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Implementation of a Classical Theory for Superfluids

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Abstract

The superfluidity of helium-4 is explained until today by a quantum theory: the Bose-Einstein condensation. This theory is rather satisfactory in describing the superfluid state of helium-4 because this one is a system made up of bosons (particles of integer spin). However, the discovery of the superfluidity of helium-3 in 1971 called into question the veracity of this quantum theory. In fact, helium-3 being a system composed of fermions (particles of half-integer spin), it cannot be subject to Bose-Einstein condensation. It is to correct this deficiency that we introduce here a classical (non-quantum) theory of superfluids. This new theory makes no difference between the λ transition of bosons and that of fermions. It is based on a fundamental law: "in a superfluid, density is conserved". In this work, we have shown that this simple law explains not only the zero viscosity of superfluids but also the surprising phenomena observed in the superfluid state, I quote the liquidity of helium at normal pressure down to 0 K, vaporization without boiling, high thermal conductivity, the fountain effect, the ability to go up one side of the wall of a container to come down on the other side and the existence of a critical velocity.

Keywords

Superfluid, Constant Density, Zero Viscosity, Temperature Gradient, Pressure Gradient

1. Introduction

When the temperature of helium-4 (H_2^4) liquid is lowered until it exceeds a certain value T_λ called the lambda (λ) temperature, under normal atmospheric pressure, a state transition occurs: the liquid passes from the normal state to the superfluid state. This transition is usually called the lambda (λ) transition. As the word "superfluid" indicates, below T_λ, the fluid has zero viscosity. Unable to find an adequate explanation for this phenomenon, physicists directly concluded that it must necessarily have a quantum origin. They then associated the λ transition with the Bose-Einstein condensation [1]. The latter is a quantum condensation of a system composed of bosons (particles of integer spin). This quantum description was satisfactory because indeed, the helium-4 atom is a boson (zero spin). However, in 1971, a few years after the discovery of the superfluidity of helium-4, the superfluidity of helium-3 (H_2^3) was demonstrated [2]. This last discovery made all the difference because the helium-3 atom is not a boson but a fermion (spin 1/2). Its superfluidity therefore could not be explained by Bose-Einstein condensation. But, physicists have nevertheless advanced a new quantum reason to justify the existence of helium-3 in a superfluid state. In fact, superfluid helium-3 presents some physical properties which differ enormously from those of superfluid helium-4, in particular magnetic properties (for example, the existence of spin waves as in ferromagnetic materials). Physicists claim that these differences are due to the fact that the helium-3 atom is a fermion while the helium-4 atom is a boson [3]. That's not true at all! Indeed, when we observe the Clapeyron diagram $P(T)$ (P is the pressure and T, the temperature) of stable helium, we notice that there is a zone where the solid-liquid equilibrium curve becomes almost parallel to the abscissa axis (see **Figure 1**) [4].

We recall that according to Clapeyron's relation:

$$\left(\frac{\mathrm{d}P}{\mathrm{d}T}\right)_{L-S} = \frac{S_l - S_s}{v_l - v_s} \quad (1)$$

where:

- S_l is the entropy in the liquid state;
- S_s is the solid state entropy;
- v_l is the volume in the liquid state;
- v_s is the solid state volume.

The horizontality of the liquid-solid equilibrium curve at low temperature is given by:

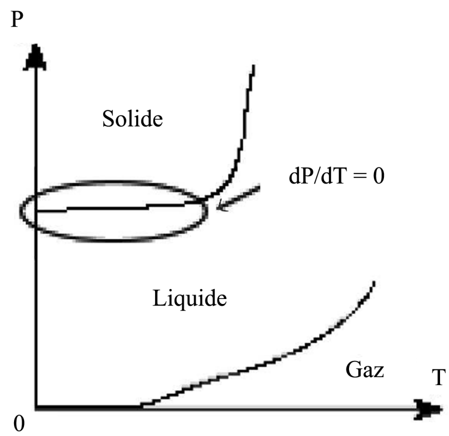

Figure 1. Particularity of the Clapeyron diagram of helium at the solid-liquid interface.

$$\left(\frac{\mathrm{d}P}{\mathrm{d}T}\right)_{L-S} = 0$$
$$\Rightarrow S_l \cong S_s \tag{2}$$

This means that the entropy of the liquid is almost equal to the entropy of the solid at low temperature. This is quite unusual since a solid is generally more spatially ordered than a liquid and therefore has lower entropy. Here since $S_l \cong S_s$, the liquid is as ordered as the solid in the superfluid phase. This implies an alignment of the magnetic spin moments of the atoms in the superfluid as in a ferromagnetic solid. But for this alignment of spin moments to appear, the spins of the atoms of the system must first be non-zero. Superfluid helium-3 atoms exhibit magnetic properties because they have non-zero spins, not because they are fermions. In other words, a non-zero spin-boson system will also exhibit magnetic properties in the superfluid state. It is therefore clear that the Bose-Einstein condensation is no longer sufficient to completely describe the superfluid state at low temperature.

We then introduce here a new classical (non-quantum) theory that makes no difference between the λ transition of bosons and that of fermions. Moreover, it explains not only the zero viscosity of superfluids but also the unusual physical phenomena observed in the superfluid state, I quote [5]: the liquidity of helium at normal pressure down to 0 K, vaporization without boiling, high thermal conductivity, the fountain effect, the ability to go up one side of the wall of a container to come down on the other side and the existence of a critical velocity.

2. The Classical Theory

2.1. Description

Who says viscosity, says friction; and who says friction, says collisions between particles. Viscosity is therefore a physical quantity that characterizes the collisions between particles in a fluid. And besides, we can even see it through the unit of measurement used. Indeed, in the international system of units, the kinematic viscosity is measured in $m^2 \cdot s^{-1}$. This simply means that the viscosity is proportional to the cross-section and the frequency of collision of the fluid particles. Zero viscosity therefore means "absence of any collision". So we have to show here that below T_λ, no more collision is possible in the superfluid. To demonstrate this, we consider a superfluid electron gas, for example in a superconductor. Indeed, it is the superfluidity of the electron gas in the superconducting metal below the critical temperature that is responsible for the infinite conductivity and the Meissner effect of the material [6]. The superfluid state of the gas appears when the interelectronic distance (the distance between two neighboring electrons) becomes minimum. We have called this minimum interelectronic distance: the lambda (λ) distance. In other words, in the superfluid state, it is impossible for two electrons to get closer to a distance less than the λ distance. How to explain the existence of this λ distance in the superfluid state? To answer this question, consider two neighboring electrons in the electron gas.

One of the electrons is at rest while the other is in motion. If the mobile electron moves towards the fixed electron with speed V_e, then its kinetic energy is given by: $E_c = \frac{1}{2} m_e V_e^2$ (with m_e, the mass of the electron). When this mobile electron is at a distance r from the fixed electron then it undergoes a Coulomb repulsion whose energy is given by: $E_p = \frac{4\pi\varepsilon_0 e^2}{r}$ (with e, the charge of the electron and ε_0, the electric permittivity of the vacuum). Two cases can then be distinguished:

- If $E_c > E_p$, then there will be a collision.
- If $E_c < E_p$, then there will never be a collision.

In the superfluid state, the velocity V_e of the electron is so small that we always have $E_c < E_p$. Hence the zero viscosity of superfluids. The superfluid being as ordered as a solid, each electron is surrounded by four other electrons: one electron on the left, another on the right, another above and another below. Thus, in the superfluid, the electron is as frozen in a fixed point because it is repelled from all four sides. The distance between two electrons in the superfluid therefore never varies. It is this distance that we have named λ distance. From this, then follows a law that completely defines the superfluid state: **in a superfluid, density is conserved**. We also call this constant density in the superfluid state the lambda (λ) density. The atom is made up of a cloud of negatively charged electrons that surrounds a positive nucleus, the same reasoning that we have just applied to an electron gas can also be applied to a system composed of neutral atoms. Let us now show that this simple law of constant density in the superfluid state explains all the extraordinary phenomena observed in this state.

2.2. Applications

1) Critical velocity

For the viscosity to be zero during a flow of the superfluid on a fixed solid plate (*i.e.*, no collision between the atoms of the superfluid and the atoms of the plate), it is necessary that the energy of the Coulomb repulsion E_p between the atoms of the superfluid and those of the plate is greater than the kinetic energy of the atoms of the superfluid. In this case, the superfluid moves on the solid plate without touching it by levitating above it. This then imposes the existence of a limit flow velocity called the critical velocity above which $E_p < E_c$ and the flow becomes normal (non-zero viscosity). When the flow velocity is equal to the critical velocity, we therefore have: $E_p = E_c$.

This critical velocity is highlighted when trying to rotate a superfluid. Indeed, if a box containing a superfluid is slowly rotated, the walls of the box rotate but the superfluid remains motionless in a state of rest [7]. This is because the Coulomb repulsion between the atoms of the superfluid and those of the wall prevails over the kinetic energy of rotation of the atoms of the wall (there is therefore no contact between the superfluid and the wall: zero viscosity). But when the speed of rotation exceeds the critical velocity, the superfluid begins to rotate (there is a collision between the atoms of the superfluid and those of the wall: non-zero viscosity).

2) Liquidity of helium down to 0 K at normal pressure

Helium is a permanent liquid. This is a unique property of helium; it is the only compound that remains liquid down to absolute zero under normal pressure [8]. Unlike a usual classical diagram, we do not observe a triple point in the Clapeyron diagram of helium (see **Figure 2**) [4]. Indeed, helium is the only element that becomes superfluid at normal pressure. Its λ temperature is 2.17 K. Thus, as solidification (transition from liquid to solid state) corresponds to an increase in density (decrease in volume) and as density does not vary in the superfluid state, helium will remain liquid as long as it is superfluid. This is why helium remains liquid even at 0 K at normal pressure. This is proof of the veracity of the classical theory of superfluids introduced in this work.

3) Vaporization without boiling

In a liquid in the normal state, boiling occurs because heat travels through it by convection. The fluid near the heat source heats up and its density decreases (increase in volume). Then, following the principle of Archimedes, the less dense fluid rises to make room for the denser fluid. It is this density gradient in the liquid that leads to the formation of bubbles. However, in a superfluid, the λ density is constant and therefore uniform. Hence there is no possible convection in the superfluid and therefore no formation of bubbles: the liquid vaporizes without boiling (evaporation).

4) High thermal conductivity

There are only three means of heat propagation in a system: conduction, convection and radiation. We have just seen that in a superfluid, there cannot be convection. Radiation is also to be excluded because there is no production of any photon in the superfluid. There is, therefore, only one possible means of heat propagation in a superfluid: conduction. However, experiments have shown that the heat conduction in superfluid helium is 1000 times greater than that of copper. Indeed, the conductivity in a superfluid differs completely from the conductivity in a metal. In a solid material, heat is conducted through free electrons (conduction electrons). These electrons transmit kinetic energy from one end of the metal to the other by collision. However, in a superfluid, there can be no collision between particles (zero viscosity). The heat then propagates from

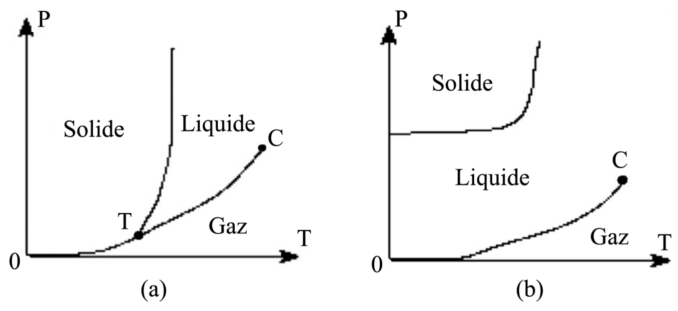

Figure 2. (a) The classic diagram; (b) The helium diagram.

one end of the superfluid to the other in the form of an elastic wave at the speed of sound. In fact, the absence of collision in the superfluid allows us to consider it, in first approximation, as an ideal fluid. The ideal fluid equation is given by:

$$PV = nRT \tag{3}$$

where:

- P is the pressure;
- V is the volume;
- T is the temperature;
- n is the number of moles;
- R is the ideal fluid constant.

As the mass of the superfluid is conserved ($n = Cte$), the volume is also conserved because the λ density of the superfluid is constant.

We then arrive at the equation:

$$P = CT \quad \text{(with } C \text{, a constant)}$$

$$\Rightarrow \Delta P = C\Delta T \tag{4}$$

Which means any variation or gradient in temperature implies a variation or gradient in pressure. It is this pressure gradient that propagates like a wave in the superfluid and which is responsible for the high thermal conduction of superfluids.

5) Fountain effect

When a tube containing superfluid helium is heated, there is a continuous emission of a jet of helium which can reach several tens of centimeters in height at the exit of the tube. This is the fountain effect. The emission ceases when the tube is no longer heated. The height of the jet is variable and increases with the temperature gradient imposed on the system (see **Figure 3**).

Indeed, Equation (4) tells us that any temperature gradient implies a pressure gradient in the superfluid. If the temperature gradient is high, then the pressure gradient will also be high and can then produce a force of pressure greater than the force of gravity. Hence the fountain effect. As the pressure force increases with the pressure gradient, the jet height will increase with the temperature gradient. As soon as the temperature gradient becomes zero, the pressure gradient also becomes zero and the fountain effect disappears.

6) Ability to go up one side of the wall of a container to go down the other side

When a tube containing superfluid helium is immersed in another container containing superfluid helium, we notice that a thin film located near the wall of the tube rises on one side and descends on the other side. The ascent ceases as soon as equilibrium is regained where the levels inside and outside the tube are identical (See **Figure 4**).

In fact, when the tube is immersed in the container, a pressure gradient (a pressure difference) is created between the two sides of the tube wall. According to Equation (4), this pressure gradient implies a temperature gradient. The latter then leads to a small-scale fountain effect near the wall of the tube. Hence the

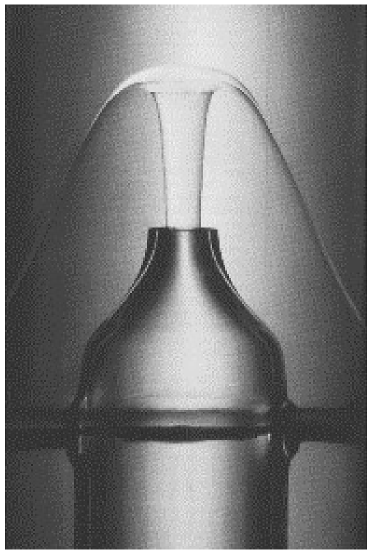

Figure 3. Fountain effect.

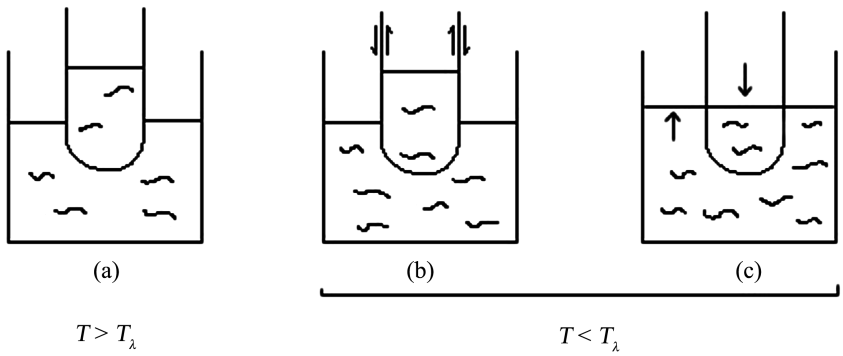

Figure 4. (a) This is normal helium in a test tube immersed in a container also containing liquid helium, nothing happens. (b) Same experiment but with superfluid helium, the helium flows along the walls of the tube until the levels equalize. (c) The levels inside and outside the tube are identical in the superfluid phase, nothing more happens.

rise of the film is located near the wall. As soon as equilibrium is found, the pressure gradient becomes zero and therefore, the temperature gradient also becomes zero. The fountain effect near the wall disappears and the rise of the film ceases.

3. Conclusions

In this work, we have introduced a classical theory that better describes the superfluid state than the quantum theory based on Bose-Einstein condensation. This classical theory is based on a fundamental law stated as follows: in a superfluid, density is conserved. We have shown that this law explains not only the zero viscosity of superfluids but also the surprising phenomena that occur in the superfluid state, I quote:

- The critical speed V_λ corresponds to a fluid flow velocity V for which the kinetic energy E_c of the atoms of the superfluid is equal to the Coulomb repulsion energy E_p between the atoms of the superfluid and the atoms of the fixed plate. If $V < V_\lambda$, then $E_c < E_p$ and the viscosity is zero. If $V > V_\lambda$, then $E_c > E_p$ and the viscosity is no longer zero.

- The liquidity of helium down to 0 K under normal pressure is due to the fact that superfluid helium retains its density even at 0 K.
- Vaporization without boiling is explained by the uniformity and constancy of the density of the superfluid. No convection is therefore possible in the superfluid state.
- The high thermal conductivity is due to the fact that in the superfluid the heat propagates as an elastic wave at the speed of sound.
- The fountain effect is observed when the superfluid is heated because any temperature gradient implies a pressure gradient in the superfluid state.
- The ability to move up one side of the wall of a vessel and down the other side is nothing more than a small-scale fountain effect near the wall. Indeed, any pressure gradient implies a temperature gradient in the superfluid state.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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