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Static Aspect of Heterogeneous Competition in Duopoly

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Abstract

The study gives a comparison between price policy and quantity policy in duopoly producing differentiated goods with different production costs and indicates which is more beneficial. Further, it is investigated that in a non-linear duopoly with differentiated goods and two different policies, firms may earn more profit if they choose a quantity policy in a stable economy when the marginal production cost of both the firms is the same. If the production cost of both firms is different, then the price policy is better only when the firm is efficient.

Keywords

Oligopoly, Duopoly, Iso-Elastic Demand, Linear Model, Non-Linear Model, Static Model

1. Introduction

There are mainly two types of market systems, one is Monopoly and the other is Perfect Competition and they are completely contrary to each other in character [1] [2]. Cournot proposed a new market system named Oligopoly [3]. Oligopoly is the transitional case of monopoly and perfect competition [4] [5]. Duopoly is the sub-case of oligopoly. A duopoly is a market system that is controlled by two firms. There are two classical models in the theory of duopoly. One is of Cournot and the other is of Bertrand [6]. Both mentioned models firms were generating the same kind of goods and adopting homogeneous strategies. Cournot model firms use quantity strategies while Bertrand model firms use price strategies [7] [8]. Numerous researchers have established mathematical models to deal with diverse situations of duopoly [9] [10]. In most established studies, researchers assumed that in a duopoly, both firms produce homogeneous goods and adopt the same kind of strategy either quantity strategy or price strategy [11] [12]. But in real-life situations, firms may produce different kinds of products which provide customers a variety of goods to choose from them and it provides opportunities for firms to improve their products because of competition. Also, it is not necessary that both firms adopt homogeneous strategies [13]. Instead, in order to get maximum benefit and avail market situations, firms use different strategies. Various studies have been conducted to give a realistic outlook to the classical model [14] [15]. Many theories are established on the idea of product differentiation [16] [17] as well. Some researchers established the relation between relative profit objective and the advertising competition model [18] [19]. But, there is not abundant work on the non-linear duopoly model with the heterogeneous cost of production and heterogeneous strategies. In this paper, the strategic behavior in a heterogeneous nonlinear differentiated duopoly is observed. For this, a duopoly model is examined after introducing nonlinearity through heterogeneous strategies and differentiated goods while the cost of production of both firms is also heterogeneous. The main objective of the paper is to analyze the static effect of non-linearity on the behavior of the competitors. Non-linearity in the duopoly model is due to various reasons like heterogeneity in production cost, heterogeneity in strategy, and heterogeneity in competition, which means that different firms choose different strategies. This paper is systematized like this: In Section 2, there is an outline of the general demand function in dissimilar duopoly; Section 3 describes the differentiated duopoly with linear demand function; Section 4 explains the duopoly with non-linear demand with heterogeneous stratagems as well as heterogeneous manufacturing cost and a conclusion is given in Section 5.

2. Duopoly with General Demand Function

As in a duopoly, two firms compete against each other. So, let there be two firms firm 1 and firm 2. Let us suppose that firm 1 produces x_1 quantity of a product with price p_{x_1} and firm 2 produces x_2 quantity of product with price p_{x_2}. The motive of both firms is to earn maximum profit. Let us consider the case when both players follow heterogeneous strategy. In that case, one of the players follows quantity strategy taking price variable of other as given and other follows price strategy taking quantity variable of other firm as given. In other words, one of the players follows Cournot and the other Bertrand. From here forth, we write CB competition, if the first player follows Cournot and second Bertrand competition. Similarly, BC competition indicates that the first firm follows Bertrand competition and the other Cournot. The inverse demand function [20] is given by

$$p_{x_1} = f(x_1, x_2),$$

$$p_{x_2} = g(x_1, x_2)$$

Above equation can also be written as

$$x_1 = f^{-1}\left(p_{x_1}, x_2\right) \quad (2.1)$$

$$x_2 = g^{-1}\left(x_1, p_{x_2}\right) \quad (2.2)$$

If c_1 and c_2 are marginal production costs of firm 1 and firm 2 respectively, their corresponding profits are

$$\pi_{x_1} = \left(p_{x_1} - c_1\right)x_1 \quad (2.3)$$

$$\pi_{x_2} = \left(p_{x_2} - c_2\right)x_2 \quad (2.4)$$

In CB competition, firm 1 is quantity setter and firm 2 is price setter. So, variables are x_1 and p_{x_2}. Therefore, rewriting Equations (2.3) and (2.4), we get

$$\pi_{x_1} = \left(f\left(x_1, x_2\right) - c_1\right)x_1$$

$$\pi_{x_1} = f\left(f\left(x_1, g^{-1}\left(x_1, p_{x_2}\right)\right) - c_1\right)x_1$$

$$\pi_{x_1} = \Pi_{x_1}\left(x_1, p_{x_2}\right)$$

and

$$\pi_{x_2} = \left(g\left(x_1, x_2\right) - c_2\right)x_2$$

$$\pi_{x_2} = g\left(g\left(x_1, g^{-1}\left(x_1, p_{x_2}\right)\right) - c_2\right)g^{-1}\left(x_1, p_{x_2}\right)$$

$$\pi_{x_2} = \Pi_{x_2}\left(x_1, p_{x_2}\right)$$

Thus, profit in CB competition of both the firms is a function of two variables, where quantity is variable for the first firm and price for the second firm, as both the firms want to earn maximum profit. For that take partial derivatives of these profit functions w.r.t x_1 and p_{x_2} respectively and then equate to zero. Reaction functions are obtained by solving these equations for value of x_1 and p_{x_2} for which profit is maximum. Moreover, these values of x_1 and p_{x_2} give CB equilibrium [20]. Similarly, in BC competition, firm 1 is the price setter and firm 2 is quantity setter. Value of x_1 is substituted in Equations (2.3) and (2.4) respectively as follows:

$$\pi_{x_1} = \left(f\left(x_1, x_2\right) - c_1\right)x_1$$

$$\pi_{x_1} = \left(f\left(f^{-1}\left(p_{x_1}, x_2\right), x_2\right) - c_1\right)f^{-1}\left(p_{x_1}, x_2\right)$$

$$\pi_{x_1} = \Pi_{x_1}\left(p_{x_1}, x_2\right)$$

and

$$\pi_{x_2} = \left(g\left(x_1, x_2\right) - c_2\right)x_2$$

$$\pi_{x_2} = \left(g\left(f^{-1}\left(p_{x_1}, x_2\right), x_2\right) - c_2\right)x_2$$

$$\pi_{x_2} = \Pi_{x_2}\left(p_{x_1}, x_2\right)$$

As discussed above, take partial derivatives of these profit functions w.r.t. p_{x_1}

and x_2 respectively and then equate to zero. Reaction functions are obtained by solving these equations for values of p_{x_1} and x_2 for which profit is maximum. Moreover, these values of p_{x_1} and x_2 give BC equilibrium.

3. Differentiated Duopoly Model with Linear Demand Function

Here we consider that both the firms are dealing in differentiated goods. Let θ_1 be the measure of extent to which product of both firms differentiate. Linear inverse demand function is given by

$$p_{x_1} = p_m - x_1 - \theta_1 x_2 \tag{3.1}$$

$$p_{x_2} = p_m - \theta_1 x_1 - x_2 \tag{3.2}$$

where p_m denotes the maximum price firms want to gain, $\theta_1 = 1$ means that goods are perfect substitutes and $\theta_1 = 0$ means independent products. Here we take assumption that $0 < \theta_1 < 1$ [21]. Now we want to find CB equilibrium and maximum profits of both the firms. So, we need to find the values of x_1 and x_2 from Equations (3.1) and (3.2). For this multiply Equation (3.2) by θ_1 and subtract from (3.1), which gives

$$\left(1-\theta_1^2\right) x_1 = \left(1-\theta_1\right) p_m - p_{x_1} + \theta_1 p_{x_2}$$

$$\Rightarrow x_1 = \frac{1}{1-\theta_1^2}\left[\left(1-\theta_1\right) p_m - p_{x_1} + \theta_1 p_{x_2}\right] \tag{3.3}$$

Similarly, Multiplying (3.1) by θ_1 and subtract from (3.2), which gives

$$\left(1-\theta_1^2\right) x_2 = \left(1-\theta_1\right) p_m + \theta_1 p_{x_1} - p_{x_2}$$

$$\Rightarrow x_2 = \frac{1}{1-\theta_1^2}\left[\left(1-\theta_1\right) p_m + \theta_1 p_{x_1} - p_{x_2}\right] \tag{3.4}$$

As in CB competition x_1 and p_{x_2} are the variables. So, eliminate x_2 from Equations (3.1) by substituting value of x_2 from (3.2)

$$\Rightarrow p_{x_1} = p_m - x_1 - \theta_1\left(p_m - \theta_1 x_1 - p_{x_2}\right)$$

$$\Rightarrow p_{x_1} = \left(1-\theta_1\right) p_m - \left(1-\theta_1^2\right) x_1 + \theta_1 p_{x_2} \tag{3.5}$$

$$\Rightarrow \pi_{x_1} = \left(p_{x_1} - c_1\right) x_1 \tag{3.6}$$

by using Equation (3.5), Equation (3.6) reduces to

$$\Rightarrow \pi_{x_1} = \left(\left(1-\theta_1\right) p_m - \left(1-\theta_1^2\right) x_1 + \theta_1 p_{x_2} - c_1\right) x_1 \tag{3.7}$$

Also, by using Equation (2.4), we have

$$\pi_{x_2} = \left(p_{x_2} - c_2\right) x_2$$

Further, using Equation (3.4), we get

$$\Rightarrow \pi_{x_2} = \left(p_{x_2} - c_2\right) \frac{1}{1-\theta_1^2}\left\{\left(1-\theta_1\right) p_m + \theta_1 p_{x_1} - p_{x_2}\right\}$$

$$\Rightarrow \pi_{x_2} = \frac{1}{1-\theta_1^2} p_{x_2}(1-\theta_1) p_m + \frac{\theta_1 p_{x_1} p_{x_2}}{1-\theta_1^2} - \frac{1}{1-\theta_1^2}(p_{x_2})^2 - c_2 \frac{(1-\theta_1) p_m + \theta_1 p_{x_1} - p_{x_2}}{1-\theta_1^2}$$

Substituting value of p_{x_1} from Equation (3.5) in π_{x_2}, we get

$$\pi_{x_2} = \frac{p_{x_2}(1-\theta_1) p_m}{1-\theta_1^2} + \frac{\theta_1\left((1-\theta_1) p_m - x_1(1-\theta_1^2) + \theta_1 p_{x_2}\right) p_{x_2}}{1-\theta_1^2} - \frac{1}{1-\theta_1^2}(p_{x_2})^2 - \frac{c_2\left[(1-\theta_1) p_m + \theta_1\left((1-\theta_1) p_m - x_1(1-\theta_1^2) + \theta_1 p_{x_2}\right) - p_{x_2}\right]}{1-\theta_1^2} \tag{3.8}$$

Now from Equations (3.7) and (3.8), it is clear that π_{x_1} and π_{x_2} are functions of variables x_1 and p_{x_2}. So, reactions functions can be obtained by taking

$$\frac{\partial \pi_{x_1}}{\partial x_1} = 0$$

and

$$\frac{\partial \pi_{x_2}}{\partial p_{x_2}} = 0$$

we get

$$(1-\theta_1) p_m - 2(1-\theta_1^2) x_1 + \theta_1 p_{x_2} - c_1 = 0 \tag{3.9}$$

$$\frac{1}{1-\theta_1^2}(1-\theta_1) p_m + \frac{1}{1-\theta_1^2}\left\{\theta_1\left((1-\theta_1) p_m - x_1(1-\theta_1^2) + 2\theta_1 p_{x_2}\right)\right\} - 2\frac{p_{x_2}}{1-\theta_1^2} - \frac{c_2\left[(\theta_1)^2 - 1\right]}{1-\theta_1^2} = 0$$

$$\Rightarrow p_m - \theta_1 x_1 - 2 p_{x_2} + c_2 = 0 \tag{3.10}$$

If we consider the case that production cost of both firms is positive and same *i.e.* homogeneous production cost. Take c_1 and c_2 both zero, and this is typical postulation of linear models. Solving reaction functions given in Equations (3.9) and (3.10) simultaneously, we get equilibrium output in CB competition.

$$(2-\theta_1) p_m = (4-3\theta_1^2) x_1$$

$$\Rightarrow x_1 = \frac{(2-\theta_1) p_m}{4-3\theta_1^2} \tag{3.11}$$

Substituting this value of x_1 in Equation (3.11), we get

$$\Rightarrow p_{x_2} = \frac{(2-\theta_1-\theta_1^2) p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_2} = \frac{(2+\theta_1)(1-\theta_1) p_m}{4-3\theta_1^2} \tag{3.12}$$

Now we need to find x_2 and p_{x_1}, for that substitute value of x_1 and p_{x_2} from (3.11) and (3.12) in Equation (3.5), which is

$$p_{x_1} = (1-\theta_1)p_m - (1-\theta_1^2)x_1 + \theta_1 p_{x_2} \tag{3.13}$$

Then we get

$$p_{x_1} = (1-\theta_1)p_m - \frac{(1-\theta_1^2)(2-\theta_1)p_m}{4-3\theta_1^2} + \frac{\theta_1(2+\theta_1)(1-\theta_1)p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_1} = \frac{\left[(1-\theta_1)(4-3\theta_1^2) - (1-\theta_1^2)(2-\theta_1) + \theta_1(2+\theta_1)(1-\theta_1)\right]p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_1} = \frac{\left[(1-\theta_1)\left\{(4-3\theta_1^2) - (1+\theta_1)(2-\theta_1) + \theta_1(2+\theta_1)\right\}\right]p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_1} = \frac{\left[(1-\theta_1)(2+\theta_1-\theta_1^2)\right]p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_1} = \frac{(1-\theta_1)(1+\theta_1)(2-\theta_1)p_m}{4-3\theta_1^2}$$

$$\Rightarrow p_{x_1} = \frac{(1-\theta_1^2)(2-\theta_1)p_m}{4-3\theta_1^2} \tag{3.14}$$

Next for value of x_2, use values of (3.12) and (3.13) in Equation (3.4), so that

$$x_2 = \frac{1}{1-\theta_1^2}\left[(1-\theta_1)p_m + \frac{\theta_1(1-\theta_1^2)(2-\theta_1)p_m}{4-3\theta_1^2} - \frac{(2+\theta_1)(1-\theta_1)p_m}{4-3\theta_1^2}\right]$$

$$\Rightarrow x_2 = \frac{1-\theta_1}{1-\theta_1^2}\left[\frac{\left((4-3\theta_1^2) + \theta_1(1+\theta_1)(2-\theta_1) - (2+\theta_1)\right)p_m}{4-3\theta_1^2}\right]$$

$$\Rightarrow x_2 = \frac{1}{1+\theta_1}\frac{(2+\theta_1-2\theta_1^2-\theta_1^3)p_m}{4-3\theta_1^2}$$

$$\Rightarrow x_2 = \frac{1}{1+\theta_1}\left[\frac{(\theta_1+1)(1-\theta_1)(2+\theta_1)p_m}{4-3\theta_1^2}\right]$$

$$\Rightarrow x_2 = \frac{(1-\theta_1)(2+\theta_1)p_m}{4-3\theta_1^2} \tag{3.15}$$

So, CB competition Equations (3.11) and (3.14) give equilibrium output and Equations (3.12) and (3.13) give equilibrium price. Substitute these values in profit function to get CB profit

$$\pi_{x_1}^{CB} = \frac{(2-\theta_1)^2(1-\theta_1)^2 p_m^2}{(4-3\theta_1^2)^2}$$

and

$$\pi_{x_2}^{CB} = \frac{(2+\theta_1)^2(1-\theta_1)^2 p_m^2}{(4-3\theta_1^2)^2}$$

of both the firms respectively. When we take the ratio of the output, prices and profit of both the firms, then observations are as follows:

$$\frac{x_1}{x_2} = \frac{2-\theta_1}{2-\theta_1-\theta_1^2} > 1 \quad [0 < \theta_1 < 1]$$

$$\frac{p_{x_1}}{p_{x_2}} = \frac{2+\theta_1-\theta_1^2}{2+\theta_1} < 1$$

and

$$\frac{\pi_{x_1}}{\pi_{x_2}} = \frac{4-3\theta_1^2+\theta_1^3}{4-3\theta_1^2-\theta_1^3} > 1$$

Similarly, in BC competition, the case is reversed only. In this case, firm 1 is price setter and takes price as variable considering output of the other is given and firm 2 is quantity setter, takes quantity as variable considering price of other as given. So, we need to just replace x_1 by x_2 and p_{x_1} by p_{x_2} in the above drawn results. Then observations are as follows:

$$\frac{x_1}{x_2} = \frac{2-\theta_1-\theta_1^2}{2-\theta_1} < 1$$

$$\frac{p_{x_1}}{p_{x_2}} = \frac{2+\theta_1}{2+\theta_1-\theta_1^2} > 1$$

$$\frac{\pi_{x_1}}{\pi_{x_2}} = \frac{4-3\theta_1^2-\theta_1^3}{4-3\theta_1^2+\theta_1^3} < 1$$

Above observations clearly indicate that in both CB and BC competition, out of two firms producing differentiated goods and following heterogeneous strategy, the quantity setter firm produces more output, have fewer prices and enjoys more profits.

4. Duopoly Model with Non-Linear Demand

Here we take an assumption that demand is iso-elastic [22]. Then inverse non-linear demand function is given by

$$p_{x_1} = \frac{1}{x_1+\theta_1 x_2}$$

and

$$p_{x_2} = \frac{1}{\theta_1 x_1 + x_2}$$

$$\Rightarrow x_1 p_{x_1} + \theta_1 p_{x_1} x_2 = 1 \tag{4.1}$$

and

$$\Rightarrow \theta_1 x_1 p_{x_2} + p_{x_2} x_2 = 1 \tag{4.2}$$

Solve these equations for the output of firms x_1 and x_2 Then

$$\Rightarrow x_1 = \frac{1}{1-\theta_1^2}\left(\frac{1}{p_{x_1}} - \frac{\theta_1}{p_{x_2}}\right) \tag{4.3}$$

and

$$x_2 = \frac{1}{1-\theta_1^2}\left(\frac{\theta_1}{p_{x_1}} - \frac{1}{p_{x_2}}\right) \tag{4.4}$$

In CB competition with non-linear demand function, reaction functions will be obtained as in case of linear demand functions. We will get CB reaction functions for firm 1 and firm 2 as

$$\theta_1 p_{x_2} = c_1\left(\left(1-\theta_1^2\right)x_1 p_{x_2} + \theta_1\right)^2 \tag{4.5}$$

$$\theta_1 x_1 p_{x_2}^2 = c_2 \tag{4.6}$$

Here reactions functions are function of two variables x_1 and p_{x_2} as mentioned above. But for sake of convenience, convert the reaction functions in variables x_1 and x_2, and substitute the value of p_{x_2} in Equation (4.5), we get

$$\frac{\theta_1}{\theta_1 x_1 + x_2} = c_1\left(\frac{\left(1-\theta_1^2\right)x_1}{\theta_1 x_1 + x_2} + \theta_1\right)^2$$

$$\theta_1\left(\theta_1 x_1 + x_2\right) = c_1\left(x_1 + \theta_1 x_2\right)^2 \tag{4.7}$$

Similarly, we get

$$\theta_1 x_1 = c_2\left(\theta_1 x_1 + x_2\right)^2 \tag{4.8}$$

Dividing (4.7) by (4.8)

$$\theta_1 + \frac{x_2}{x_1} = \frac{c_1}{c_2}\left(\frac{1+\frac{\theta_1 x_2}{x_1}}{\theta_1 + \frac{x_2}{x_1}}\right)^2$$

Take $y = \frac{x_2}{x_1}$ and $\alpha = \frac{c_2}{c_1}$, Then

$$\alpha\left(\theta_1 + y\right) = \left(\frac{1+\theta_1 y}{\theta_1 + y}\right)^2 \tag{4.9}$$

Let the right hand side of Equation (4.9) be denoted by $F(y)$ and left hand side be denoted by $G_{CB}(y)$. Let the solution of Equation (4.9) be denoted by *Y*. So, intersection of $F(y)$ and $G_{CB}(y)$ gives CB equilibrium.

$$Y = \frac{x_2}{x_1} \tag{4.10}$$

In order to express the value of CB output in terms of *Y*. Use (4.7) to get

$$\theta_1\left(\theta_1 x_1 + x_2\right) = c_1\left(x_1 + \theta_1 x_2\right)^2$$

$$\Rightarrow \theta_1 x_1\left(\theta_1 + \frac{x_2}{x_1}\right) = c_1 x_1^2\left(1 + \frac{\theta_1 x_2}{x_1}\right)^2$$

$$\Rightarrow \theta_1 x_1\left(\theta_1 + Y\right) = c_1 x_1^2\left(1 + \theta_1 Y\right)^2$$

$$\Rightarrow x_1 = \frac{\theta_1(\theta_1 + Y)}{c_1(1+\theta_1 Y)^2}$$

Also from (4.8)

$$\theta_1 = c_2 x_1 \left(\theta_1 + \frac{x_2}{x_1}\right)^2$$

$$\Rightarrow x_1 = \frac{\theta_1}{c_2(\theta_1 + Y)^2} \tag{4.11}$$

Similarly

$$\Rightarrow x_2 = \frac{Y\theta_1(\theta_1 + Y)}{c_1(1+\theta_1 Y)^2} = \frac{Y\theta_1}{c_2(\theta_1 + Y)^2} \tag{4.12}$$

Also express prices and profits of both the firms in terms of parameter Y so that it can be compared. For this substitute values of x_1 and x_2 in p_{x_1} and p_{x_2} to get

$$\Rightarrow p_{x_1} = \frac{c_2(\theta_1 + Y)^2}{\theta_1(\theta_1 Y + 1)}$$

$$\Rightarrow p_{x_1} = \frac{1}{x_1^{CB}(\theta_1 Y + 1)} \tag{4.13}$$

Similarly

$$\Rightarrow p_{x_2} = \frac{1}{x_1^{CB}(\theta_1 + Y)} \tag{4.14}$$

Now, profit in CB equilibrium of firm 1 is

$$\pi_{x_1} = \left(p_{x_1} - c_1\right) x_1^{CB}$$

$$\pi_{x_1} = \left(\frac{1}{(1+\theta_1 Y) x_1^{CB}} - c_1\right) x_1^{CB}$$

$$\pi_{x_1} = \frac{1}{1+\theta_1 Y} - \frac{\theta_1(\theta_1 + Y)}{(1+\theta_1 Y)^2}$$

$$\pi_{x_1} = \frac{1-\theta_1^2}{(1+\theta_1 Y)^2}$$

In the similar way

$$\pi_{x_2} = \frac{Y^2 \alpha}{1-\theta_1^2}\left(\frac{\theta_1 Y + 1}{\theta_1 + Y}\right)^2$$

Taking ratio of output, prices and profits of both the firms, we get

$$\frac{x_2}{x_1} = Y, \frac{p_{x_2}}{p_{x_1}} = \frac{\theta_1 Y + 1}{\theta_1 + Y} \text{ and } \frac{\pi_{x_2}}{\pi_{x_1}} = \frac{Y^2(\theta_1 Y + 1)^2}{1-\theta_1^2(\theta_1 + Y)^2} \tag{4.15}$$

From (4.9) and (4.10), it is clear that

$$G_{CB}(Y) = F(Y)$$

Now

$$\frac{\theta_1 Y+1}{\theta_1+Y} \geq 1 \text{ or } \frac{\theta_1 Y+1}{\theta_1+Y} \leq 1$$

According as $Y \leq 1$ or $Y \geq 1$, $F(1)=1$, and considering $G_{CB}(1)=1$, we get a critical value of production cost ratio α. Let this value be denoted by $\bar{\alpha}$, then

$$\bar{\alpha}=\frac{1}{1+\theta_1}<1, \quad 0<\theta_1<1$$

As mentioned above, If $Y>1$, $\frac{\theta_1 Y+1}{\theta_1+Y}<1$. So, from Equation (4.9), it is obtained that

$$\alpha(\theta_1+Y)<1$$

$$\Rightarrow \alpha<\frac{1}{\theta_1+Y}<\frac{1}{\theta_1+1}=\bar{\alpha}$$

Similarly, for $Y<1$, $\frac{\theta_1 Y+1}{\theta_1+Y}>1$. Then, Equation (4.9) gives

$$\alpha(\theta_1+Y)>1$$

$$\Rightarrow \alpha>\frac{1}{\theta_1+Y}>\frac{1}{\theta_1+1}=\bar{\alpha}$$

So, $\frac{x_2^{CB}}{x_1^{CB}}=Y$, <1, =1 or >1 And $\frac{p_{x_2}}{p_{x_1}}=\frac{\theta_1 Y+1}{\theta_1+Y} \geq 1$ or $\frac{\theta_1 Y+1}{\theta_1+Y} \leq 1$

According as $\alpha>\bar{\alpha},=\bar{\alpha}$ or $\alpha<\bar{\alpha}$.

This means that the parameter region $\{(\theta_1,\alpha): 0<\theta_1<1, \alpha>0\}$ has two sub-regions, below and above the curve $\alpha=\bar{\alpha}$. $x_1^{CB}>x_2^{CB}$ and $p_{x_1}^{CB}<p_{x_2}^{CB}$ for (θ_1,α) in the region above the curve. $x_1^{CB}<x_2^{CB}$ and $p_{x_1}^{CB}>p_{x_2}^{CB}$ below the curve $\alpha=\bar{\alpha}$. This can be interpreted that whichever firm produces more output sells at lower prices.

Bertrand Cournot Competitions

In BC competition, firm 1 is the price setter and the second is quantity setter. Following the above stated procedure, the reaction functions are

$$\theta_1 x_2 p_{x_1}^2=c_1 \tag{4.16}$$

$$\theta_1 p_{x_1}=c_2\left(\theta_1-\left(1-\theta_1^2\right)x_2 p_{x_1}\right)^2 \tag{4.17}$$

The reaction functions given in above equations are defined in (p_{x_1}, x_2) space. BC equilibrium will be converted in quantity space (x_1, x_2) by substituting values of p_{x_1} in these equations. Equation (4.16) becomes

$$\theta_1 x_2=c_1\left(x_1+\theta_1 x_2\right)^2 \tag{4.18}$$

Similarly, Equation (4.17) becomes

$$\theta_1\left(x_1+\theta_1 x_2\right)=c_2\left[\left(1-\theta_1^2\right)x_2+\theta_1\left(x_1+\theta_1 x_2\right)\right]^2$$

i.e.

$$\theta_1\left(x_1+\theta_1 x_2\right)=c_2\left[x_2+\theta_1 x_1\right]^2 \tag{4.19}$$

Equations (4.18) and (4.19) are defined in $\left(x_1,x_2\right)$ space. Dividing (4.18) and (4.19) gives

$$\frac{\theta_1\left(x_1+\theta_1 x_2\right)}{\theta_1 x_2}=\frac{c_2}{c_1}\left(\frac{x_2+\theta_1 x_1}{x_1+\theta_1 x_2}\right)^2$$

Taking $\alpha=\frac{c_2}{c_1}, y=\frac{x_2}{x_1}$, above equation becomes,

$$\frac{\alpha y}{1+\theta_1 y}=\left(\frac{\theta_1 y+1}{\theta_1+y}\right)^2 \tag{4.20}$$

Right hand side of Equation (4.20) and Equation (4.9) is the same.

As above, denote right hand side by $F\left(y\right)$ and left hand side by $G_{BC}\left(y\right)$. Solution of Equation (4.20) will be given by intersection of these two functions.

This solution is BC equilibrium.

Since $G_{BC}\left(y\right)$ is monotonically increasing and $\lim_{y\to\infty}G_{BC}\left(y\right)=\frac{\alpha}{\theta_1}$. So, $G_{BC}\left(y\right)$ is bounded above. Also, $\lim_{y\to\infty}F\left(y\right)=\theta_1^2$.

Here, $\lim_{y\to\infty}G_{BC}\left(y\right)>\lim_{y\to\infty}F\left(y\right)$.

This is equivalent to $\alpha>\theta_1^3$.

So, here conjecture is $\alpha>\theta_1^3$.

Let roots of Equation (4.20) be denoted by *Z*, which is a function of θ_1 and α.

$$Z=\frac{x_2^{BC}}{x_1^{BC}} \text{ and } \frac{\alpha Z}{1+\theta_1 Z}=\left(\frac{1+\theta_1 Z}{\theta_1+Z}\right)^2 \tag{4.21}$$

Substituting x_1^{BC} and $x_2^{BC}=Zx_1^{BC}$ in Equations (4.18) and (4.19), in order to get explicit form of BC output in terms of parameters. Then

$$x_1^{BC}=\frac{\theta_1 Z}{c_1\left(1+\theta_1 Z\right)^2}=\frac{\theta_1\left(1+\theta_1 Z\right)}{c_2\left(\theta_1+Z\right)^2}>0 \tag{4.22}$$

and for finding value of x_2^{BC}, take $x_1^{BC}=\frac{x_2^{BC}}{Z}$ in Equations (4.18) and (4.19).

$$x_2^{BC}=\frac{\theta_1 Z^2}{c_1\left(1+\theta_1 Z\right)^2}=\frac{\theta_1 Z\left(1+\theta_1 Z\right)}{c_2\left(\theta_1+Z\right)^2}>0 \tag{4.23}$$

Substituting values of x_1^{BC} and x_2^{BC} from Equations (4.22) and (4.23) in equations

$$p_{x_1}=\frac{1}{x_1+\theta_1 x_2} \text{ and } p_{x_2}=\frac{1}{x_2+\theta_1 x_1}$$

BC prices are

$$p_{x_1}^{BC} = \frac{1}{(1+\theta_1 Z)x_1^{BC}} \text{ and } p_{x_2}^{BC} = \frac{1}{(Z+\theta_1)x_1^{BC}}$$

Their ratios are

$$\frac{p_{x_2}^{BC}}{p_{x_1}^{BC}} = \frac{1+\theta_1 Z}{\theta_1 + Z} \tag{4.24}$$

As mentioned above, $G_{BC}(1)=1$, this will give a critical value of production cost ratio, denote it by $\bar{\alpha}$. From expression (4.20), using $G_{BC}(1)=1$, it is obtained that $\bar{\alpha}=1+\theta_1$.

If $Z>1$

$$1+\theta_1 Z \le Z+\theta_1 \quad [0<\theta_1<1]$$

i.e. $\frac{1+\theta_1 Z}{Z+\theta_1} \le 1$

From Equation (4.21), it is clear that

$$\frac{\alpha Z}{1+\theta_1 Z} < 1$$

i.e. $\alpha < \frac{1+\theta_1 Z}{Z} = \frac{1}{Z}+\theta_1$

i.e. $\alpha < 1+\theta_1 \quad \left[\frac{1}{Z}<1\right]$

i.e. $\alpha < \bar{\alpha}$

Similarly, for $Z<1$, $\frac{1+\theta_1 Z}{Z+\theta_1} \ge 1$, $\alpha > \bar{\alpha}$.

This means output of firm 1 is more than that of firm 2, and its prices are lesser above the critical line $\bar{\alpha}=1+\theta_1$. But in the region below the critical line, firm 2 produces more output than the firm 1 and set lower prices.

When repeating the above procedure, we reach the conclusion that efficient firm produces more output and sells at lower price. Here efficient means production cost is less.

5. Conclusions

In a duopoly market system when both firms choose heterogeneous strategies *i.e.* one of the firms chooses quantity strategies and the other chooses price strategies, it is observed that when production costs are homogeneous, then quantity setter firms produce more output, face lower prices, and make larger profits than price setter firm. So, the quantity strategy is more profitable. When strategies and production costs are heterogeneous, the efficiency of the firms is the determining factor to decide which firm will produce more and sell at a lower price.

This paper studies the static aspects of the non-linear heterogeneous duopoly model. There is a future scope for investigating the dynamical aspects with numerical simulation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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