Singular Value Noise Reduction Method for Full-Spectrum Input Gamma Energy Spectrum

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Abstract

The traditional smoothing method is complicated to use when the number of road sites is large, and multiple smoothing is required in the case of large noise, aiming at this problem, this paper uses the singular value decomposition denoising method to smooth the gamma energy spectrum. This paper expounds the basic principle of the singular value decomposition method, and the mathematical models of different reconstruction matrices and compares the effects of different reconstruction matrices on full-spectrum noise reduction. The results show that the signal-to-noise ratio is improved by 4.27% compared with the traditional method after the singular value decomposition, selection, reconstruction and reduction after using the Hankel matrix for energy spectrum allosteric.

Subject Areas

Nuclear Physics

Keywords

Noise Reduction, Gamma-Ray Spectrometry, Spectral Smoothing, NASVD

1. Introduction

In recent years, due to Japan's discharge of radioactive waste into the ocean, the issue of environmental monitoring has been increasingly mentioned by the public. In radioactive detection, the removal of noise is the primary issue for measurement. The current mainstream noise subtraction method is: Multi-point smoothing method, but this method requires a long and complex calculation time for full spectrum input. Currently, a noise reduction and smoothing of energy spectrum can be completed efficiently and quickly.

The singular value decomposition method is often used for signal noise re-

duction, and there is also signal interference and noise in radiometric measurements, these noises are usually caused by detectors, cosmic rays, or other radioactive contamination [1]. The noise in the gamma energy spectrum brings great difficulties to the analysis and identification of the gamma energy spectrum, and even leads to erroneous results (especially for low-level radioactivity analysis and the identification of nuclear materials with little difference [2]). In 1995, P. P. Kanjilal *et al.* proposed a new concept of decomposing the signal into component periodic waveforms, that is, singular value decomposition (SVD) to separate different signal components to achieve the function of noise reduction [3] [4]. Brian Minty and Jens Hovgaard evaluated two methods, the Noise Adjusted Singular Value Decomposition (NASVD) and the Maximum Noise Fraction (MNF), in terms of the accuracy and precision of synthesizing noise reduction spectra in gamma-ray spectroscopy [5]. Sascha Reinhardt *et al.* introduced the NASVD method in the full spectrum analysis of environmental radioactivity, and proved that the full spectrum analysis based on the noise-adjusted singular value decomposition method is a possible analysis method. It can be used to describe time-varying background and adjustment calculations, applied to time series of gamma spectra, which may have advantages compared to peak-based analysis [6].

Although the SVD method is a relatively convenient and quick method to reduce noise, it is currently only used for a single characteristic peak in the energy spectrum processing, and this article will use the SVD method for full-spectrum smoothing. By comparing other methods, a faster and better noise reduction method can be obtained.

2. SVD Decomposition Method

Dan Kalman expounded the basic method of SVD as a mathematical model [7], that is to decompose the matrix, but unlike eigendecomposition, SVD does not require the matrix to be decomposed to be a square matrix.

Assuming that matrix A is an $m \times n$ matrix, then we define the SVD of matrix A as:

$$A = U \Sigma V^{\mathrm{T}} \tag{1-1}$$

Among them, U is an $m \times m$ matrix, and Σ is an $m \times n$ diagonal matrix, that is, except for the elements on the diagonal, all others are 0, and his singular values are arranged on the diagonal from large to small. V is an $n \times n$, and $UU^{\mathrm{T}} = I_m$, $VV^{\mathrm{T}} = I_n$, the singular value is the diagonal element in the Σ matrix and $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \sigma_p \geq 0$, $p = \min(m,n)$.

According to the principle of singular value decomposition: decompose the different components and arrange them in descending order, then in the actual decomposition process, only the first 10% of the singular values need to be retained, That is, it is divided into two groups $(\sigma_1, \sigma_2, \cdots, \sigma_x)$ and $(\sigma_x, \sigma_{x+1}, \cdots, \sigma_p)$ The former is considered to be the singular value that retains the information contained in the original matrix, and the latter is considered to be caused by noise.

In the subsequent processing, the singular value of the latter is removed to complete the removal of noise.

3. Gamma Energy Spectrum Variable Matrix Structure

Since the gamma energy spectrum is a one-dimensional model, that is, its mathematical model is $A_\gamma \in R^{1 \times P}$ and P is the number of track addresses, it needs to be increased in dimension when performing singular value decomposition on it. It is constructed into an $m \times n$ matrix and then performs singular value decomposition.

3.1. Continuous Truncated Signal Matrix Construction Method

This is a relatively simple method, the principle of which is to directly truncate the one-dimensional energy spectrum somewhere, take two positive integers m and n, and truncate m segments of the same length at n points each time for this sequence, so that this m-segment construction matrix Y_P sets the gamma energy spectrum signal as:

$$Y_P = \left[x_1, x_2, x_3, \cdots, x_p \right] \tag{2-1}$$

After selecting multiple breakpoints, the energy spectrum is truncated and transformed into

$$Y_P = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ x_{m+1} & x_{m+2} & \cdots & x_{mp} \\ \vdots & \vdots & \ddots & \vdots \\ x_{(n-1)m+1} & x_{(n-1)m+2} & \cdots & x_{nm} \end{bmatrix} \tag{2-2}$$

And $p = m * n$, $m \geq 2$, $n \geq 2$.

Regarding the selection of m and n, the entire matrix should be transformed into a square matrix as far as possible. In general, when there are 1024 channels, $m = n = 32$, and if it is 2048 channels, the choice is $m = 64$, $n = 32$ or $m = 32$, $n = 64$.

This method can quickly generate an allosteric matrix, and can greatly compress the size of the matrix to ensure a faster speed in subsequent SVD calculations. However, when the truncation point is at the peak position, part of the characteristics of the source spectrum will be lost, so this method needs to avoid the peak position and select a reasonable matrix size. When reconstructing the restored energy spectrum, this method only needs to splicing the separated lines to obtain the gamma energy spectrum.

3.2. Hankel Matrix

Hankel Matrix refers to a matrix with equal elements on each sub-diagonal [8]. A symmetric matrix can be obtained by constructing a Hankel matrix from the gamma energy spectrum, and the gamma energy spectrum is on the main diagonal.

Constructing the Hankel matrix from formula (2-1), we get

$$H_P = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_2 & x_3 & \cdots & x_{n+1} \\ \vdots & \vdots & \cdots & \vdots \\ x_{p-n+1} & x_{p-n+2} & \cdots & x_p \end{bmatrix} \qquad (2\text{-}3)$$

When using SVD to decompose it, it will be more computationally expensive than the direct truncation method, but there is a correlation between adjacent elements in the matrix, and the gamma energy spectrum can be regarded as a continuous spectrum, so the use of Hankel to construct the matrix is a more reasonable method.

As with the truncation matrix, the selection of n should try to convert the matrix into a square matrix to reduce the computational complexity of SVD.

3.3. Selection of Singular Values and Energy Spectrum Restoration

After the energy spectrum is transformed into a multi-dimensional matrix by an allosteric matrix, and then decomposed by the SVD method, the three matrices *U*, Σ, and *V* are obtained, where $\Sigma = diag\left(\sigma_1, \sigma_2, \cdots, \sigma_p\right)$, by its basic principle It can be seen that after selecting the first 10% of the singular values in Σ, and then setting the remaining singular values to zero, a new diagonal matrix $\Sigma_1 = diag\left(\sigma_1, \sigma_2, \cdots, \sigma_n, 0, \cdots, 0\right)$ is obtained, and then the formula (1-1) becomes an allosteric matrix. The allosteric matrix is obtained by the energy spectrum truncation method. When the energy spectrum is reconstructed and restored, the gamma energy spectrum can be obtained only by splicing the separated lines. Regarding the reconstruction and restoration of Hankel, according to its transformation method, it can be known that the decomposed gamma energy spectrum can be obtained by directly calculating the elements of the main diagonal of the matrix.

Figure 1 is the flow chart of SVD calculation, which includes how to select singular values. For the reduction problem of allosteric matrix, for the truncated matrix, the processed energy spectrum can be obtained by directly connecting each row end to end. For the Hankel matrix, the processed energy spectrum can be obtained by directly selecting the elements on the main diagonal of the allosteric matrix.

4. Experimental Results and Discussion

The Cs-137 radioactive source was measured in an open environment, and the energy spectrum was not smoothed, as shown in Figure 2, for the original energy spectrum without processing, Figure 3 for the smoothed energy spectrum using the direct truncation method, Figure 4 In order to use the Hankel matrix allosteric to calculate and reconstruct the smooth energy spectrum after SVD calculation, Figure 5 shows the smoothed curve using the traditional five-point smoothing method.

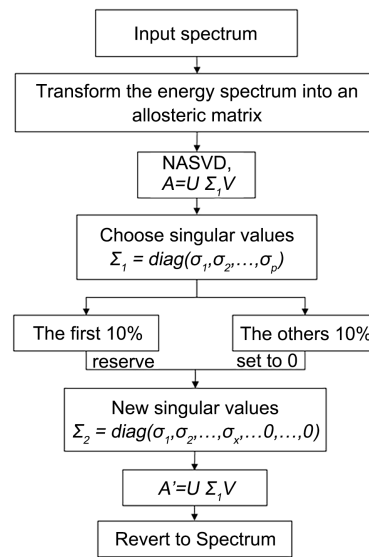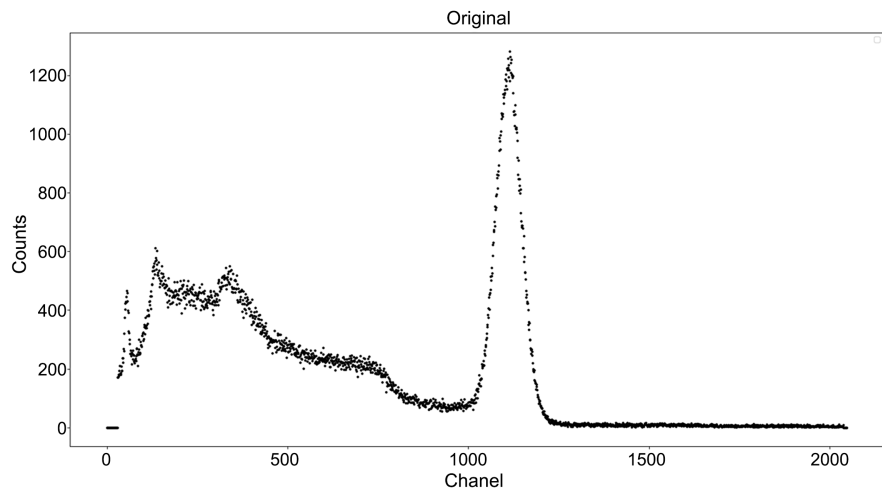**Figure 1.** SVD calculation flow chart.
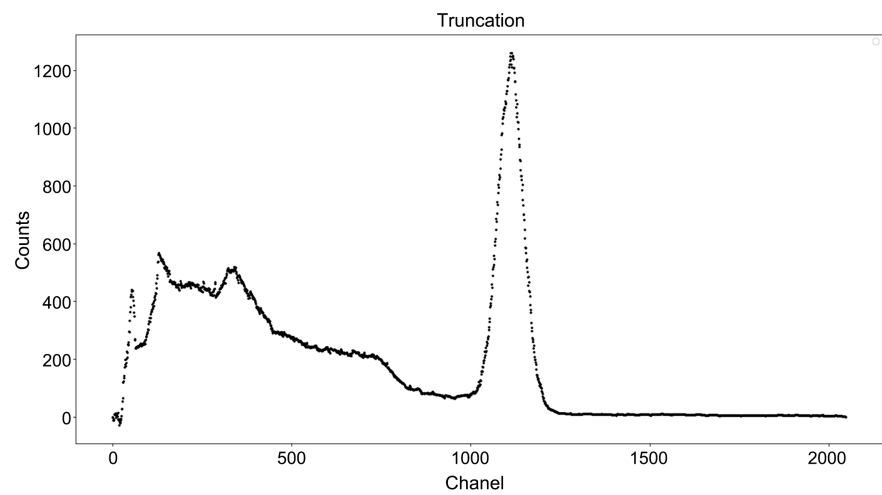

Figure 2. Raw energy spectrum.

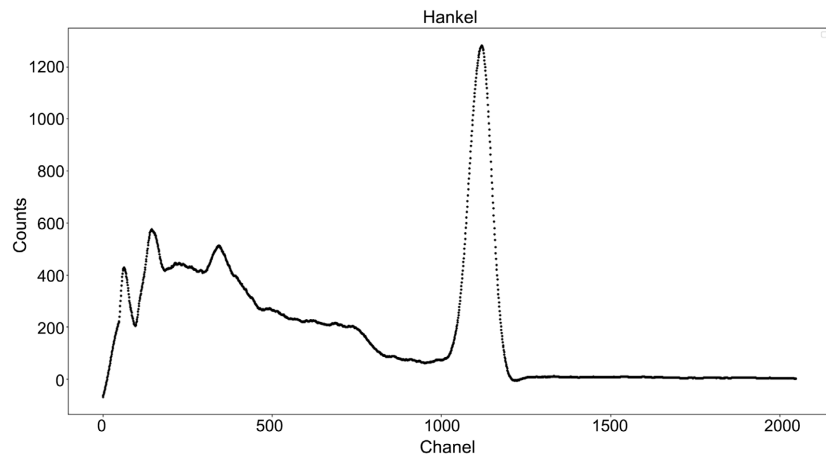**Figure 3.** Spectrum direct truncation.
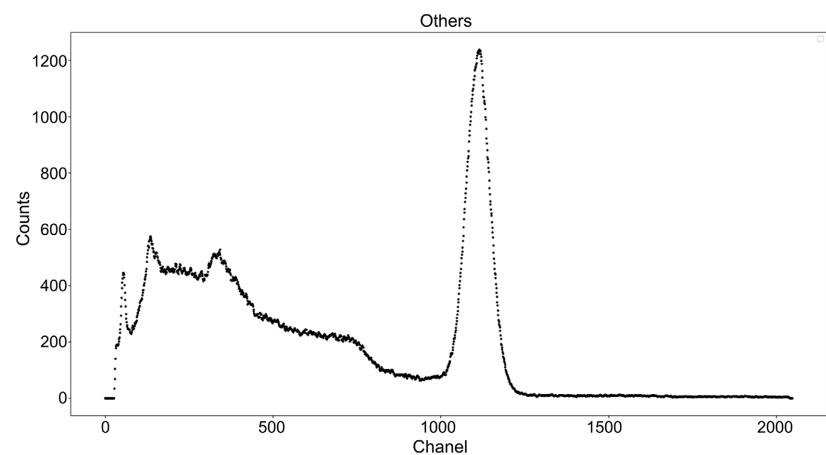
Figure 4. Hankel matrix.

Figure 5. Five-point sliding smoothing.

According to the above 4 figures, it is not difficult to draw the conclusion that when the energy spectrum is smoothed by the singular value decomposition method, the effect is obviously better than that of the traditional smoothing method; but different allosteric matrices have different smoothing effects on the energy spectrum. According to **Figure 2** compared with **Figure 3**, the smoothing effect of the Hankel matrix on the energy spectrum is better than that of the direct truncation method, and in the direct truncation method, it can be seen that there are small discontinuities in the smoothed energy spectrum, that is, the smoothing effect of some positions is not good. The reason is that it is located right near the truncation point.

Next, calculate their respective signal-to-noise ratios. Since the full spectrum is smoothed, according to the formula

$$\text{SNR} = 10 \lg \left[\frac{\dfrac{\sum_{i=0}^{N-1}(y_i)^2}{N_n}}{\dfrac{\sum_{j=0}^{N-1}(y_j)^2}{N_s}} \right] \tag{3-1}$$

Table 1. Signal-to-noise ratios corresponding to different smoothing methods.

	Five-points	Direct truncation	Hankel
SNR	1.00%	0.85%	5.27%

Among them, N_n is the track site after smoothing, x_i is the corresponding peak area of the track site, N_s is the track site before smoothing, and y_i is the corresponding peak area.

Table 1 shows the signal-to-noise ratios of different smoothing methods. Because it is calculated for the full spectrum, the signal-to-noise ratio (SNR) is numerically small. However, from the table comparison, it can be found that the effect obtained after allosteric reorganization of the energy spectrum using Hankel is excellent. It is 4.27% higher than other methods, but the effect of the truncation method at the same level is not as good as the smooth effect of the traditional method.

Compared with the traditional method, that is, the five-point method in Figure 4 and Table 1, it can be seen from the figure that the method based on Hankel matrix is obviously better than the noise reduction effect of the five-point smoothing method. The smoothing method is slightly less effective, but Hankel's method outperforms the other two methods.

Using the SVD method to denoise the energy spectrum compared with the traditional method, the final results obtained by using different allosteric matrices are different. Through experiments, the effect of the direct truncation method is 0.15% inferior to the traditional method, but the effect after using the Hankel matrix, the results increased by 4.27%. Subsequent studies will be conducted on the selection of n of the Hankel matrix to explore the influence of the value of n on the results.

5. Conclusions

In this paper, based on the basic principle of singular value decomposition, the gamma energy spectrum is decomposed by constructing different allosteric matrices to convert one-dimensional data into high-dimensional data. Comparing different methods, the following conclusions are drawn:

1) According to the singular value decomposition method, the noise reduction of the gamma energy spectrum can better reduce the noise of the energy spectrum, and the degree of reduction is related to the selection of different singular values.

2) Different allosteric matrices, such as Hankel and directly truncated matrices, have different noise reduction effects; among them, the Hankel matrix has the best effect and can deduct noise well.

Conflicts of Interest

The authors declare no conflicts of interest.

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