Open Journal of Applied Sciences, 2022, 12, 1547-1558
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

A New Mathematical Justification for the Hydrodynamic Equilibrium of Jupiter

Mahammad A. Nurmammadov

Shemakhi Astrophysical Observatory Named after N. Tusi of the Ministry of Science and Education of the Republic Azerbaijan, Shemakha, Azerbaijan

Email: nurmamedov_55@mail.ru

How to cite this paper: Nurmammadov, M.A. (2022) A New Mathematical Justification for the Hydrodynamic Equilibrium of Jupiter. *Open Journal of Applied Sciences*, **12**, 1547-1558.
https://doi.org/10.4236/ojapps.2022.129105

Received: August 25, 2022
Accepted: September 23, 2022
Published: September 26, 2022

Abstract

In this paper, the case of Jupiter being found in hydrodynamic equilibrium is for the first time investigated solely by mathematical methods. With the help of the hydrodynamic method, formulas of energy balance for oval and vortex are found, which are summed as permanent kinetic energy and constantly provide equilibrium for the stable rotational movements of Jupiter. To find the total kinetic energy of the oval and vortex in turbulent mode, Green's function methods with special definitions and flow functions that describe the movement of the vortex are applied. The results are expressed in lemmas and theorems. For the hydrodynamic equilibrium of Jupiter, the necessary and sufficient conditions for the preservation of the cyclone and the anticyclone are mentioned. The relationships between the angular velocity and the gradient of pressure and the Corolias parameter are also given. The Rossby number is given for steady rotational motion. These facts show the existence of necessary and sufficient conditions for maintaining the stability of rotational motion and prove the hydrodynamic equilibrium of Jupiter. In this case using stream function and constructing generalized Green's function and accordance energy conservation laws, the hydrodynamic equilibrium of Jupiter is proved.

Keywords

GRS, Jupiter, Cyclones, Anticyclones, Circulation, Rotation

When I die, I go to heaven and ask God two questions:
What is quantum electrodynamics and what is turbulence?
In terms of first, I'm more optimistic.

Horace Lamb, Hydrodynamics (1932).

1. Introduction

It is known (see [1] [2] [3] [4] [5]) that the equatorial plane of the planet is close to the plane of its orbit (the inclination of the axis of rotation is 3.13˚ against 23.45˚ for the Earth, so there is no change of seasons on Jupiter). Jupiter orbits its axis faster than any other planet in the solar system. The rotational period at the equator is 9 h 50 min 30 s, and at mid-latitudes it is 9 h 55 min 40 s. Due to its rapid rotation, Jupiter's equatorial radius (71,492 km) is 6.49% larger than the polar radius (66,854 km); thus, the planet's contraction is 1: 51.4%. The model of the internal structure of Jupiter is contained under the clouds, there is a layer of a mixture of hydrogen and helium with a thickness of about 21 thousand km with a smooth transition from the gaseous to the liquid phase. This is followed by a layer of liquid and metallic hydrogen with a depth of 30 - 50 thousand km. Inside Jupiter there may also be a solid core with a diameter of about 20 thousand km. Inside may be a solid core with a diameter of about 20 thousand km. The following model of Jupiter's inner structure has been recognized: The atmosphere is divided into three layers: the outer layer consisting of hydrogen; the middle layer consisting of hydrogen (90%) and helium (10%); and the bottom layer consists of hydrogen, helium and ammonia impurities, ammonium hydrosulfide and water, which form three layers of clouds: above, clouds of frozen ammonia (NH_3). It has a temperature of approximately 145˚C and a pressure of approximately 1 atm; below, a cloud of ammonium hydrosulfide (NH_4HS); and at the bottom, water ice and possibly liquid water. The pressure in this layer is about 1 atm and the temperature is about 130˚C (143 K). Below this level, the planet is opaque. There is a layer of metallic hydrogen. The temperature of this layer varies from 6300 to 21,000 K and the pressure from 200 to 4000 GPa. The other part consists of the Stone Core. The question of the long-term existence of GRS and many processes are able to disperse atmospheric vortices similar to the Great Red Spot. Turbulence and atmospheric waves in the Red Spot region absorb the energy of its winds. The vortex loses energy by radiating heat. It should be noted that the absorption of smaller eddies by the Great Red Spot may be one of the mechanisms for maintaining its life and explains the long age of the largest atmospheric formation in the solar system. However, current models show that this is not enough. 3D models that take into account both horizontal and vertical gas flows show that when the slick loses energy, a temperature difference occurs, causing hot gas from the lower atmosphere to enter (vertically) into the GRS, which allows you to recover some of the lost energy. Thus, Jupiter's red spot is "fed" in (see [4] [5] [6]). As it turns out, vertical movement is the key to the "long life" of the Great Red Spot. The model also indicates the existence of a radial flow that "pulls" the wind from the high-speed currents and again directs them towards the center of the vortex. Therefore, it makes sense to build a global model to cover the above processes. Namely, our consideration studies this process in the energy sense and gives more clarity than previous studies. In the work, it is noted (see [7]) that hydrodynamic models are almost small applied to

Jupiter because it is very difficult to construct hydrodynamic models for cyclones, anticyclones, turbulence, rotation, and the energetic budget of Jupiter. But last year, many investigators started studying mathematical and hydrodynamic models for GRS and Jupiter, which are being developed as new approaches. Our work (see [8] [9] [10]) is new approaches, and this presented work for the first time has the application of hydrodynamic models for ovals and vortex which summarize energy, finally helping to provide equilibrium in Jupiter.

2. Statement of Well-Posed Problems

Mathematical methods substantiate that Jupiter is in hydrodynamic equilibrium. The main assumptions underlying it: 1) Jupiter is in hydrodynamic equilibrium; 2) Jupiter is in thermodynamic equilibrium. If you add the laws of conservation of mass and energy to these provisions, you get a system of basic equations. In addition, Jupiter's magnetic field circuit, like any field magnet, produces radio and X-ray radiation. Note that around Jupiter, as well as around most planets of the solar system, there is a magnetosphere, a region in which the behavior of charged particles, plasma, is determined by a magnetic field. In the case of Jupiter, the sources of such particles are the solar wind and its satellite Io. Volcanic ash emitted by Io volcanoes is ionized by solar ultraviolet light. Thus sulfur and oxygen ions are formed: S^+, O^+, S^{2+}, and O^{2+}. These particles leave the satellite atmosphere. These particles leave the satellite's atmosphere, but remain in orbit around it, forming a torus. This torus was discovered by the Voyager-1 apparatus; it lies in the plane of the equator of Jupiter and has a radius of 1 RJ in cross section and a radius from the center (in this case from the center of Jupiter) to the surface generatrix of 5.9 RJ. It is he who determines the dynamics of Jupiter's magnetosphere. Therefore, in this case, we need to justify exclusively by mathematical methods the location of Jupiter in hydrodynamic equilibrium, as well as the location of Jupiter in thermodynamic equilibrium. To do this, consider the study so far only by justifying the hydrodynamic equilibrium of Jupiter. To ensure the constant action of the cyclone and the anticyclone, the relations between the angular velocity and the pressure gradient and the Coriolis parameter are given. The Rossby number is explained as follows ([8]-[15]) flow (See APPENDIX A):

The Coriolis force, or the deflecting force of rotation, appears in the equations of relative motion and is a fictitious force that describes the effect of the movement of the coordinate system associated with Jupiter: $\boldsymbol{K} = -2\Omega \times \boldsymbol{V}$. The component $-2\Omega \times \boldsymbol{V}$ along the coordinate axes:
$x: -2\Omega v(\sin\varphi), y: 2\Omega u(\sin\varphi)$, if the *x*-axes is directed to the East, but *y*-to the North, *z*-vertically upwards and the wind speed component U, V, W along these axes. In this case $w \ll u$. The quantity $f = 2\Omega(\sin\varphi)$, is called the Coriolis parameter $K_x = -2\Omega v(\sin\varphi) = fv$, $K_y = 2\Omega u(\sin\varphi) = -fu$, where Ω, the rotation velocity of Jupiter, φ, along latitude. The ratio of the inertial force to the Coriolis force is called the Rossby number: $Ro = \frac{(\mathrm{d}V/\mathrm{d}t)_x}{fU} = \frac{U}{Lf}$, scales, ho-

rizontal L, vertical H, (the atmosphere is anisotropic, and these scales differ significantly), the velocity scale U, the time scale for horizontal displacements LU^{-1} for vertical HU ones, and the characteristic Coriolis parameter $2\Omega(\sin\varphi) = f$. Here as a characteristic value of the inertial force equal to the acceleration of the particle, the characteristic value of the nonlinear term $u\mathrm{d}u = \mathrm{d}x$ is taken, and this characteristic value is equal to $\frac{U^2}{L}$. If the Coriolis force is small *i.e.* $Ro = \frac{(\mathrm{d}V/\mathrm{d}t)_x}{fU} = \frac{U}{Lf}$ is large, which means that the Corioli force can be neglected. As we can see, this depends both on the scale of motion (namely, the Coriolis force is negligible at small scales) and on the characteristic velocity: the larger it is, the larger Ro. At normal atmospheric velocities, the scale of the Coriolis force is not taken into account at mid-latitudes. Comparing the force of inertia with the force of friction, we find as a measure of their comparative significance the dimensionless Reynolds number for horizontal turbulent viscosity $Re_h = \frac{UL}{\vartheta_1}$, and for vertical turbulent viscosity $Re_\vartheta = \frac{H^2U}{L\vartheta_2}$. A fluid is rotating at constant angular velocity ω about the vertical axes a cylindrical counter. The variation of pressure in the radial is given by

$\frac{\mathrm{d}P}{\mathrm{d}r} = r\omega^2\rho$ The pressure at the axes of rotation is P_c Therefore, the required pressure at the point r is $P = P_c + \frac{1}{2}r^2\omega^2\rho$.

3. The Mathematical Justification for the Hydrodynamic Equilibrium of Jupiter

First, let's start with the fact that White Ovals, small vortices transfer their energies to a large vortex, including the GRS, as result of which is provided with constant kinetic energy. For the purpose we will try to build a visual description of this process by a mathematical formula, the justification of which is concrete and clear. Consider a number of isolated free vortices of force

$F_i\,(i = 1,2,3,\cdots,n)$ at points $M_i(x,y,z)(i = 1,2,3,\cdots,n)$ of an incompressible fluid moving rotationally in region D. These boundaries $\gamma_k\,(k = 1,2,3,\cdots,m)$ that the belt of Jupiter near of the GRS. IF we denote the usual flow function flow function of the fluid motion as

$$\Psi = \Psi(x,y;x_i,y_i,z_i) = \Psi((x_1,y_1),\cdots,(x_n,y_n),h), z_i = h = const, \tag{3.1}$$

which is iindependents of time t, then the components of the i-th vortex ($i = 1,2,3,\cdots,n$) (for example $n = 100$ ovals on Jupiter) have the following form:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = u_i = -\left.\frac{\partial\Psi^{(i)}}{\partial y}\right|_{M_i}, \frac{\mathrm{d}y_i}{\mathrm{d}t} = \upsilon_i = \left.\frac{\partial\Psi^{(i)}}{\partial y}\right|_{M_i} \tag{3.2}$$

$$\text{where } \Psi^{(i)} = \Psi - \frac{F_i}{2\pi}\ln r_i, r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \tag{3.3}$$

Here, in formulas (3.2), (3.3), the value in points is taken. Since the boundary on Jupiter is not solid, there is no stable flow in the region D. Therefore, in the hydrodynamic process, the flow of the function exists in the motion of i-th vortex and will have the following form:

$$F_i \frac{\mathrm{d}x_i}{\mathrm{d}t} = F_i u_i = -\frac{\partial E}{\partial y_i}, F_i \frac{\mathrm{d}y_i}{\mathrm{d}t} = F_i \upsilon_i = -\frac{\partial E}{\partial x_i} \tag{3.4}$$

Now, starting from the system (3.4), we can determine the Green's function for a point in the region D. After that, it is easy to determine in the flow function by the definition of the Green's function, finding the harmonic function that expresses the system (3.2). The Green's function [16] must satisfy the condition

$$G^*(x, y, x_0, y_0) = G(x, y, x_0, y_0) - \frac{1}{2\pi}\ln r_0, r_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2} \tag{3.5}$$

It is not difficult to see that the function $G^*(x, y, x_0, y_0)$ for the point $M(x, y, h), M_0(x_0, y_0, h) \in D$ is harmonic: $\frac{\partial^2 G^*}{\partial x^2} + \frac{\partial^2 G^*}{\partial y^2} = 0$. In addition, if $\frac{\partial G^*}{\partial n}$ is a normal derivative on a variable swirling from (x, y), then

$$G = G|_{\Upsilon_k} = C_k, \int_{\gamma_k} \frac{\partial G}{\partial n}\mathrm{d}s = 0, (k = 1, 2, 3, \cdots, m) \tag{3.6}$$

Since the boundary of the external liquid closed contour is located in the domain D, then it is enough to have a circle of a large radius which satisfy

$$\begin{aligned} &G(x, y; x_0, y_0)|_{\Upsilon_0} = 0, \frac{\partial G}{\partial n} = O\left(\frac{1}{r_0^2}\right) + \frac{1}{2\pi r_0}, \\ &G(x, y; x_0, y_0) = \frac{1}{2\pi}\ln r_0 + O\left(\frac{1}{r_0}\right), \frac{\partial G}{\partial s} = O\left(\frac{1}{r_0^2}\right), \end{aligned} \tag{3.7}$$

where the $\frac{\partial G^*}{\partial s}$ is taken as tangential derivative along the circle line. So, using the above, it seems possible to prove the symmetry property of this Green's function using standard methods. .Therefore, we summarize the results obtained in the following lemma.

LEMMA 3.1 The function $G(x, y; x_0, y_0)$ is defined by conditions (3.6), (3.7) and equality of $\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = 0$ and there exists in a unique generalized Green's function that satisfies the reciprocity $G(x, y; x_0, y_0) = G(x_0, y_0; x, y)$, and the reciprocity immediately leads to the following important result (see formula (2), (3)):

$$\begin{aligned} \frac{\partial G^*(x_0, y_0; x_0, y_0)}{\partial x_0} &= 2\lim_{M \to M_0} \frac{\partial G(x, y; x_0, y_0)}{\partial x}, \\ \frac{\partial G^*(x_0, y_0; x_0, y_0)}{\partial y_0} &= 2\lim_{M \to M_0} \frac{\partial G(x, y; x_0, y_0)}{\partial y}. \end{aligned} \tag{3.8}$$

Let's now apply the function $G(x, y; x_0, y_0)$ to the hydrodynamic problem of the motion of a vortex with the stream function

$$\Psi = \Psi(x, y; x_i, y_i, z_i) = \Psi((x_1, y_1), \cdots, (x_n, y_n), h), z_i = h = const \tag{3.9}$$

Note that the Laplace $\Delta G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = 0$ equation has in the following fundamental solution

$$G(x, y, x_0, y_0) = \frac{1}{r} = \frac{1}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}, \quad \forall M(x, y, z), M_0(x_0, y_0, z_0) \in D. \tag{3.10}$$

The motion given by the flow function can be called the motion $\Psi_0 = F_0 G(x, y; x_0, y_0)$ due to the vortex F_0 at the point $N_0(x_0, y_0)$. This is possible potential movement in the region D with the required singularity, which is has no circulation along of the any internal boundaries. In addition, if the region of D extended to infinity (more far as far) the flow through the arc of a circle with radius $r_0 = const$ (constant) (contributing to the external flow) tends to zero, and the flow along it (contributing to the circulation) is finite (or tends to zero), since it tends to infinity. In this case, the motion expressed in terms of stream functions is called such a motion that at the point $M_0(x_0, y_0)$ is the motion. If we subtract the stream function $\Psi_0 = F_0 G(x, y; x_0, y_0)$ due to all vortices F_0 from total stream function (3.1), then the remainder is the stream function that specifies possible potential movement in the region D. Now this movement is uniquely determined when circulation around each of the vortices is given curves $G|_{\gamma_k}$ and the fluid velocity at infinity. Since these conditions are the same for the full stream function as for this part, this motion is in fact independent of the potential (variable) coordinates of the vortices. It depend on (constant) force only if the conditions that given it are. Thus, we can call it a movement "caused by extended factors" we summarize our result in the following main lemma.

LEMMA 3.2. If in region D there are $i = 1, 2, 3, \cdots, n$ ($n = 100$, for Jupiter) vortices with incompressible fluid forces $F_i (i = 1, 2, 3, \cdots, n)$ at points $M_i(x, y, z)(i = 1, 2, 3, \cdots, n)$. In common region bounded by fixed boundaries, the fluid motion current function is given

$$\begin{aligned} \Psi &= \Psi(x, y; x_i, y_i, z_i) = \Psi((x_1, y_1), \cdots, (x_n, y_n), h) \\ &= F_0 \Psi(x, y; x_0, y_0) + \sum_{i=1}^{n} F_i G(x, y; x_i, y_i) \end{aligned} \tag{3.11}$$

where $G(x, y,; x_i, y_i)$ are given in Lemma3.1 a $\Psi(x, y; x_0, y_0)$ is the function in current of motion due to external influences, independent of $M_i(x, y, z)(i = 1, 2, 3, \cdots, n)$ and $F_i(i = 1, 2, 3, \cdots, n)$. With this result in hand, we can immediately establish the existence of the Kircoff-Rauth function (see, [17] [18] [19] and therein).

THEOREM 3.1. For the motion of vortices with force ($n = 100$, for Jupiter) in general domain which contains all ovals and vortices' having boundaries $\gamma_k\left(k=1,2,3,\cdots,m\right)$, there exists a function $E_k=E\left(\left(x_1,y_1\right),\cdots,\left(x_n,y_n\right),h\right)$ such that

$$F_i u_i=-\frac{\partial E}{\partial y_i}, F_i \upsilon_i=-\frac{\partial E}{\partial x_i}, \tag{3.12}$$

where $M_i\left(x,y,z\right)\left(i=1,2,3,\cdots,n\right)$ are the instantaneous positions of the vortices. The function defined in the following indicated immediately as kinetic energy :

$$E_k=\sum_{i=1}^{n} F_i \Psi\left(x_i,y_i;x_0,y_0\right)+\sum_{\substack{i,j=1\\(i>J)}}^{n} F_i F_j G\left(x_i,y_j;x_j,y_j\right)+\sum_{i=1}^{n} F_i^2 G\left(x_i,y;x_i,y_i\right) \tag{3.13}$$

This can be immediately seen by comparing the results obtained similarly to the results (3.2), (3.3) and (3.4). Note that the system of equation (3.12) is a Hamiltonian system of differential equations in the system of variables $\sqrt{F_i x_i}$ and $\sqrt{F_i y_i}$ in case of $n=100$, (for Jupiter).Equality (3.13) is the kinetic energy of Jupiter's liquid motion for all vortices, so equation (3.13) leads to the energy conservation laws $E_k=const$. It is appropriate to note that in the work (see [8] [9] [10]) of the considering section "Mathematical description of the rotational details and motion process for the dynamic of the GRS on Jupiter" models for Jupiter were built on the basis of "spheroids" rotating differentially, whose semi axes are independent of each other: a problem that was solved using the law of rotation derived from a generalization of Bernoulli's theorem (for ideal gas $P=t^{\frac{S}{\upsilon}}\rho^{\gamma},\gamma=\frac{c_h}{c_\upsilon}$. At constant *entropy, the Bernoulli equation takes place* $\frac{\upsilon^2}{2}+\frac{c^2}{\gamma-1}=const$) and using Lamb function (Lamb function $H=\frac{\upsilon^2}{2}+\frac{P}{\rho}-U$, where $\upsilon=\left|\boldsymbol{V}\right|$, *P*-pressure, ρ-density) also(see [9] [10]), which is valid only for axisymmetric masses. In this case, the term "quasi-potential" is additionally, introduced to pressure the rotation model. Each layer details with common boundaries of Jupiter rotate with its own angular velocity profile. The law of rotational has a simple dependence on the derivative the gravitational potential ([8] [9] [10]). Despite the fact that no approved observational data has yet been found, that all layers have a common angular velocity profile which decreases from the pole to the equator, the angular velocities (the value of the angular velocities depending on this period of rotation changes) are clearly related by equality to pressures gradients and Carioles parameters. Therefore, the mathematical substantiation really allows finding out the laws of hydro dynamical properties of the equilibrium of the GRS and Jupiter. (See APPENDIX A).

Lemma 3.3 (see proof corresponding notation of Lemmas 3.1 in [10]). Around GRS on a closed fluid circuit the time derivatives of the velocity circulation are equal to the acceleration circulation on this circuit.

Theorem 3.2 (see proof corresponding notation of Theorem 3.1 in [10]). (About conservation of vortex lines). Particles of liquid, around GRS forming vortex lines, at any time, and at all times of motion form vortex lines, coming from their origin, through ovals and swirled parts of liquid.

Theorem 3.3 (see proof corresponding notation of Theorem 3.2 in [10]) (about conservation of intensity of vortices). The intensity of any vortex (in particular the tube) remains constant at all times (See-APPENDIXA).

Theorem 3.4 (see proof corresponding notation of Theorem 3.3 (main) in [10]) Let conditions of Lemma 3.1, Theorem 3.1 and Theorem 3.2 be fulfilled, and the Rossby conditions of free, cyclone, anticyclone, and, besides, if quasi-laminar and turbulent fluid flow around the GRS exists, then the necessary and sufficient conditions for the existence of stability of constant GRS rotation and the Jupiter' s balance, the internal and external energy balances of Jupiter are preserved flow (See APPENDIX A).

4. Conclusions

The article first presents the facts and compares some results to create new continuous theoretically substantiated treatises for the assumption of maintaining the stability of Jupiter's rotational motion. With the help of rigorous, mathematically substantiated methods, the assumption of the stability of rotational motion and that Jupiter is in hydrodynamic equilibrium is proved. This is based on new work by authors who have recently considered the influence of cyclones, anticyclones, circulation and rotation factors on the stable dynamics of Jupiter, as well as new mathematical treatises on the dynamics of the GRS. Based on these results, as well as previously known methods of hydrodynamics and non-classical approaches, a theorem is substantiated in the form of lemmas that cyclones, anticyclones, GRS circulations and rotational motions create conditions for the occurrence of vortex motions along a closed GRS fluid contour. In addition, using the theory of circulation and torque, based on the equations of hydrodynamics, Coriolis force, momentum statistics, formulate a lemma and theorems on the acceleration of circulation, a theorem on the conservation of vortex lines, which describes the full dynamics of Jupiter's rotational motion (Lemma 3.1, 3.2, 3.3, Theorem 3.1, 3.2, 3.3, 3.4). Considering the output directly above, we can present the results obtained as follows (see APPENDIX A, for all figure, illustration):

- the movement of gas and liquid on the GRS is divided into three processes that combine laminar (or approximate, so-called quasi-laminar) and transitional flow along ovals with turbulent flow (See APPENDIX A);
- in cyclones, the Coriolis force is directed from the center of the vortex, therefore, a decrease is formed in it, and in anticyclones, on the contrary, an increase in the gas density;
- anticyclones are much longer-lived than cyclones, what is associated with the increased density inside them and, therefore, other things being equal, the total angular momentum of the anticyclone turns out to be higher than that of the cyclone, so it is more difficult for it to disintegrate;

- Rossby vortices slowly drift along the parallel to the west with a speed not exceeding $V_{dr} = V_R$ where V_R is the phase velocity of Rossby waves.

By means of stream function and Green's function constructed energy for one vortex motion and after summarized all ovals and energy for $i = 1, 2, 3, \cdots, n$ ($n = 100$, for Jupiter) (in particularly cases, ovals and vortex) vortexes motion. By the energy conservation laws, this summarized energy is constantan. It means that total motion of Jupiter rotation under indicated assumption always will be stability and therefore, the hydrodynamical equilibrium of Jupiter is proved..

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Beebe, R. (1996) Jupiter: The Giant Planet. Smithsonian Institution Press, Washington DC.

[2] Bagenal, F., Dowling, T.E. and McKinnon, W.B. (2004) Jupiter: The Planet, Satellites, and Magnetosphere. Cambridge University Press, Cambridge.

[3] McAnally, J.W. (2008) Jupiter and How to Observe It. Springer, London. https://doi.org/10.1007/978-1-84628-727-5

[4] Spiegel, D.S. and Madhusudhan, N. (2012) Jupiter Will Become a Hot Jupiter: Consequences of Post-Main-Sequence Stellar Evolution on Gas Giant Planets. Astrophysics. *The Astrophysical Journal*, **756**, 132. https://doi.org/10.1088/0004-637X/756/2/132

[5] Simon-Miller, A.A., Gierasch, P.J., Beebe, R.F., Conrath, B., Flasar, F. and Achterberg, R.K. (2002) New Observational Results Concerning Jupiter's Great Red Spot. *Icarus*, **158**, 249-266. https://doi.org/10.1006/icar.2002.6867

[6] Youssef, A. and Marcus, P.S. (2003) The Dynamics of Jovian White Ovals from Formation to Merger. *Icarus*, **162**, 74-93. https://doi.org/10.1016/S0019-1035(02)00060-X

[7] Вдовиченко В.Д. and Кириенко Г.А. (2014) Атмосфера Юпитера. Большое Красное Пятно. LAP LAMBERT Academic Publishing.

[8] Nurmammadov, M.A. and Atai, A.A. (2020) Influences the Factors of Cyclones, Anticyclones, Circulation and Rotation to the Steady Dynamic of Great Red Spot of Jupiter. *Journal of Natural Sciences*, **8**, 16-20. https://doi.org/10.15640/jns.v8n2a2

[9] Nurmammadov, M.A. and Atai, A.A. (2021) New Mathematical Tractates about Dynamics of Great Red Spot on the Jupiter. *International Journal of Physics and Astronomy*, **9**, 1-7. https://doi.org/10.15640/ijpa.v9n1a1

[10] Nurmammadov, M.A. and Atai, A.A. (2021) A New Mathematical Justification of Necessary Criteria and Sufficient Conditions for Marinating Stability of Jupiter's Rotational Motion Around of Jupiter's the Sun. *International Journal of Current Advanced Research*, **10**, 12422-12428. https://www.journalijcar.org/

[11] Rossby, C.G. (1939) Relation between Variations in the Intensity of the Zonal Circulation of the Atmosphere and the Displacements of the Semi-Permanent Centers of Action. *Journal of Marine Research*, **2**, 38-55.

[12] Ingersoll, A.P., Dowling, T.E., Gierash, P.J., *et al.* (2004) Dynamics of Jupiter's At-

mosphere. Jupiter Planet, Satellites Magnetosphere, 105-128.

[13] Appleby, J., Orton, G., Martonchik, J. and Aumann, H. (1981) Properties of NH_3 Gas and Ice in the Jovian Atmosphere from Investigation of the Far Infrared Spectrum. *Bulletin of the American Astronomical Society*, 13, 736.

[14] Marcus, P.S. (2004) Prediction of a Global Climate Change on Jupiter. *Nature*, **428**, 827-831.

[15] Sanchez-Lavegab, L.J. (2005) Jupiter's Cyclones and Anticyclones Vorticity from Voyager and Galileo Images. *Icarus*, **174**, 178-191.

[16] Bleistein, N. (1984) Mathematical Methods for Wave Phenomena. Academic Press, Cambridge, MA.

[17] Kirchhoff, G. (1932) Vorlesungen Liber Mathematics Physik, Mechanik, p. 255 ff. See also Lamb, H., Hydrodynamics, 230.

[18] Kirhhoff, G. (1896) Vortesingen Uber Mathematische Physic. Teubner, Leipzeig.

[19] Barsch, T. and Pisoia, A. (2015) Critical Points of the *N*-Vortex Hamiltonian in Bounded Planar Domains and Steady State Solutions of the Incompressible Euler Equations. *SIAM Journal on Applied Mathematics*, **75**, 726-744. https://doi.org/10.1137/140981253

Appendix A: Some Extra Material (Figure Which is Used Author of This Paper)

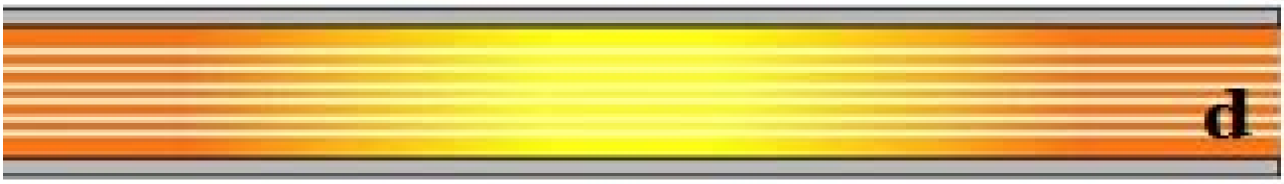

Figure A1. The Laminar rejime (see [9] [10]).

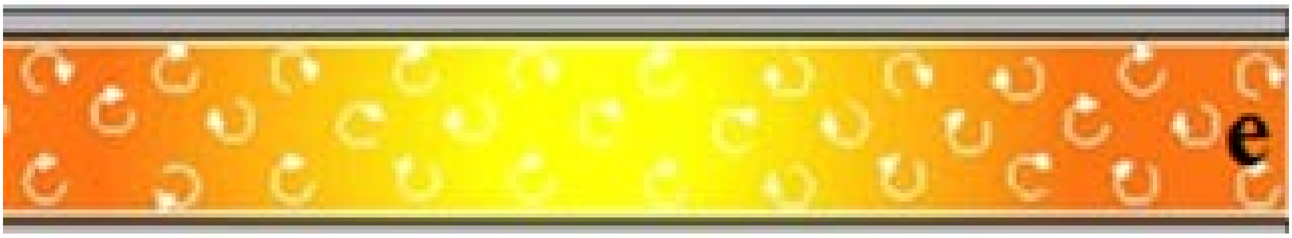

Figure A2. The formation Ovals and turbulence formation (see [9] [10]).

Figure A3. Close up of the Great Red Spot imaged by the Juno Spacecraft in April 2018 From NASA.

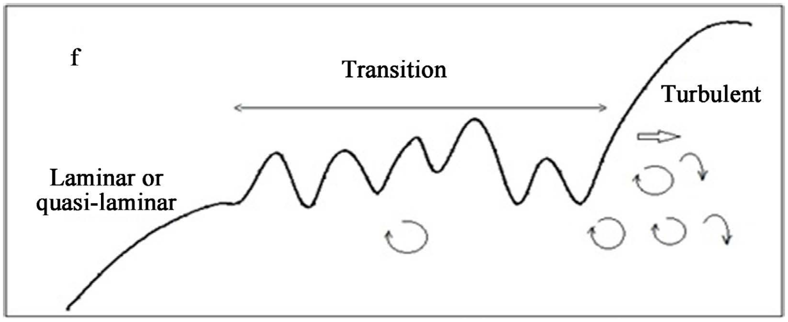

Figure A4. Illustration of scheme from laminar + quasi-laminar to transition turbulence regime (see [9] [10]).

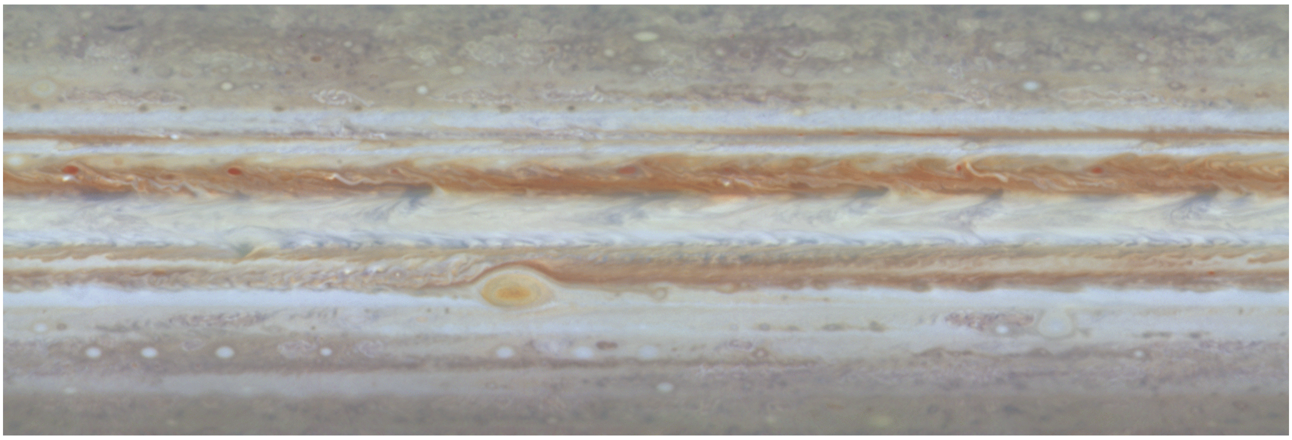

Figure A5. Zones belts and vortices on Jupiter. The wide equatorial zone is visible in the center surrounded by two dark equatorial belts (SEB and NEB).

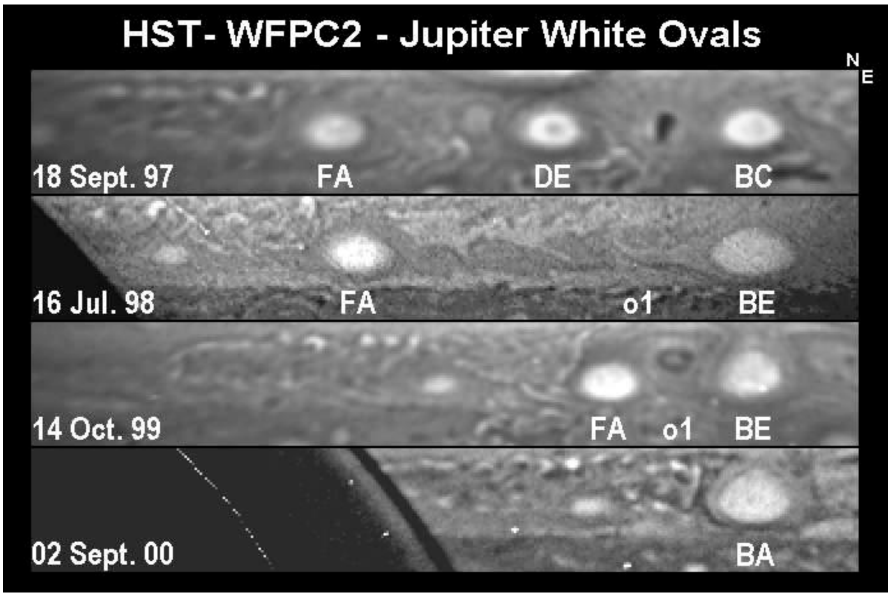

Figure A6. WHITE OVAL DE, JUPITER About 10 hours before closest approach to Jupiter, Voyager 1 acquired three 1 × 3 narrow angle green filtered mosaics of one of the three big, white ovals that were present in the South Temperate Zone at latitude 33˚S during the Voyager flybys. These ovals formed in 1939-1941 and had been shrinking since then.