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New Approach to the Creation of General Theory of Relativity

Valentin V. Mdzinarishvili

Tbilis, Georgia
Email: ValentinmdzinariSvili@gmail.com

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Abstract

It has to be noted that Albert Einstein used the extremal property of geodesic line when developing the general theory of relativity (GTR). In the following years, after the development of the GTR, it became known that the expression for a geodesic line can be replaced by the Euler-Lagrange equation. In the present work, the extremal property of the separatrix of a mathematical pendulum is used to develop a new approach to the general theory of relativity. The pendulum separatrix satisfies the Euler-Lagrange equation. A transcendental equation has been obtained, the solution of which makes it possible to determine the angle of rotation of the separatrix φ_s as a function for the angle of rotation of the mathematical pendulum. It makes possible for a fixed point $\varphi_{s,i}$ located on the separatrix, to find the corresponding value of the function $\mathrm{th}\left(\varphi_{s,i}\right)$ used in determining the speed v_i of movement of this point according to the general theory of relativity. The proposed approach is invariant to the distance along which the point moves. The examples illustrating the proposed approach have been considered.

Subject Areas

Classical Physics, Modern Physics

Keywords

Mathematical Pendulum, Separatrix, Hamiltonian Equation, Riemannian Surface, Pseudo-Riemannian Surface, Lorentz Transformation

1. Introduction

Einstein used the extremal (minimal) property of geodesic line when developing the general theory of relativity (GTR). Geodesic lines are curves in R^3 whose arc length between two given points has a minimum value. In this case, the variation of the arc length δs between these points should be equal to zero. The square of the infinitely small distance between these points on the arc is

$$\begin{aligned}(\mathrm{d}s)^2 = g_{11}(\mathrm{d}x_1)^2 + g_{12}\mathrm{d}x_1\mathrm{d}x_2 + g_{13}\mathrm{d}x_1\mathrm{d}x_3 + g_{21}\mathrm{d}x_2\mathrm{d}x_1 + g_{22}(\mathrm{d}x_2)^2 \\ + g_{23}\mathrm{d}x_2\mathrm{d}x_3 + g_{31}\mathrm{d}x_3\mathrm{d}x_1 + g_{32}\mathrm{d}x_3\mathrm{d}x_2 + g_{33}(\mathrm{d}x_3)^2,\end{aligned} \tag{1}$$

where g_{ij} - *i, j*th component of "metric tensor" of body curvature, each component of which is characterized by three coordinates x_1, x_2, x_3. Let us compose from the components of the curvature tensor a square **Table 1** with the size 3 × 3:

Of the nine components of the curvature tensor given in **Table 1**, only six will be independent—three of them are located on the diagonal of **Table 1** and another half of the six components $g_{21} = g_{12}$, $g_{31} = g_{13}$, $g_{23} = g_{32}$, symmetrically located on both sides of the diagonal, *i.e.*, a total of six components will be subject to determination.

In cosmic space, the position of the body is characterized by four coordinates x_1, x_2, x_3, t (t is current time). In this case, instead of **Table 1**, we will have a table consisting of the components of the curvature tensor, 4 × 4 in size with ten independent components to be determined [1]. Some of these components will already depend on time: they will not be constant, but change forming a surface called a "field". However, these ten components of the curvature tensor impose too many restrictions on the equations that define them. As a result, the equations of gravitation for a region without matter are obtained [2]. This circumstance became the reason that Einstein was not fully satisfied with the GTR developed by him.

The purpose of this work is to substantiate the possibility (the proof) of using the extremal property of the separatrix of a mathematical pendulum instead of the geodesic line used when developing the GTR.

2. Considerations for Justifying the Replacement of the Geodesic Line by the Euler-Lagrange Equation

In following years after developing the GTR, it became known that variational methods make it possible to replace the expression of a geodesic line with the Euler-Langrage equation (see, for example [3]). As is known, the Euler-Langrage equation is equivalent to the Hamilton equation:

$$0 = \frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\cdot\frac{\partial L}{\partial p} \equiv \frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial H}{\partial q} = 0, \tag{2}$$

where $L(q,p) = T - U$ Lagrangian, $H = T + U$ -Hamiltonian, $T(q,p)$ kinetic energy, $U(q)$ potential energy, q generalized coordinate, $p = \dot{q}$ generalized

Table 1. The components of curvature tensor.

g_{11}	g_{12}	g_{13}
g_{21}	g_{22}	g_{23}
g_{31}	g_{32}	g_{33}

impuls.

Let's establish the connection of the geodesic line with the Euler-Lagrange equation. As noted above, the variation in arc length between two points belonging to R^3 must equal zero

$$\delta\int_{t_0}^{t_1} \mathrm{d}s = 0. \tag{3}$$

According to the expression (1) the linear element of the arc is define as:

$$\mathrm{d}s = \sqrt{\sum_{i,k} g_{ik}\,\mathrm{d}u_i\,\mathrm{d}u_k} = \sqrt{\sum_{i,k} g_{ik}\dot{u}_i\dot{u}_k}\,\mathrm{d}t,$$

where

$$\dot{u}_i = \frac{\mathrm{d}u_i}{\mathrm{d}t},\ \dot{u}_k = \frac{\mathrm{d}u}{\mathrm{d}t}.$$

Therefore, geodesic lines are defined from Equation (3)

$$\delta\int_{t_0}^{t_1} \sqrt{\sum_{i,k} g_{ik}\dot{u}_i\dot{u}_k}\,\mathrm{d}t = \delta\int_{t_0}^{t_1} L\mathrm{d}t = 0. \tag{4}$$

The expression L depends on u_i and u_k, which in turn are functions of the parameter t. If we calculate the variation (4), we get

$$\delta\int_{t_0}^{t_1} L\mathrm{d}t = \int_{t_0}^{t_1}\sum_{i,k}\left(\frac{\partial L}{\partial u_i}\delta u_i + \frac{\partial L}{\partial \dot{u}_i}\frac{\mathrm{d}}{\mathrm{d}t}\delta \overset{\circ}{u}_i\right)\mathrm{d}t.$$

Integrating by parts the last expression, we obtain

$$\delta\int_{t_0}^{t_1} L\mathrm{d}t = \left[\sum_{i,k}\frac{\partial L}{\partial \dot{u}_i}\delta u_i\right]_{t_0}^{t_1} + \int_{t_0}^{t_1}\sum_{i,k}\left(\frac{\partial L}{\partial u_i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{u}_i}\right)\delta u_i\mathrm{d}t. \tag{5}$$

Due to the fact that only the lengths of the arcs of the curves between two fixed points are compared with each other, the variation of δu_i is equal to zero and, therefore, the component in square bracket (5) is equal to zero; hence, we have $\delta\int L\mathrm{d}t = 0$. So, we have Equations (2) and (5)as

$$\delta\int L\mathrm{d}t = 0.$$

3. Determination of the Separatrix of the Mathematical Pendulum Used in the Development of the GTR

In monograph [4], it is noted that Euler-Lagrange equation is satisfied by an extremal, and in monograph [5], it is shown that the integral from a soliton is an extremal. It was also established there that the integral from a soliton is equivalent to the separatrix of a mathematical pendulum. This means that the separatrix is an extremal, *i.e.*, the separatrix satisfies the Euler-Lagrange equation. In order to determine the separatrix of a mathematical pendulum, it is necessary to have the equations of the mathematical pendulum itself.

The equations of a mathematical pendulum have the form

$$\dot{P} = -F\cos\varphi,\ \ \dot{\varphi} = GP, \tag{6}$$

where $F = hmg$, $G = 1/mh^2$, mg is force of gravity acting on mass m, h denotes the pendulum length.

The Hamiltonian of the pendulum is the sum of the kinetic energy $\frac{1}{2}Gp^2$ and potential energy $U = -F\cos\varphi$:

$$H = \frac{1}{2}Gp^2 - F\cos\varphi = E, \tag{7}$$

where E is a total energy of mathematical pendulum (6).

If E is greater than the maximum value of the potential energy, then the impulse is always other than zero. This leads to an unlimited change φ, *i.e.*, to the rotation. In this case, $p > 0$ motion is from left to right with energies E_u. For $E < F$, the motion is limited (within potential pit) and corresponds to the oscillations of the pendulum. If $E = F \equiv E_s$, then the motion occurs along the separatrix.

The coordinate φ, and impulse p of the mathematical pendulum satisfy the Hamiltonian Equation (2)

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\partial H}{\partial p} = Gp, \quad \frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial H}{\partial \varphi} = -F\sin\varphi. \tag{8}$$

Now let us find the separatrix equation, using the Hamiltonian (7) and condition $E = F$:

$$p_s = \frac{2^{1/2}\omega_0}{G}\left(1 + \cos\varphi_s\right)^{1/2}, \tag{9}$$

where $\omega_0 = (FG)^{1/2}$, and index s corresponds to the values of the variables on the separatrix.

From (9) it follows

$$p_s = \pm\frac{2\omega_0}{G}\cos\frac{\varphi_s}{2}, \tag{10}$$

where plus and minus correspond to the upper and lower branches of the separatrix.

Application of the first Hamiltonian Equation (8), with account of (10) gives

$$\frac{\mathrm{d}\varphi_s}{\mathrm{d}t} = \pm 2\omega_0\cos\frac{\varphi_s}{2}. \tag{11}$$

Solving Equation (11) with respect to dt and integrating with the initial condition $\varphi = 0$ for $t = 0$, we will have

$$\pm\omega_0 t = \int_0^{\varphi_{\pm s}} \frac{\mathrm{d}(\varphi/2)}{\cos(\varphi/2)} = \mathrm{Intg}\left|\frac{\varphi_{\pm s}}{4} + \frac{\pi}{4}\right|. \tag{12}$$

Expression (12) requires a joint (integral) representation of the masses ω_0 and time t.

The formula (12) can be written separately for the plus and minus signs in the function)

$$\mathrm{Intg}\left[\left|\pm\left(\frac{\varphi_{\pm s}}{4}+\frac{\pi}{4}\right)\right|\right]:$$

$$+\omega_0 t=\int_0^{\varphi_{+s}}\frac{\mathrm{d}(\varphi/2)}{\cos(\varphi/2)}=\mathrm{Intg}\left(\frac{\varphi_{+s}}{4}+\frac{\pi}{4}\right), \tag{12a}$$

$$-\omega_0 t=\int_0^{\varphi_{-s}}\frac{\mathrm{d}(\varphi/2)}{\cos(\varphi/2)}=\mathrm{Intg}\left[\left|-\left(\frac{\varphi_{-s}}{4}+\frac{\pi}{4}\right)\right|\right]. \tag{12b}$$

The choice of the sign in formulas (12a) and (12b) is made is accordance with the direction of the separatrixmotion shown in Figure 1.

After the reversal of the formulas (12a) and (12b), we will have

$$\varphi_{+s}=4\mathrm{arctg}\left[\exp(\omega_0 t)\right]-\pi\text{, for }\ \omega_0 t\subset[\pi,2\pi], \tag{13a}$$

$$\varphi_{-s}=4\mathrm{arctg}\left[+\exp(-\omega_0 t)\right]+\pi\text{, for }\ \omega_0 t\subset[0,\pi]. \tag{13b}$$

In Figure 1 the section BA of the separatrix ABC, is equal to

$$\varphi_{|-s|}=\left|-4\mathrm{arctg}\left[\exp(-\omega_0 t)\right]\right|+\pi\text{, for }\ \omega_0 t\subset[0,\pi]. \tag{13c}$$

Without taking into account the direction of motion of the separatrix (7).

Consequently, the formula (13c) is considered on the descending section of the separatrix.

The segment BC of the separatrix ABC is determined by the formula (13a). On the whole, the entire separatrix ABC (Figure 1) is the sum of these sections:

$$\begin{gathered}\varphi_{SABC}=\varphi_{|-s|}+\varphi_{+s}=\left|-4\mathrm{arctg}\left[\exp(-\omega_0 t)\right]\right|,\\ \omega_0 t\subset[0,\pi]+4\mathrm{arctg}\left[\exp(\omega_0 t)\right],\ \omega_0 t\subset[\pi,2\pi].\end{gathered} \tag{13d}$$

Let us turn to the segment of the separatrix AB, shown in Figure 1 which corresponds to the formula (13d). On the interval $[0,\pi]$ the separatrix segment ABC is characterized by a concave, descending branch of the BA, *i.e.*, by some line of a pseudo-Riemannian surface; it is defined by the formula (13b) with account of sign "–":

$$\varphi_{-s}=-4\mathrm{arctg}\left[\exp(-\omega_0 t)\right], \tag{14}$$

without taking into consideration term π.

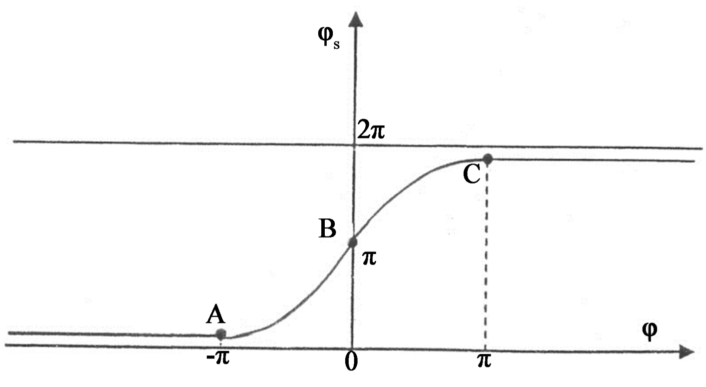

Figure 1. The separatrix ABC.

On the interval $[\pi, 2\pi]$ the separatrix segment ABC is a convex ascending branch of BC, *i.e.* a certain line of the Riemannian surface; it is defined by the formula (13a) with account of sign "+":

$$\varphi_{+S} = 4\operatorname{arctg}\left[\exp(\omega_0 t)\right], \tag{15}$$

without taking into consideration the term π.

Consequently, instead of formulas (14) and (15), the separatrix ABC can be described by a single formula

$$\varphi_S \equiv \varphi_{SABC} = \begin{cases} \left|-4\operatorname{arctg}\left[\exp(-\omega_0 t)\right]\right|, & \text{for } \omega_0 t \subset [0, \pi], \\ 4\operatorname{arctg}\left[\exp(\omega_0 t)\right], & \text{for } \omega_0 t \subset [\pi, 2\pi]. \end{cases} \tag{16}$$

Now, let us find the asymptotes of the separatrix ABC (**Figure 1**). The solution of the lower Equation (16) for $\omega_0 t = \varphi_S$ allows us to determine the upper asymptote $\varphi_{S\max} = 6.275659\cdots$. We used a personal computer to solve the transcendental Equation (16) in an iterative way. Saturation of six digits after the decimal began after six iteration steps, starting with the first step equal to one. Since the curve ABC is symmetric about the axis $O\varphi_S$, the lower asymptote is determined by the difference $\varphi_{S\min} = 2\pi - \varphi_{S\max} = 0.006525\cdots$

4. Modern Interpretation of the Theory of Relativity [6]

The first step towards the creation of GTR was made by Lorentz. He wrote down the transformation that belonged to him in relation to the mass m of a body moving at a speed of v:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}, \tag{17}$$

where m is the body mass in the state of rest, *i.e.*, when $v = 0$, c is speed of light in vacuum.

To determine the body mass m moving with a velocity v commensurate with the speed of light c, it is necessary to use formula (17). For this purpose, the value of the ABC curve should be displayed on the hyperbolic tangent function, *i.e.*, for a fixed point $\varphi_{s,i}$ of the curve ABC, you need to find the corresponding value of the function $\operatorname{th}(\varphi_{s,i}) = v_i/c$. Correspondingly, the velocity of the body at a point I will be defined from relation

$$v_i = c\left\{\operatorname{th}\left[4\operatorname{arctg}\left(e^{\varphi_i}\right)\right]\right\}, \quad -\pi \le \varphi_i \le \pi, \tag{18}$$

whose mass at the point I is an expression (17)

$$m_i = m_0 \operatorname{ch}\left[4\operatorname{arctg}\left(e^{\varphi_i}\right)\right], \quad -\pi \le \varphi_i \le \pi. \tag{19}$$

The use of formulas (18) and (19) is illustrated in five examples.

1) Assume $\varphi = -2$, where $\varphi_{S,-2} = 4\operatorname{arctg}\left(e^{-2}\right) = 0.538072$. Speed of the body will be $v = c\left[\operatorname{th}(0.538072)\right]$, and the body weight will be equal to $m = m_0 \operatorname{ch}(0.538072)$.

2) Assume $\varphi = -1$, where $\varphi_{S,-1} = 4\operatorname{arctg}\left(e^{-1}\right) = 1.410054$. Speed of the body will be $v = c\left[\operatorname{th}\left(1.410054\right)\right]$, and the body weight will be equal to $m = m_0 \operatorname{ch}\left(1.410054\right)$.

3) If $\varphi = 0$, then we will have: $\varphi_{S,0} = 4\operatorname{arctg}\left(e^{0}\right) = \pi = 3.141592$. In such a case speed of the body is equal to $v = c\left[\operatorname{th}\left(3.141592\right)\right]$, and the body weight will be: $m = m_0 \operatorname{ch}\left(3.141592\right)$.

4) If $\varphi = 1$, we have $\varphi_{S,1} = 4\operatorname{arctg}\left(e^{1}\right) = 4.873132$. The speed of the body $v = c\left[\operatorname{th}\left(4.873132\right)\right]$, and the corresponding body weight is equal to $m = m_0 \operatorname{ch}\left(4.873132\right)$.

5) If we assume that $\varphi = 2$, then we will have: $\varphi_{S,2} = 4\operatorname{arctg}\left(e^{2}\right) = 5.745113$, speed of the body will be $v = c\left[\operatorname{th}\left(5.745113\right)\right]$, and the body weight will be: $m = m_0 \operatorname{ch}\left(5.745113\right)$.

Thus, it follows from the Lorentz transformations that the state (*i.e.*, the value) of a physical quantity (in this case mass) depends on its speed, if this speed is commensurable with the speed of light. For an arbitrary physical quantity (length, time, etc.), the above can be generalized by using hyperbolic functions.

Let us find the length ℓ_i of a certain body moving with the velocity v_i corresponding to point *i* of the function ABC (see **Figure 1**), and commensurable to the speed of light *c*. The velocity of this body, expressed in terms of the speed of light, is found from formula (18). The length of a given body, determined with the use of the Lorentz transformation, is reduced in accordance with expression

$$\ell_i = L \operatorname{sech}\left[4\operatorname{arctg}\left(e^{\varphi_i}\right)\right] < L, \quad -\pi \le \varphi_i \le \pi,$$

where *L* is the length of the body at rest, *i.e.* when $v = 0$.

In the case when the system moves at a speed corresponding to the first point of the ABC function (see **Figure 1**), not only the length of this system (body) decrease, but the time flow τ_i also slows down according to formula

$$\tau_i = t \operatorname{sech}\left[4\operatorname{arctg}\left(e^{\varphi_i}\right)\right] < t, \quad -\pi \le \varphi_i \le \pi,$$

where *t* is the time flow in a stationary system ($v = 0$).

Consequently, the use of the above technique for determining the changes in physical quantities (mass increase, shortening and slowing down of the time flow) of a system moving with a speed commensurate with the speed of light gives the same results as classical GTR.

5. Results

The creation of general theory of relativity proposed by Einstein has been significantly improved. An improved approach to the creation of GTR is carried out in two stages.

At the first stage, the transcendental Equation (16) is solved. This allows us to determine the type of separatrix on the plane $\left(\varphi_s, \varphi\right)$, *i.e.*, to determine the angle of rotation of the separatrix φ_s as a function from the current angle of rotation of the mathematical pendulum φ.

At the second stage, a specific point of the separatrix is mapped onto the hyperbolic tangent function, *i.e.*, for a fixed point i of the separatrix φ_{si}, the corresponding value of the function $\text{th}\left(\varphi_{s,i}\right)$ is found. This makes possible to determine the speed and mass of the body located at the point *i* of the separatrix. During the practical implementation of both stages, it is advisable to use a computer.

6. Conclusion

It is necessary to pay attention to the fact that the form of the separatrix does not depend on the distances at which the speed of the moving body is determined. Therefore, the present approach to GTR is invariant to the distances over which the body moves. Therefore, the author of this article does not agree with the opinion that GTR can be applied only at large distances [7].

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Conflicts of Interest

The author declares no conflicts of interest.

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