapered elastic tubes), (ii) spatial variations of the external pressure (representing, for example, a localized compression as in sphygmomanometry), and (iii) variations of the vessel's undeformed cross-sectional area.

In particular, properties (i) and (ii) have been used to describe flow limitation in the lung (for instance, by Elad *et al.* 1987). During forced expiration, a sub—to supercritical flow transition arises due to non-uniform airway properties, and a super—to subcritical transition can take place further downstream via an elastic jump. The elastic jump's position is impacted by changes to the downstream boundary conditions, but the overall flow is unaffected. The jugular vein of the giraffe serves as a similarly analogous example. According to tests, the vein is substantially contracted while the giraffe is standing upright, and the flow across it is supercritical. According to [7], the downstream flow rate Q and the vein's downstream cross-sectional area (located at the confluence with the distended superior vena cava) are connected to where an elastic leap occurs in the vein. Additionally, they demonstrated how a rise in Q causes the elastic leap to travel farther downstream and came to the conclusion that, in a constant flow, the value of $Q_{\max}$ cannot be greater than the value of Q for which the elastic jump occurs at the downstream end of the jugular vein. If $\rho g \sin\phi$ is negative, then the flow is upstream and equation 3.16 becomes

$$\frac{\partial \alpha}{\partial \xi} = \frac{L\left(\rho g \sin\phi + R(A)Q\right)}{c_0^2 \rho C^2 \left(S^2 - 1\right)} \tag{36}$$

which changes the meaning of the values associated with S. therefore, the flow becomes supercritical when $S < 1$ and sub-critical when $S > 1$.

If $\rho g \sin\phi$ then the flow of blood is downstream then $S > 1$ the flow is supercritical and subcritical when $S < 1$.

4.4.3. Without Downstream Boundary Conditions

Case 1: When $Q > Q^*$ (Supercritical flow)

The flow is similar as when S>1 since the values of α are less than those of α_1 and the flow is supercritical. The signs of the function $\frac{\partial \alpha}{\partial \xi}$ are as in Figure 3.

From Table 1 we observe that all values that lies within the region area $\alpha < \alpha_1$ tends to move to point α_{lim} while the values within the region $\alpha > \alpha_1$ moves away from the same point. This means that the solution trajectories that are subjected to the upstream boundary conditions $\alpha_o < \alpha_{lim}$ approaches α_{lim} from below and once the point α_{lim} is reached, there is no further change. If, the boundary condition of the upstream is such that $\alpha_{lim} < \alpha_{lim} < \alpha_1$, then the solution trajectories approaches α_{lim} in which the the vessel becomes collapsed uniformly in the remaining entire length. For this state, the flow remains supercritical throughout, and the speed index S approaches S_{lim}.

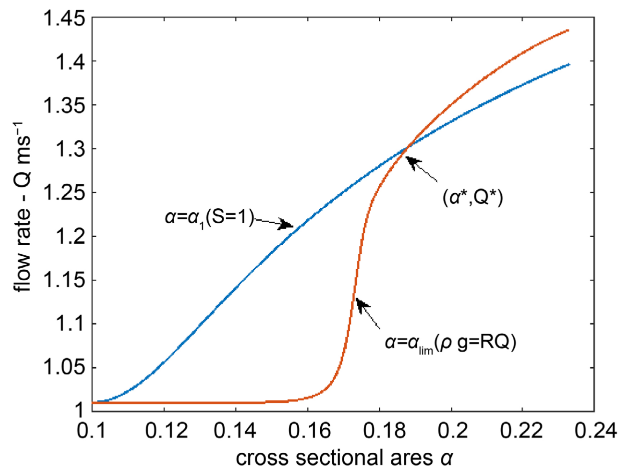


Figure 3. Plots of flow rate Q versus cross-sectional area α for $\alpha = \alpha_{lim}$ (when $\rho g = RQ$) and $\alpha = \alpha_1$ (when $S = 1$).

Table 1. Summary of signs for the flow region $Q > Q^*$ for the function $\frac{\partial \alpha}{\partial \xi}$ of Equation (35).

	$\alpha < \alpha_{lim}$	$\alpha_{lim} < \alpha_{lim} < \alpha_1$	$\alpha > \alpha_1$
$Lg\rho \sin\phi - RQ$	negative (−ve)	positive (+ve)	positive (+ve)
$\frac{\partial \alpha}{\partial \xi}$	positive (+ve)	negative (−ve)	positive (+ve)
positive $1 - S^2$	negative (−ve)	negative (−ve)	positive (+ve)

For the Boundary condition $\alpha > \alpha_1$, the solution moves away from α_1 making the cross-sectional area to continue growing. for this case, the flow has moved to sub-critical and the speed index approaches zero. Since the change is gradual, the presence of inertial forces has got very little influence.

Case 2: When $Q < Q^*$ (Critical flow)

As in case 1 above, the signs of the function $\frac{\partial \alpha}{\partial \xi}$ are summarized as in Figure 4.

From Table 2, three possibilities arise. We have seen in case1 that α_1 acts like an attractor, then any solution trajectory that has upstream boundary condition as $\alpha_o < \alpha_1$ will tend towards α_1. The flow starts with supercritical ($S > 1$) but becomes critical ($S = 1$) before the friction-gravity balance is achieved. This condition means that the gradient function of $\frac{d\alpha}{d\xi}$ becomes infinite. This

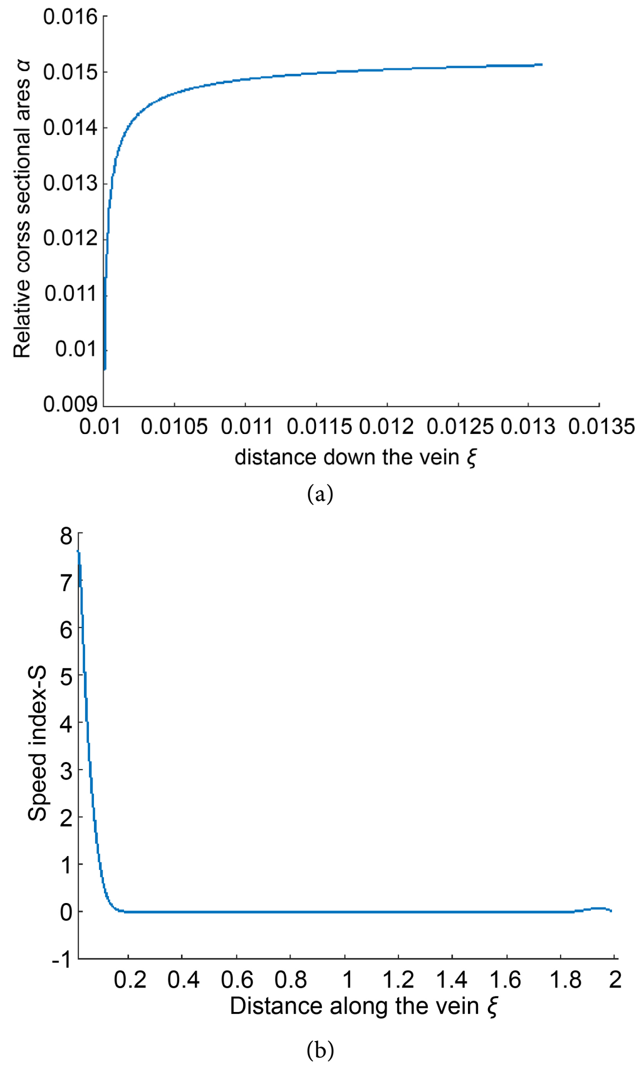


Figure 4. Plots of α and S against the non-dimensional distance ξ along the vein for the case $Q > Q^*$ at different range of values of α at $\xi = 0$.

Table 2. Summary of signs for the flow region $Q < Q^*$ for the function $\frac{\partial \alpha}{\partial \xi}$ of Equation (35).

	$\alpha < \alpha_{lim}$	$\alpha_{lim} < \alpha_{lim} < \alpha_1$	$\alpha > \alpha_1$
$Lg\rho \sin\phi - RQ$	negative (−ve)	negative (−ve)	positive (+ve)
$\frac{\partial \alpha}{\partial \xi}$	positive (+ve)	negative (−ve)	positive (+ve)
$1 - S^2$	negative (−ve)	positive (+ve)	positive (+ve)

condition is very difficult to be achieved hence the flow rate with the upstream area condition cannot occur. Actually, the fluid speed exceeds wave speed hence not possible for the wave to propagate upstream hence the need to change the conditions at the inlet. At this point, an elastic jump (similar to a shock in gas dynamics and hydraulic jumps in the open channels) is experienced.

Taking the upstream boundary condition as $\alpha_{lim} < \alpha_{lim} < \alpha_1$, the attraction to α_1 is experienced and the flow fast becomes critical before friction-gravity forces balances. The infinite negative gradient achieved is not possible and at this point, the flow is said to be chocked. The flow approaches the critical point at $S = 1$ from the initial sub-critical point. The wave now propagates upstream adjusting the inlet conditions and avoiding the choked condition. Actually, the conditions will adjust themselves in case the flow becomes critical and resistivity forces balance gravity so that the flow goes through the sub-critical to supercritical transitions smoothly. The choking state provides a mechanism in which conditions in collapsible vessels determine the flow rates upstream (an important mechanism that provides for the flow in air limitation in the airways (lungs)).

Case 3: When $Q = Q^*$ (Sub-critical flow)

As in case 1 above, the signs of the function $\frac{\partial \alpha}{\partial \xi}$ are summarized as in Table 3 and Figure 5.

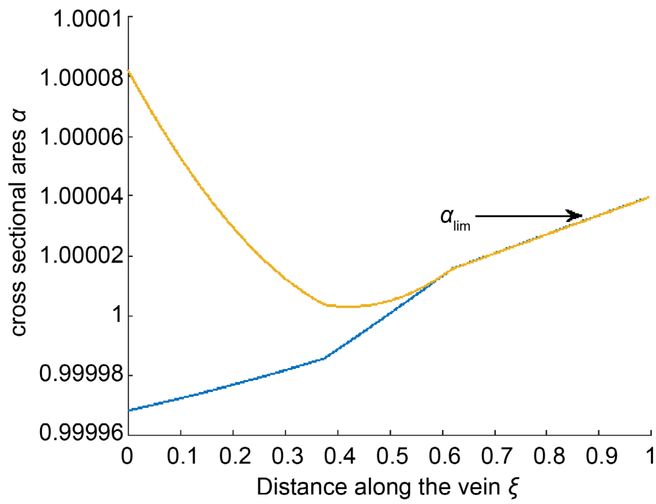
This is the flow region $\alpha^* = \alpha_{lim} = \alpha_1$. The gradient is always positive hence no attractors, and the area is always growing. If the upstream boundary condition is supercritical ($\alpha_o < \alpha^*$). The area increases such the $\alpha_0 = \alpha^*$ at a finite distance from the inlet. For this condition, the values of the numerator and denominator approach zero at the same time providing a smooth transition from super to sub-critical flow thereby avoiding an elastic jump. If $\alpha > \alpha^*$, the flow is said to be increasingly sub-critical as is also for the case $\alpha_o > \alpha^*$

4.4.4. With Downstream Boundary Conditions

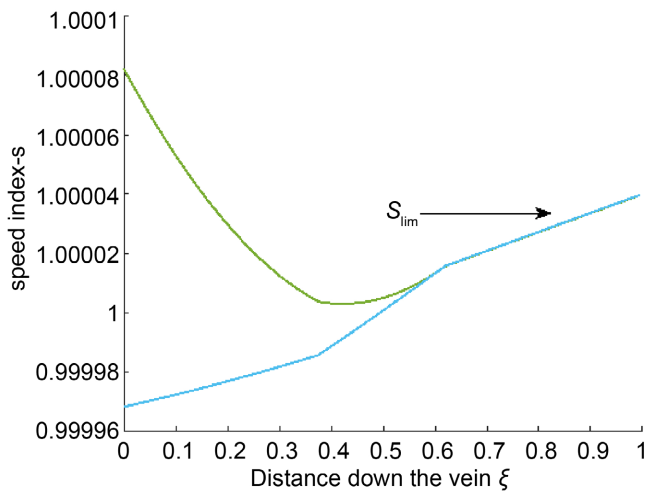
Downstream boundary conditions are important in the study of collapsible tubes. In laboratory experiments, it is possible to hold a given number of parameters constant as others are varied. In our introduction, we discussed the circulatory system which applies to both giraffes and human beings. In the discussion, we

Table 3. Summary of signs for the flow region $Q < Q^*$ for the function $\frac{\partial \alpha}{\partial \xi}$ of Equation (35).

	$\alpha < \alpha^*$	$\alpha > \alpha^*$
$Lg\rho \sin\phi - RQ$	negative (−ve)	positive (+ve)
$\frac{\partial \alpha}{\partial \xi}$	positive (+ve)	positive (+ve)
$1 - S^2$	negative (−ve)	positive (+ve)



(a)



(b)

Figure 5. Plots of α and S against the non-dimensional distance ξ along the vein for the case $Q > Q^*$ at different range of values of α at $\xi = 0$.

stated the importance of the right atrium pressure which is kept constant most of the time. Hence need to investigate the relationship/effect it has to blood flow and collapsibility of the jugular vein. If we consider the flow rate $Q > Q^*$, the solutions obtained as above are either wholly sub-critical or wholly supercritical if no downstream boundaries are specified. A wholly supercritical flow does not allow information to propagate upstream via pressure waves due to the high speed of the fluid which exceeds that of the wave. For wholly sub-critical flows, pressure waves can propagate upstream. This means that a fixed downstream pressure together with a steady flow rate will ensure that the conditions at the upstream boundary are adjusted so that the correct pressure at the outlet is reached. In summary, it can be said that it is either the elastic jump which is required to satisfy the downstream boundary condition or the flow is sub-critical and hence the upstream pressure can be adjusted allowing the downstream boundary condition to be satisfied.

The formation of elastic jumps (An elastic jump is a transition region in which the quasi one dimensional assumption has broken down and lies between two regions in which the assumption is valid) plays a key role in downstream boundary conditions. [13] and [11], the equations of momentum and conservation of mass can be written relating the jump, upstream, and downstream boundary.

Consider an elastic jump moving steadily with constant shape and in a frame fixed in the jump. Writing α_u, U_u and p_u for the no dimensional upstream area, velocity and pressure respectively and α_d, U_d and p_d for the corresponding values downstream, the equation of mass conservation can be written as;

$$\alpha_u U_u = \alpha_d U_d = Q \tag{37}$$

and the momentum equation is

$$\alpha_u U_u^2 - \alpha_d U_d^2 = \alpha_d p_d - \alpha_u p_u = \int_{\alpha_u}^{\alpha_d} p(\alpha)\,\mathrm{d}\alpha \tag{38}$$

Using the tube law as in Equation (38) and pressure p non-dimensionalized such that

$$p = \frac{1}{K_p}\left(p_e + K_p F(\alpha)\right) \tag{39}$$

substituting Equation (38) and (37) into (37) we get

$$\chi(\alpha_u) = \chi(\alpha_d) \tag{40}$$

where

$$\chi(\alpha) = \frac{Q^2}{\alpha} + \alpha F(\alpha) - \Gamma(\alpha) \tag{41}$$

$$\Gamma(\alpha) = \int_{\alpha_u}^{\alpha_d} p(\alpha)\,\mathrm{d}\alpha \tag{42}$$

The steady flow rate is known and conditions downstream are known. This means if we solve Equation (41) the upstream conditions are obtained. From experimental results of [14] elastic jumps generated on collapsible tubes concurred with the results obtained.

4.5. Solution

The governing Equation (19) is an ordinary differential equation which is solved using the following boundary conditions:

$$\alpha(\xi=0)=\alpha_i, \alpha(\xi=1)=\alpha_d \quad (43)$$

The differential equation is solved using a numerical fourth order Range-Kutta method. The integration is carried out backward for the sub-critical flows (This means we start with the downstream end and calculate the solution to the upstream) allowing the specifications of the downstream conditions. The upstream condition is part of the solution. For wholly supercritical flows, the upstream condition is specified and integration is done forward in ξ. The inclusion of elastic jump is done if a sub-critical boundary condition is required at the outlet.

Solution Procedure

Start with the upstream end with the supercritical flow and integrate forward till some arbitrary point ξ introduces an elastic jump. At this point, the cross-sectional area upstream of the jump is known and 40 is used to calculate the jump and gives the sub-critical flow. After this, the integration is continued with the same flow rate. Figure 7 shows an initially supercritical flow followed by an elastic jump and the sub-critical flow in which the vessel expands until the outlet.

From the Figure 6, the position of the jump is determined by the downstream boundary condition. When an arbitrary jump is introduced at some point ξ_1 means that the area at the outlet α_d^1 will either be less than or greater than the required boundary condition ξ_d. At this point a comparison is made between α_d^1 and ξ_d and one can increase or decrease ξ_1 to ξ_2 at the upstream for supercritical flow. This process is repeated with now the elastic jump shifted to ξ_2 to give a downstream area α_d^2. This procedure is now repeated until $\alpha_d^n - \alpha_d$ is sufficiently small. For the regions where the flow tends to be critical flow, the integration is stopped as long as the speed index sufficiently gets close to 1. The pressure waves for such flows will propagate upstream thus adjusting the inlet conditions and thus the flow smoothly transits to supercritical flow. Such flows are actually time-dependent.

5. Results

The specific morphological data that is available for the giraffe jugular vein together with various parameters are used to show that the flow limitation is theoretically possible. To estimate some of the parameters we use the pressure equation used in the siphon mechanism experiment given as;

$$P_1-(P_2-\rho gh)=LRQ \quad (44)$$

5.1. Possible Mechanism for Flow Limitation

As discussed earlier, in normal circumstances the pressure in the right atrium in

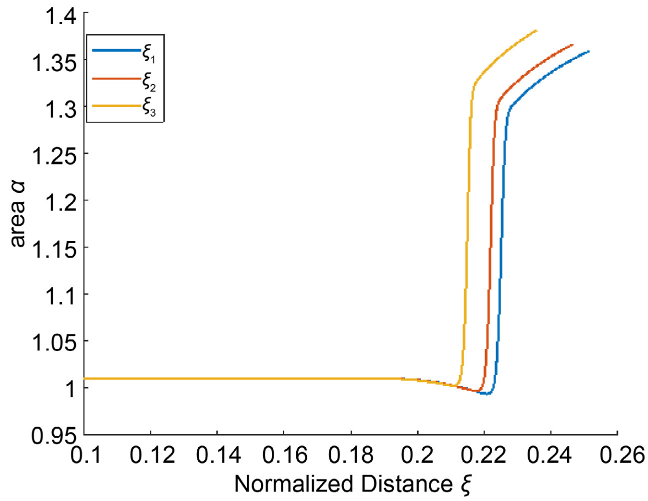


Figure 6. Plots of cross-sectional area versus distance down the vein showing how the location of the jump depends on the downstream boundary conditions.

giraffes and hence in the Superior vena cava (SVC) is close to the atmospheric pressure. Using the tube law, this gives a fixed downstream value α_d for the area. Taking the SVC pressure as 1 mm Hg (130 Pa), greater than the external pressure P_e, then the tube law for the uniform tube gives $\alpha_d \approx 1.39$. The tube is distended and stiff and Equation (26) gives the wave speed as $c(\alpha) = 1.16\ \text{m} \cdot \text{s}^{-1}$. Now for each flow value of Q, Equation (13) must be solved subject to that boundary conditions.

As in our previous discussion, we take a case when $Q > Q^*$, for any given flow rate Q, the flow is either wholly sub-critical or wholly supercritical. The pressure (area) of the right atrium is such that the flow is likely to be sub-critical, so if supercritical upstream, then along the vein, there must be an elastic jump to return the flow to sub-critical velocity. The location of the jump depends on the downstream boundary as in **Figure 7**.

Therefore it is possible to achieve the required downstream area either with the solution or an initially supercritical flow followed by a jump. From equation 44, it is not possible for the case of the jugular vein which is collapsed since the condition requires a steadily increasing area. This leaves us to be concerned with the focus on the solution of the supercritical-jump-sub-critical flow for the region.

From Equation (33) we find that α_{lim} increases with increased values of Q.

Figure 7 plots of cross-sectional area versus distance down the vein showing various flow rates Q in which all lead to downstream area. From the figure, any value of $Q_1 > Q^*$, the area α_{lim} is reached within a short distance from the inlet which means that in order to reach the required downstream area, a jump is located at some point x_1 along the vein. If the jump is located before this point, then the area at the outlet would be greater than the required one. Again if this point is located at a point beyond x_1 it will give an area smaller at the outlet as in

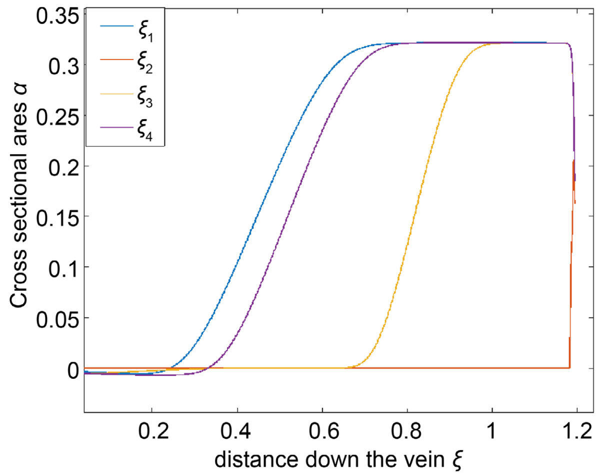


Figure 7. Plots of cross-sectional area versus distance down the vein showing various flow rates (where $Q^* < Q_1 < Q_2 < Q_3$).

Figure 6. As the flow rate is increased, the value of α_{lim} increases thereby increasing the values of the jump further downstream. The flow rate continues to increase until a point Q_2 is reached for which the jump has to be located in order to achieve the required outlet area.

For flow rates $Q < Q^*$ flows are sub-critical of chocked depending on the inlet area. Using equation 33, we see that α_{lim} is smaller for small values of Q and from equation **Figure 3** it is clear that for these flow rates, $\alpha_{lim} > \alpha_1$. Hence for any inlet area greater than α_{lim}, any downstream area greater than α_{lim} can be reached sub-critically. The flow rate estimates in the jugular vein used are one used by (Brondum E, 2013) which is approximated as 1.67 cms^{-1}. Using the data given as in Equation (44), **Figure 3** shows $Q^* \approx 1.3\,\text{cm}\cdot\text{s}^{-1}$. From this model, we realize that gravity-friction plays a major role but it is absent in the expiratory flow. Other numerical values of downstream area α_d are as shown in the figures.

5.2. Physiological and Parameter Variations

Results for Giraffe

In the computation of the results above we have used the following morphological data as Brondum *et al.* 2009.

- $A_O = 5\,\text{cm}^2$, $Q = 1.67\,\text{cm}\cdot\text{s}^{-1}$ and $Q = 1.3\,\text{cm}\cdot\text{s}^{-1}$;
- $K_p = 5\,\text{Pa}$, found from equation 19 by taking the Poisson ratio as $\sigma = 0.5$, (for in-compressible materials) the Young's modulus used is the same as the one used by [7];
- The length L of the vein is given as 2 m;
- Blood density used is $\rho = 1.03\times10^3\,\text{kg}\cdot\text{m}^3$ and $\mu = 0.004\,\text{Pa}\cdot\text{s}^{-1}$ while $g = 9.8\,\text{m}\cdot\text{s}^{-2}$ and the angle of inclination is varied as $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$;
- The central venous pressure used is as approximated by (Brondum 2008) and is equal to 1 mmHg (133 Pa). This is the pressure assumed to be external

pressure P_e neglecting any other pressures.

Figure 8 shows the results of the maximum achievable flow rate against the downstream cross-sectional area. Actually, the morphology of the jugular vein affects the pressure difference in that changing the perivascular pressure p_e affects the downstream boundary condition. In the computation, it has been assumed that the pressure at the right atrium and hence in the superior vena cava is 1Hg greater than the external pressure. If the external pressure is changed, the transmural pressure changes hence the downstream area α_d will be smaller than 1.0. In this figure, it is clear that the maximum flow rate decreases with the decrease in the downstream area. Therefore, the effect on the Maximum flow rate of increasing the external pressure decreases the maximum flow rate thus making it possible for flow limitation to occur hence the conditions of the jugular vein determine the blood flows to the head of the giraffe.

For Humans

Actually, the flow limitation calculations above can be performed for the human jugular vein. fo this section we use the parameters and geometric properties as given by comoet 1997. As in the giraffe jugular vein, the following properties are adapted,

- $A_O = 9\,\text{cm}^2$, $Q = 6.67\,\text{cm}\cdot\text{s}^{-1}$ and $Q = 6.3\,\text{cm}\cdot\text{s}^{-1}$ as estimated to be the flow rate through the carotid artery;
- $K_p = 0.5\,\text{Pa}$, found from equation 19 by taking the Poisson ratio as $\sigma = 0.5$, (for in-compressible materials) the Young's modulus used is the same as the one used by [7]. This gives young's modulus as $E \approx 4\times10^5\,\text{Pa}$;
- The length L of the vein is given as 2 m;
- Blood density used is $\rho = 1.03\times10^3\,\text{kg}\cdot\text{m}^3$ and $\mu = 0.004\,\text{Pa}\cdot\text{s}^{-1}$ while $g = 9.8\,\text{m}\cdot\text{s}^{-2}$ and the angle of inclination is varied as $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$.

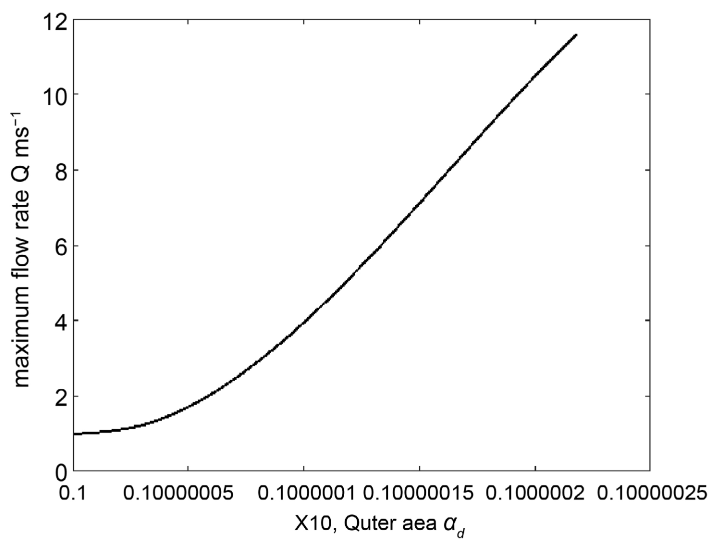


Figure 8. Plots of computed maximum achievable flow rate against downstream cross-sectional area α_d.

Using Equation (33), if we use $\phi = \frac{\pi}{2}$ the above parameters give the critical value $\alpha_{lim} = 0.071$ with corresponding values of $Q = 1.64\,\text{m}\cdot\text{s}^{-1}$ in an upright posture. Since the human jugular vein is known to be collapsed while standing upright, it is assumed that the steady solutions are most likely to be in a Super-critical-jump-supercritical state.

6. Conclusions

This paper models the giraffe jugular veins as a uniform collapsible tube existing from some rigid skull. The equations governing one-dimensional steady flow through such a tube for various conditions have been developed. The effect of inertial and angle of inclination which has not been discussed previously has been included (as in the previous paper by [15]). It has been shown that different flows for a uniform tube (vein) are possible. However, this flow matches that of a jugular vein which is supercritical and the steady solution has been given by the balance between the driving forces of gravity and the viscous resistance to the flow. The flow at the right atrium of the heart must be sub-critical for a fixed right atrium pressure which means that an elastic jump is required to return the flow to sub-critical from the supercritical flow upstream. This type of relationship gives rise to flow limitation.

At the same time, given any right atrium fixed pressure, there exists a maximum flow rate which when exceeded, the boundary conditions of the flow do not hold (boundary conditions at the right atrium are not satisfied) hence making the steady flow impossible. This mechanism of flow limitation is slightly different from the other one in that causes airways through forced expiration. From the observation made, it is clearly shown that there is an intravascular pressure difference with a change in height.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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