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Addressing Traffic Congestion by Using of the Counter-Intuitive Phenomenon of Braess' Paradox in Transportation Networks

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Abstract

Vehicular traffic congestion is an extremely dangerous problem in urban areas, where transportation networks are becoming more complicated to design and execute. However, the apparent normal idea to build more roads in order to solve the traffic congestion can be sometimes quite a bad idea. The Braess' paradox, which is a counter-intuitive phenomenon that can occur in transportation networks, states that under certain circumstances the addition of a new road to a traffic network can increase the travel times for all network users. This could impact the design of new traffic networks and the extension of existing ones. By utilizing Braess' paradox, transportation planners can analyze the traffic flow situation in a road network before adding new roads as the redistribution of the traffic flow may increase the average travel time, and hence, making the traffic congestion even worse. This paper explains this phenomenon in order to avoid possible negative consequences resulting from the construction of new roads, since the capacity of the road networks of many cities has long been reached and space for the construction of new roads is limited.

Subject Areas

Applied Statistical Mathematics, Mathematical Analysis, Network Modeling and Simulation

Keywords

Braess' Paradox, Transportation Networks, Equilibrium of Traffic Flow, Traffic Congestion, Transportation Planning

1. Introduction

Many real-world phenomena can be explained in terms of networks. For example, when considering the traffic congestion within a city or country, the road network shows the possible routes that cars and trucks might take, and it provides limits on the traffic volume that can be managed. An electric network describes how electricity can flow. The social network of friends might describe who could communicate with whom to spread news. The computer networks are connected to one another by physical cables.

A network consists of objects with connections between them. In a road network, the objects may be the cities, while the connections might present the roads between them. The properties of the objects could correspond to the number of cars in the cities, while the properties of the connections could correspond to the lengths of the roads, the number of lanes they consist of, the amount of traffic, etc [1] [2] [3].

In mathematics a network is called a graph, and objects are called vertices (or nodes) and the connections are called edges or routes. For example, when we show the road network of a country as a graph, the vertices are the cities under consideration, while the edges list all the roads between them.

A transportation network, for a single mode (for example, air, rail, road, or water) can be specified by [2]-[14]:

- A network, where a network consists of links between nodes. Links can be one-way or two-way.
- A cost for traversing each link. The cost can be a function of the demand (the amount of traffic traversing that link).
- Demands on the network, as specified by source nodes for users and the destination of each user.
- Objective functions for the users, such as the minimization of trip time or the maximization of the probability that total trip time will not exceed a given maximization.

Braess' paradox states that sometimes adding one or more roads to an existing road network can slow down overall traffic flow throughout the network. The paradox was discovered by German mathematician Dietrich Braess in 1968. Under this paradox, an improvement to a transportation network, and thus an increase in the number of choices available to users of the network, can result in decreased performance [15] [16]. The arising question is that does add a route choice always bring more congestion? Or does road building generate traffic? The correct answer is "not necessarily". For example, one research project looked at routes through the city of Boston and found that of the 246 possible links on a journey between Harvard Square and Boston Common, closing one of six particular links did display the Braess' Paradox of improving traffic flow, but closing one of the other 240 did make things worse [7] [8]. So specifically, how can road additions induce the Braess' Paradox? Essentially, they occur when an improvement attracts a significant volume of traffic that the approaches cannot handle. If the induced congestion on these approaches affects other routes, then the whole system suffers as a result [17]-[29]. There are many simulation software's that can be used to apply the Braess' paradox to a traffic network, such as the "MassMotion" model to test the impact of crowdy traffic flow on the design and operations before and after a change in traffic network.

Therefore, transportation engineers, and planners can effectively make use of Braess' paradox to decide whether to add new roads to the network under consideration to address traffic congestion.

2. Background History

Braess' paradox has been used to explain many real-life cases of improved traffic flow when existing major roads are closed. Furthermore, this paradox does not only hold in transportation networks but may have analogies in electrical power grids and biological systems as well [3] [5] [8] [12].

For instance, in Stuttgart, Germany, after big investments into the road network in 1969, the traffic flow did not improve until a section of newly built road was closed for traffic. On Earth Day in 1990, the traffic authorities in New York City decided to close down the 42nd Street for the parade, an always congested street, predicting to see chaos and congestion in the city. But, as it turned out, this prediction was completely false, the traffic situation actually improved when the street was closed down. Twenty years later, in 2010, the authorities of New York decided to ban vehicles on Broadway from 47th to 42nd Streets and from 35th to 33rd Streets. The two main motivating reasons behind this action were improving traffic flow and pedestrian safety. They observe that some streets get more congested while other streets get less congested. But on average the whole road network does not get much more congested than what it was before [30].

In 1999 one of the three main traffic tunnels in South Korea's capital city was shut down for maintenance. Despite this road being heavily used for traffic, the result was not the predicted chaos and jams, instead the traffic flows improved in the city. Inspired by this phenomenon, Seoul's city planners demolished a motorway leading into the heart of the city and experienced the same strange result, with the added benefit of creating a 1000 acre park for the local inhabitants [31]. It is counter-intuitive that you can improve commuters' travel times by reducing route options, because all planners normally want to improve things by adding routes. In 2008 Youn, Gastner and Jeong demonstrated specific routes in Boston, New York City and London where that might actually occur and specified roads that could be closed to reduce travel times [32].

In 2012, an international team of researchers showed that Braess' paradox may occur in mesoscopic electron systems. In particular, they showed that adding a path for electrons in a nanoscopic network paradoxically reduced its conductance. That was shown both by simulations as well as experiments at low temperature using as scanning gate microscopy [33] [34].

Adilson E. Motter and collaborators demonstrated that Braess' paradox outcomes may often occur in biological and ecological systems. Motter suggests removing part of a perturbed network could rescue it. For resource management of endangered species food webs, in which extinction of many species might follow sequentially, selective removal of a doomed species from the network could bring about the positive outcome of preventing a series of further extinctions. [35] [36]

Also, in basketball, a team can be seen as a network of possible routes to scoring a basket, with a different efficiency for each pathway, and a star player could reduce the overall efficiency of the team, analogous to a shortcut that is overused increasing the overall times for a journey through a road network. A proposed solution for maximum efficiency in scoring is for a star player to shoot about the same number of shots as teammates [37].

3. Nash Equilibrium of a Road Network and the User Equilibrium

In traffic assignment studies, network equilibrium models are used for the prediction of traffic patterns in transportation networks that are subject to congestion. Since traveling can be modeled as a game in which all actors independently wish to maximize their payoff (e.g., minimize their travel time), the situation can be seen as a case of Nash equilibrium. As we saw that Braess' paradox states that, counterintuitively, adding a road to a road network could possibly impede its flow (e.g., the travel time of each driver); equivalently, closing roads could potentially improve travel times. This is because the Nash equilibrium of such a system is not necessarily optimal. While the road network is not in a Nash equilibrium, individual drivers are able to improve their respective travel times by changing the routes they take. In the case of Braess' paradox, drivers will continue to switch until they reach Nash equilibrium despite the reduction in overall performance [1] [2] [3] [4].

A Nash Equilibrium is a set of rules in game theory that players act out, with the property that no player benefits from changing their strategy. Intuitively, this means that if any given player were told the strategies of all their opponents, they still would choose to retain their original strategy.

Similarly, in transportation networks, there are many researchers who worked on this analysis concept, such as, Wardrop who stated two principles that formalize different notions of equilibrium, and introduced the alternative behavior for the minimization of the total travel costs [2] [3] [4] [5] [38];

Wardrop's first principle of route choice, also known as "user equilibrium", described the spreading of trips over alternate routes due to congested conditions. It states that the journey times in all routes actually used are equal or less than those that would be experienced by a single vehicle on any unused route. The traffic flows that satisfy this principle are usually referred to as "user equilibrium" (UE) flows, since each user chooses the route that is the best. Specifically, a user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action. A variant is the stochastic user equilibrium (SUE), in which no driver can unilaterally change routes to improve his/her perceived, rather than actual, travel times.

Wardrop's second principle, also known as "system optimal", states that at equilibrium, the average journey time is at a minimum. That implies that all users behave cooperatively in choosing their routes to ensure the most efficient use of the whole system. For example, this would be the case if a central authority could command them all which routes to take. Traffic flows satisfying Wardrop's second principle are generally deemed system optimal (SO). Researchers have argued that it can be achieved with marginal cost road pricing, or by a central routing authority dictating route choices.

Wardrop did not provide specific algorithms for solving Wardrop equilibria, but as with Nash equilibria, simple solutions to user equilibrium can be found through iterative simulation, with each agent assigning its route given the choices of the others. Later, the Frank-Wolfe algorithm improves on this by exploiting dynamic programming properties of the network structure, to find solutions with a faster form of iteration [1] [3] [4] [5].

The user equilibrium assumes that all users choose their own route towards their destination based on the travel time that will be consumed in different route options. The users will choose the route which requires the least travel time. When the congestion occurs on roads, it will extend the delay time in travelling through the road.

The core principle of User Equilibrium is that all used routes between a given OD (Origin-Destination) pair have the same travel time. An alternative route option is enabled to use when the actual travel time in the system has reached the free-flow travel time on that route [38].

If we assume that the travel time for each person driving on a route to be equal, an equilibrium of the traffic flow will always exist [1] [2] [3] [4] [5].

Let $L_e(x)$ be the travel time of each person traveling along a route e when x people take that route. Suppose there is a traffic network with x_e people driving along rote e. Let the energy of e, $E(e)$, be:

$$\sum_{i=1}^{x_e} L_e(i) = L_e(1) + L_e(2) + \cdots + L_e(x_e)$$

Now let the total energy of the traffic network be the sum of the energies of every route in the network.

Normally, drivers take a choice of routes that minimizes the total energy. Such a choice must exist because there are many choices of routes in any traffic network. That will be an equilibrium.

But we will assume, for contradiction, this is not the case. Then, there is at least one driver who can switch the route and improve the travel time.

Suppose the original route is $(e_1, e_2, \ldots, e_n)$ while the new route is $(e_1', e_2', \ldots, e_m')$.

Let (E) be total energy of the traffic network and investigate what happens when the route $(e_1, e_2, \ldots, e_n)$ is removed. The energy of each edge route (e_i) will be reduced by $[L_{ei}(x_{ei})]$ and the energy E will be reduced by:

$$\sum_{i=0}^{n} L_{ei}(x_{ei})$$

That is the total travel time needed to take the original route. If the new route $(e_1', e_2', \ldots, e_m')$ is then added, the total energy (E) will be increased by the total travel time needed to take the new route. Because the new route is shorter than the original route, (E) must be decreased relative to the original configuration, contradicting the assumption that the original set of routes minimized the total energy. Therefore, the choice of routes minimizing total energy will always result in an equilibrium of traffic flow [2] [3] [4] [5] [38].

4. Braess' Paradox—Example 1

Consider a simple road network as shown in **Figure 1**, on which we assume that 4000 drivers wish to travel from point (Start) to point (End). There are two main paths the drivers can take. They can either travel along the path (start-A-end) or along the path (start-B-end).

The choice depends on the presence of traffic. Some roads might be narrow and get congested quickly. On these roads, the travel time for every driver depends on how many travelers (T) would pick that path.

In this network, we assume that the roads (Start-A) and (B-End) are narrow and travel time is estimated to be (T/100) minutes on average. The travel on these roads becomes slower as more and more drivers choose them.

But there might be some roads in any traffic network that they never get congested. On these roads, the travel time for every driver will be a constant number of minutes. In this network, we assume that the roads (A-End) and (Start-B) are wide and travel time is estimated to be 45 minutes on average.

Hence, the travel time in minutes on the Start-A road is the number of travelers (T) divided by 100, and on Start-B is a constant 45 minutes. If the dashed road from A to B does not exist (so that the traffic network has only 4 roads in total), then;

The time needed to drive the route (Start-A-End) with α drivers would be [(α/100) + 45].

The time needed to drive the route (Start-B-End) with β drivers would be [(β/100 + 45].

Since there are 4000 drivers, the fact that ($\alpha + \beta = 4000$) can be used to show that:

$\alpha = \beta = 2000$ when the system is at equilibrium of traffic flow. Therefore, each route will take: [2000/100 + 45 = 65 minutes] at equilibrium. If either route took less time for any reason, then a rational driver would switch from the longer route to the shorter route.

Now, we will assume that the dashed line A-B is a road with an extremely short travel time of approximately 0 minutes. Assuming that the road is opened, and one driver tries to travel through the route [Start-A-B-End]. He will find that his time is [2000/100 + 2000/100 = 40 minutes], getting a saving of 25 minutes. Soon, more of the 4000 drivers will try this new route, so that the time taken rises from 40 minutes and keeps going up.

When the number of drivers trying the new route reaches 2500 for example, with 1500 still in the (Start-B-End) route, their time will be [2500/100 + 4000/100 = 65 minutes], which is no improvement over the original route.

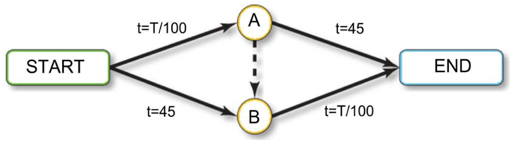

Figure 1. The road network of Example 1 to explain Braess' paradox.

Meanwhile, those 1500 drivers have been slowed to [45 + 4000/100] = 85 minutes], getting a 20-minute increase. As a result, they will be obliged to switch to the new route via A so that, it takes: [4000/100 + 4000/100 = 80 minutes].

No driver will be willing to travel on the route (A-End) or (Start-B) because any driver trying them will take 85 minutes. Thus, the opening of the dashed route would cause irreversible impact to the network, costing every driver 80 minutes instead of the original 65 minutes. If every driver were to agree not to use the A-B path, or if that route were closed, every driver would benefit by a 15-minute reduction in travel time. Figure 2 presents the travel time of both original routes and the equilibrium point of the network traffic flow.

Looking at the strange behavior of Braess' paradox in the above traffic network, one might wonder if this is a common enough phenomenon to really think about in traffic planning and design? The answer is that the Braess' Paradox is about as likely to occur as not occur. The reasoning assumptions include the following [14] [19]:

- There is just a single origin-destination pair on which the flow is being considered.
- The network is assumed to be congested; however, in non-congested networks, one can show that Braess' paradox will never occur.

Nevertheless, the paradox remains relevant, and continues to fascinate, and to inspire research in transportation planning and design. Indeed, the Braess' Paradox has served as a bridge for broadening perspectives in other scientific disciplines by enabling the advancement of the theory of the behavior of complex network systems with a vast range of important applications.

5. Braess' Paradox—Example 2

The road network shown in Figure 3 connects locations A and B. At peak hour (or maximum hour) vehicles enter the network at A at a flow rate of 1500 cars per hour, and drivers choose one of two routes to travel to B, either route 1, crossing bridge a, or route 2, crossing bridge b.

We assume that the number of cars, per hour, that take route 1 is L and the number of cars, per hour, that take route 2 is R.

We also assume that the bridges, a, and b, are bottlenecks, so they would cause the traffic to be slow. We will assume that the travel time through both bridges is directly proportional to the number of cars per hour, or flow rate of cars, so that the travel time is (L/100) minutes for bridge a, and (R/100) minutes for bridge b. We will consider that the remaining of both routes consists of wide roads with a travel time of 20 minutes each.

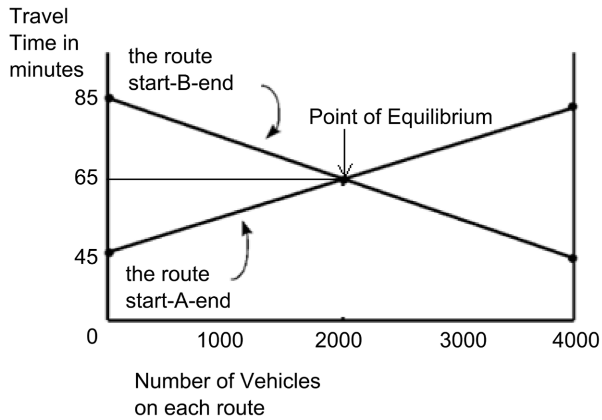

Figure 2. The Equilibrium point of the road network of Example 1.

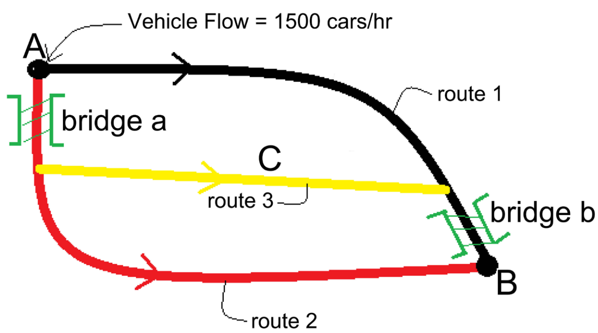

Figure 3. The road network of Example 2 to explain Braess' paradox.

We can determine the expected distribution of traffic, that is the number of cars, per hour, on each route. To do so, we assume that each driver has travelled through the network several times, and has developed a particular strategy, that is perceived as minimizing travel time. Under this assumption, the travel time must be the same for all the drivers, otherwise there would be an incentive for some of the drivers to change their strategy. This situation is called a steady-state, or Nash equilibrium of the road network, which is the same as Wardrop user equilibrium in transportation network.

Since the Nash equilibrium is dynamic, that is, it was maintained by the same number of the cars that enter the network at every hour, so that everybody achieves the same travel time, and no one is better off.

Now, the travel time, in minutes, for each of the two routes is:

$(L/100 + 20)$ for route 1, and $(R/100 + 20)$ for route 2.

At equilibrium we can assume that: $(L/100 + 20) = (R/100 + 20)$.

Also, the number of cars L, and R must be added up to the incoming flow. So, $(L + R = 1500)$.

Solving these two simultaneous equations, we find that: $L = R = 750$.

So, the traffic flow distributes equally between the two routes, with a travel time of $(750/100 + 20 = 27.5)$ minutes each.

We now assume that our road network is expanded with the addition of a crossroad C (route 3) for which the travel time is only 7 minutes for instance.

We will check the Braess' paradox to see if this addition to the network can decrease/increase the network travel time?

Drivers can now choose between three routes, the two previous ones and a new route 3 that goes from point A to bridge a, onto road C and through bridge b to point B.

As before, L is the flow of cars arriving at B via route 1 and R the flow of cars leaving A via route 2. Moreover, P is the flow of cars on road C.

Then the number of cars, per hour, going through bridge a must be $(L + P)$, while the number of cars, per hour, going through bridge b must be $(R + P)$.

So, the travel time on each of the three routes will be:

$[(L + P)/100 + 20]$ for route 1,

$[(R + P)/100 + 20]$ for route 2, and

$[(L + P)/100 + 7 + (R + P)/100 +20]$ for route 3.

As before, the traffic will have reached a steady-state, or a Nash equilibrium, when the travel time is the same for all the drivers. So, at equilibrium, we have:

$$\frac{L+P}{100}+20=\frac{R+P}{100}+20=\frac{L+P}{100}+7+\frac{R+P}{100}$$

Also, we have: $L + R + P = 1500$ cars per hour, and $L = R$ as before.

We can now find the three unknowns, L, R, P and then the common travel time for all the drivers:

$L = R = 200$ cars per hour and $P = 1100$ cars per hour.

Travel time = [(200 + 1100)/100 +20] = 33 minutes for each of route 1 and 2.

Travel time = 13 + 7+ 13 = 33 minutes for route 3.

This new travel time of 33 minutes represents 20% increase from the previous travel time of 27.5 minutes.

This indicates that adding the crossroad C would cause bad congestion and negatively affect the performance of the whole network. The drivers do not have any incentive to switch to the other routes, because they all have the same travel time. However, if the drivers agree to avoid route C completely, the travel time will decrease as before. If the road networks have controller systems directing the traffic, then in such networks the Braess' paradox will not occur. It is only observed when drivers choose their own best routes.

6. Conclusion

Braess' Paradox states that, counterintuitively, adding extra roads to a network when the moving entities selfishly choose their route, can in some cases reduce overall performance. This is because the Nash equilibrium (or user equilibrium) of such a system is not necessarily optimal. Network traveling can be modeled as a game in which all actors independently wish to maximize their payoff (e.g. minimize their travel time), therefore, the situation can be understood as a case of Nash equilibrium. In many real-life cases it has been noticed that adding a new road to an existing road network could possibly impede its flow (e.g., the travel time of each driver), and the removal of a road in a congested transportation network could result in improved flow. The paradox may have similar applications in electrical power grids and biological systems. A transportation network consists of objects with connections between them, in addition to the properties of the objects and their connections. The objects may be the cities, while the connections are given by the roads between them. The properties of the objects could correspond to the number of cars in the cities, while the properties of the connections could correspond to the lengths of the roads, the number of lanes they consist of, the amount of traffic, etc. The objects and the connections between them are modeled within the graph theory. Normally when we build more roads, we also give people incentives to use their cars more often because there is more infrastructure available. But, adding an extra road, which may seem like a shortcut, will redistribute the traffic in the road network. Braess' paradox says that this redistribution of the traffic flow may lead to an increased average travel time. While the road network is not in a Nash equilibrium (or user equilibrium), individual drivers are able to improve their respective travel times by changing the routes they take. In the case of Braess' paradox, drivers will continue to switch until they reach Nash equilibrium despite the reduction in overall performance. However, Braess's paradox is about as likely to occur as not occur when a random new route is added. Braess's paradox has a counterpart in case of a reduction of the road network (which may cause a reduction in individual travel time). Therefore, building a new road may make the traffic problem worse, especially when the road network is congested. So, in conclusion, adding route choices to a road network can improve flow through it, or it can impede it, we just have to check the traffic conditions using Braess' paradox. Therefore, Braess' paradox is a very useful tool for transportation planners to analyze the traffic networks before deciding whether to add new roads to the network under consideration.

Conflicts of Interest

The author declares no conflicts of interest.

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