

# Adaptive Two-Stage EKF for INS-GPS Loosely Coupled System with Unknown Fault Bias

Kwang Hoon Kim, Jang Gyu Lee

School of Electrical Engineering and Computer Science, Seoul National University,  
133-dong, ASRI #610, San 56-1, Sillim-dong, Gwanak-gu, Seoul, 151-742, South Korea

Chan Gook Park

School of Mechanical and Aerospace Engineering, Seoul National University,  
San 56-1, Sillim-dong, Gwanak-gu, Seoul, 151-744, South Korea

**Abstract.** This paper proposes an adaptive two-stage extended Kalman filter (ATEKF) for estimation of unknown fault bias in an INS-GPS loosely coupled system. The Kalman filtering technique requires complete specifications of both dynamical and statistical model parameters of the system. However, in a number of practical situations, these models may contain parameters, which may deviate from their nominal values by unknown random bias. This unknown random bias may seriously degrade the performance of the filter or cause a divergence of the filter. The two-stage extended Kalman filter (TEKF), which considers this problem in nonlinear system, has received considerable attention for a long time. The TEKF suggested until now assumes that the information of a random bias is known. But the information of a random bias is unknown or partially known in general. To solve this problem, this paper firstly proposes a new adaptive fading extended Kalman filter (AFEKF) that can be used for nonlinear system with incomplete information. Secondly, it proposes the ATEKF that can estimate unknown random bias by using the AFEKF. The proposed ATEKF is more effective than the TEKF for the estimation of the unknown random bias. The ATEKF is applied to the INS-GPS loosely coupled system with unknown fault bias.

**Keywords.** adaptive two-stage extended Kalman filter; covariance rescaling; unknown fault bias

## 1 Introduction

The well-known Kalman filtering has been widely used in many industrial areas. This Kalman filtering technique requires complete specifications of both dynamical and statistical model parameters of the system. However, in a number of practical situations, these models contain parameters, which may deviate from their nominal values by unknown constant or unknown random bias. These unknown biases may seriously degrade the performance of the filter or even generate the divergence of the filter. To solve this problem, a new procedure for estimating the dynamic states of a linear stochastic system in the presence of unknown constant bias vector was suggested by Friedland (Friedland, 1969). This filter is called the two-stage Kalman filter (TKF). Many researchers have contributed to this problem. Recently, the TKF to consider not only a constant bias but also a random bias has been suggested through several papers (Ignagni, 1990; Alouani, 1993; Keller, 1997; Hsieh, 1999; Ignagni, 2000).

Several researchers performed the extension of the TKF to nonlinear system (Shreve, 1974; Mendel, 1976; Caglayan, 1983). Unknown random bias of nonlinear system may also generate the large problem in the TEKF because the TEKF may be diverged if the initial estimates are not sufficiently good. So the TEKF for nonlinear system with a random bias has to assume that the dynamic equation and the noise covariance of unknown random bias are known although these are unknown in general. To solve these problems, an adaptive filter can be used. This paper proposes an adaptive two-stage extended Kalman filter (ATEKF) by using an adaptive fading extended Kalman filter (AFEKF). As a result, unknown random bias can be estimated by the ATEKF.

To verify the performance of the ATEKF, the proposed ATEKF is applied to the INS-GPS loosely coupled navigation system with unknown fault bias. In general, the INS-GPS loosely coupled system uses the EKF, which cannot effectively track time-varying parameters (Ljung, 1979). So when the navigation system has unknown fault bias, the EKF cannot estimate unknown fault bias and cannot eliminate this. This paper shows that the ATEKF can estimate and eliminate unknown fault bias in the INS-GPS loosely coupled system with unknown fault bias.

## 2 Problem Formulation

Consider the following nonlinear discrete-time stochastic system represented by

$$x_{k+1} = f_k(x_k, b_k) + w_k^x \quad (1a)$$

$$b_k = A_k b_k + w_k^b \quad (1b)$$

$$z_k = h_k(x_k, b_k) + v_k \quad (1c)$$

where,  $x_k$  is the  $n \times 1$  state vector,  $z_k$  is the  $m \times 1$  measurement vector and  $b_k$  is the  $p \times 1$  bias vector of unknown magnitude. All matrices have the appropriate dimensions. The noise sequence  $w_k^x$ ,  $w_k^b$  and  $v_k$  are zero mean uncorrelated Gaussian random sequences with

$$E \begin{bmatrix} w_k^x \\ w_k^b \\ v_k \end{bmatrix} \begin{bmatrix} w_k^x \\ w_k^b \\ v_k \end{bmatrix}^T = \begin{bmatrix} Q_k^x & 0 & 0 \\ 0 & Q_k^b & 0 \\ 0 & 0 & R_k \end{bmatrix} \delta_{kj} \quad (1d)$$

where  $Q_k^x > 0$ ,  $Q_k^b > 0$ ,  $R_k > 0$  and  $\delta_{kj}$  is the Kronecker delta. The initial states  $x_0$  and  $b_0$  are assumed to be uncorrelated with the white noise processes  $w_k^x$ ,  $w_k^b$  and  $v_k$ . Assume that  $x_0$  and  $b_0$  are Gaussian random variables with

$$E[x_0] = x_0^*, \quad E[(x_0 - x_0^*)(x_0 - x_0^*)^T] = P_0^x > 0,$$

$$E[b_0] = b_0^*, \quad E[(b_0 - b_0^*)(b_0 - b_0^*)^T] = P_0^b > 0 \text{ and}$$

$$E[(x_0 - x_0^*)(b_0 - b_0^*)^T] = P_0^{xb}$$

The function  $f_k(x_k, b_k)$  and  $h_k(x_k, b_k)$  can be expanded by Taylor series expansion as

$$\begin{aligned} f_k(x_k, b_k) &= f_k(\hat{x}_k(+), \bar{b}_k(+)) \\ &+ F_k(\hat{x}_k(+), \bar{b}_k(+))(x_k - \hat{x}_k(+)) \\ &+ B_k(\hat{x}_k(+), \bar{b}_k(+))(b_k - \bar{b}_k(+)) \\ &+ \varphi_f(x_k, \hat{x}_k(+), \bar{b}_k(+)) \end{aligned} \quad (2)$$

$$\begin{aligned} h_k(x_k, b_k) &= h_k(\hat{x}_k(-), \bar{b}_k(-)) \\ &+ H_k(\hat{x}_k(-), \bar{b}_k(-))(x_k - \hat{x}_k(-)) \\ &+ D_k(\hat{x}_k(-), \bar{b}_k(-))(b_k - \bar{b}_k(-)) \\ &+ \varphi_h(x_k, \hat{x}_k(-), \bar{b}_k(-)) \end{aligned} \quad (3)$$

where  $\hat{x}_k(\cdot)$  and  $\bar{b}_k(\cdot)$  are the state estimate and the bias estimate respectively,  $\varphi_f$  and  $\varphi_h$  mean the higher order terms and

$$F_k(\hat{x}_k(+), \bar{b}_k(+)) = \left. \frac{\partial f_k}{\partial x} \right|_{\substack{x=\hat{x}_k(+), \\ b=\bar{b}_k(+)}},$$

$$B_k(\hat{x}_k(+), \bar{b}_k(+)) = \left. \frac{\partial f_k}{\partial b} \right|_{\substack{x=\hat{x}_k(+), \\ b=\bar{b}_k(+)}},$$

$$H_k(\hat{x}_k(-), \bar{b}_k(-)) = \left. \frac{\partial h_k}{\partial x} \right|_{\substack{x=\hat{x}_k(-), \\ b=\bar{b}_k(-)}},$$

$$D_k(\hat{x}_k(-), \bar{b}_k(-)) = \left. \frac{\partial h_k}{\partial b} \right|_{\substack{x=\hat{x}_k(-), \\ b=\bar{b}_k(-)}}.$$

The problem is to design an adaptive two-stage extended Kalman filter (ATEKF) to give a solution for nonlinear stochastic system with a random bias on the assumption that the stochastic information of a random bias is unknown or partially known.

## 3 Two-stage Extended Kalman Filter in the Presence of a Random Bias

In this section, we extend the optimal TKF to nonlinear stochastic system models in order to obtain a suboptimal TEKF. Until now, the several researchers have proposed the TEKF (Mendel, 1976; Caglayan, 1983). But the proposed TEKF was based on the suboptimal TKF. Recently, several researchers have proposed the optimal TKF for linear stochastic system with unknown random bias (Keller, 1997; Hsieh, 1999; Ignagni, 2000). Thus we newly propose the TEKF based on the optimal TKF as Lemma 3.1.

**Lemma 3.1:** A discrete-time two-stage extended Kalman filter is given by the following coupled difference

equations when the information of nonlinear stochastic system (1) is perfectly known.

$$\hat{x}_k(-) = \bar{x}_k(-) + U_k(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+))\bar{b}_k(-) \quad (4a)$$

$$\hat{x}_k(+) = \bar{x}_k(+) + V_k(\hat{x}_k(-), \bar{b}_k(-))\bar{b}_k(+) \quad (4b)$$

$$P_k^x(-) = \bar{P}_k^x(-) + U_k(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+))\bar{P}_k^b(-) U_k^T(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) \quad (5a)$$

$$P_k^x(+) = \bar{P}_k^x(+) + V_k(\hat{x}_k(-), \bar{b}_k(-))\bar{P}_k^b(+) V_k^T(\hat{x}_k(-), \bar{b}_k(-)) \quad (5b)$$

where  $A_k$  and  $Q_k^b$  are known. The two-stage extended Kalman filter can be decomposed into two filters such as the modified bias free filter and the bias filter. The modified bias free filter, which is designed on the assumption that the biases are identically zero, gives the state estimate  $\bar{x}_k(\cdot)$ . On the other hand, the bias filter gives the bias estimate  $\bar{b}_k(\cdot)$ . Finally the corrected state estimate  $\hat{x}_k(\cdot)$  of the TEKF is obtained from the estimates of two filters and the coupling equation  $U_k$  and  $V_k$ . The modified bias free filter is

$$\bar{x}_k(-) = F_{k-1}(\bar{x}_{k-1}(+), \bar{b}_{k-1}(+))\bar{x}_{k-1}(+) + C_k + u_{k-1} \quad (6a)$$

$$\bar{P}_k^x(-) = F_{k-1}(\bar{x}_{k-1}(+), \bar{b}_{k-1}(+))\bar{P}_{k-1}^x(+) F_{k-1}^T(\bar{x}_{k-1}(+), \bar{b}_{k-1}(+)) + \bar{Q}_{k-1}^x \quad (6b)$$

$$\bar{K}_k^x(\bar{x}_k(-), \bar{b}_k(-)) = \bar{P}_k^x(-) H_k^T(\bar{x}_k(-), \bar{b}_k(-)) [H_k(\bar{x}_k(-), \bar{b}_k(-))\bar{P}_k^x(-) H_k^T(\bar{x}_k(-), \bar{b}_k(-)) + R_k]^{-1} \quad (6c)$$

$$\bar{P}_k^x(+) = [I - \bar{K}_k^x(\bar{x}_k(-), \bar{b}_k(-)) H_k(\bar{x}_k(-), \bar{b}_k(-))] \bar{P}_k^x(-) \quad (6d)$$

$$\bar{\eta}_k^x = z_k - H_k(\bar{x}_k(-), \bar{b}_k(-))\bar{x}_k(-) - E_k \quad (6e)$$

$$\bar{x}_k(+) = \bar{x}_k(-) + \bar{K}_k^x \bar{\eta}_k^x \quad (6f)$$

and the bias filter is

$$\bar{b}_k(-) = A_{k-1} \bar{b}_{k-1}(+) \quad (7a)$$

$$\bar{P}_k^b(-) = A_{k-1} \bar{P}_{k-1}^b(+) A_{k-1}^T + Q_{k-1}^b \quad (7b)$$

$$\bar{K}_k^b(\bar{x}_k(-), \bar{b}_k(-)) = \bar{P}_k^b(-) N_k^T [H_k(\bar{x}_k(-), \bar{b}_k(-))\bar{P}_k^x(-) H_k^T(\bar{x}_k(-), \bar{b}_k(-)) + R_k + N_k \bar{P}_k^b(-) N_k^T]^{-1} \quad (7c)$$

$$\bar{P}_k^b(+) = [I - \bar{K}_k^b(\bar{x}_k(-), \bar{b}_k(-)) N_k] \bar{P}_k^b(-) \quad (7d)$$

$$\bar{\eta}_k^b = \bar{\eta}_k^x - N_k \bar{b}_{k-1}(-) \quad (7e)$$

$$\bar{b}_k(+) = \bar{b}_{k-1}(-) + \bar{K}_k^b \bar{\eta}_k^b \quad (7f)$$

with the coupling equations

$$N_k = H_k(\bar{x}_k(-), \bar{b}_k(-)) U_k(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) + D_k(\hat{x}_k(-), \bar{b}_k(-)) \quad (8a)$$

$$U_k(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) = \bar{U}_k [I - Q_{k-1}^b [\bar{P}_k^b(-)]^{-1}] \quad (8b)$$

$$\bar{U}_k = \begin{pmatrix} F_{k-1}(\bar{x}_{k-1}(+), \bar{b}_{k-1}(+)) V_{k-1}(\bar{x}_{k-1}(-), \bar{b}_{k-1}(-)) \\ + B_{k-1}(\bar{x}_{k-1}(+), \bar{b}_{k-1}(+)) \end{pmatrix} A_{k-1}^{-1} \quad (8c)$$

$$V_k(\bar{x}_k(-), \bar{b}_k(-)) = U_k(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) - \bar{K}_k^x(\bar{x}_k(-), \bar{b}_k(-)) N_k \quad (8d)$$

$$u_k = (\bar{U}_{k+1} - U_{k+1}(\hat{x}_k(+), \bar{b}_k(+))) A_k \bar{b}_k(+) \quad (8e)$$

$$\bar{Q}_k^x = Q_k^x + U_{k+1} Q_{k+1}^b \bar{U}_{k+1} \quad (8f)$$

Here, the initial conditions:

$$\bar{x}_0(+) = x_0^* - V_0 b_0^*, \quad \bar{b}_0(+) = b_0^* \quad (9a)$$

$$\bar{P}_0^x(+) = P_0^x - V_0 P_0^b V_0^T, \quad \bar{P}_0^b(+) = P_0^b \quad (9b)$$

where  $V_0 = P_0^{xb} (P_0^b)^{-1}$ .

The equations of the TEKF are similar to the equations of the optimal TKF. But although the structure of this TEKF is possible for nonlinear stochastic system, the coupling between the modified bias free filter and the bias filter in the nonlinear case exists and all linearizations about the state and bias estimate have to be evaluated. Thus a designer has to give attention to the linearization point when the TEKF is used.

In general, the EKF may give biased estimates and be diverged if the initial estimates are not sufficiently good. So the incomplete information of nonlinear system may generate the large problem in the EKF. The TEKF has also the same problem. Thus the TEKF of Lemma 3.1 assumes that  $A_k$  and  $Q_k^b$  are known.

However, in most case, these are unknown. If this information is incomplete, the performance of the TEKF may be degraded or the TEKF may be diverged. To solve this problem, the TEKF has to be adapted to environment of incomplete random bias information. Thus an adaptive two-stage extended Kalman filter is needed

#### 4 Adaptive Two-stage Extended Kalman Filter in the Presence of a Random Bias

##### 4.1 Adaptive Fading Extended Kalman Filter using Innovation Covariance

Consider the following nonlinear discrete-time stochastic system represented by

$$x_{k+1} = f_k(x_k) + \Delta_{1,k} + w_k \quad (10a)$$

$$z_k = h_k(x_k) + \Delta_{2,k} + v_k \quad (10b)$$

where  $x_k$  is the  $n \times 1$  state vector and  $z_k$  is the  $m \times 1$  measurement vector. Moreover  $w_k$  and  $v_k$  denote sequences of uncorrelated Gaussian random vectors with zero means. Each covariance matrix is

$$E[w_k w_k^T] = Q_k \delta_{kj}, \quad E[v_k v_k^T] = R_k \delta_{kj}, \quad E[w_k v_k^T] = 0 \quad (11)$$

where  $\delta_{kj}$  is the Kronecker delta. The variable  $\Delta_{1,k}$  means the uncertainty term in the dynamic equation that is generated by incomplete process noise covariance, unknown input bias or incomplete model coefficients. On the other hand, the variable  $\Delta_{2,k}$  means the uncertainty term in the measurement equation that is generated by incomplete measurement noise covariance or unknown measurement bias. Also  $\Delta_{1,k}$  and  $\Delta_{2,k}$  are independent of  $w_k$ ,  $v_k$  and  $x_k$  respectively. If the system information is perfectly known,  $\Delta_{1,k}$  and  $\Delta_{2,k}$  are all zero.

The function  $f_k(x_k)$  and  $h_k(x_k)$  can be expanded by Taylor series expansion as

$$f_k(x_k) = f_k(\hat{x}_k(+)) + F_k(x_k - \hat{x}_k(+)) + \varphi_f(x_k, \hat{x}_k(+)) \quad (12)$$

$$h_k(x_k) = h_k(\hat{x}_k(-)) + H_k(x_k - \hat{x}_k(-)) + \varphi_h(x_k, \hat{x}_k(-)) \quad (13)$$

where  $F_k = \left. \frac{\partial f_k}{\partial x} \right|_{x=\hat{x}_k(+)}$ ,  $H_k = \left. \frac{\partial h_k}{\partial x} \right|_{x=\hat{x}_k(-)}$ . The variable  $\varphi_f$  and  $\varphi_h$  mean the higher order terms.

**Definition 4.1:** Several equations related to the innovation are arranged as follows.

$$\eta_k = z_k - h_k(\hat{x}_k(-)) \quad (14a)$$

$$C_k = E[\eta_k \eta_k^T] = H_k P_k(-) H_k^T + R_k \quad (14b)$$

$$\bar{C}_k = \frac{1}{M-1} \sum_{i=k-M+1}^k \eta_i \eta_i^T \quad (14c)$$

where  $\eta_k$  is the innovation of filter,  $C_k$  is the calculated innovation covariance and  $\bar{C}_k$  is the estimated innovation

covariance. Here  $M$  is a window size.  $P_k(-)$  is a calculated error covariance of the EKF.

The equations of the EKF are similar to those of the linear Kalman filter. Thus first we derive an optimal property from the linear Kalman filter and secondly expand this result in terms of nonlinear system. One of the important properties of the optimal linear Kalman filter is that the innovation is a white sequence when the optimal filtering gain is used. Although the EKF is a suboptimal filter, this property can be used to improve the performance of the EKF in nonlinear system (10). The following equation (15) is the auto-covariance of the innovation in the EKF:

$$E[\eta_{k+j} \eta_k^T] = H_{k+j} F_{k+j-1} [I - K_{k+j-1} H_{k+j-1}] \dots F_{k+1} [I - K_{k+1} H_{k+1}] F_k [P_k(-) H_k^T - K_k C_k] \quad (15)$$

$$, \forall j = 1, 2, 3, \dots$$

$$S_k = P_k(-) H_k^T - K_k C_k \quad (16)$$

The white sequence has to satisfy a condition that the auto-covariance is zero because whiteness means uncorrelated in time. If the common term (16) of the auto-covariance (15) for all  $j = 1, 2, 3, \dots$ , is zero, then the innovation sequence is a white sequence. If we insert the error covariance and the Kalman gain of the EKF into equation (15), the auto-covariance of the innovation is identically zero since  $S_k$  is zero for all  $j = 1, 2, 3, \dots$ . Actually, the Kalman gain of the EKF is easily obtained when  $S_k = 0$  in the equation (16). In case of the linear Kalman filter, if the auto-covariance equation is zero, we can say that the filter is optimal. But, in case of the EKF, the real innovation sequence is not a white sequence due to higher order terms to be neglected and the linearization error. Nonetheless if the auto-covariance equation (15) of the innovation in the EKF is zero, then the performance of the EKF will be improved. When the system model is incomplete, the real covariance of the innovation is different from a theoretical one given in equation (14b). Hence the real auto-covariance of the innovation may not be identically zero although  $S_k = 0$ . Under this background, this section newly proposes a new AFEKF.

**Definition 4.2:** For a design of a new AFEKF, let's define several variables. Assume that the system information is incomplete. From now,  $P_k(-)$ ,  $K_k$  and  $C_k$  are defined as an error covariance, a Kalman gain and an innovation covariance for system with incomplete information. Here,  $C_k$  is calculated from the model equation of system with incomplete information. Thus  $C_k$  is also called as the calculated innovation covariance. On the other hand,  $\bar{P}_k(-)$ ,  $\bar{K}_k$  and  $\bar{C}_k$  are defined as an error covariance, an optimal Kalman gain and an

innovation covariance for system with complete information. We have to obtain these  $\bar{P}_k(-)$ ,  $\bar{K}_k$  and  $\bar{C}_k$  by the appropriate method because the information of system to be considered is incomplete. Here,  $\bar{C}_k$  can be obtained from (14c). Thus  $\bar{C}_k$  is also called as the estimated innovation covariance. We assume  $\bar{P}_k(-) = \lambda_k P_k(-)$  where  $\lambda_k$  is a forgetting factor. If  $\bar{P}_k(-)$  can be obtained by  $\lambda_k$ , then the Kalman gain  $\bar{K}_k$  can be obtained from  $\bar{P}_k(-)$ .

Let's think the case that the information of system is unknown or partially known. If  $\bar{P}_k(-)$  can be obtained by using the forgetting factor and  $\bar{C}_k$  can be exactly estimated by (14c), then the Kalman gain  $\bar{K}_k$  makes that  $\bar{S}_k = 0$ , where

$$\bar{S}_k = \bar{P}_k(-)H_k^T - \bar{K}_k\bar{C}_k \quad (17)$$

However, in many cases, the difficulty is to get  $\bar{P}_k(-)$  and  $\bar{K}_k$ . Therefore the filter is suboptimal for system with incomplete information in general. As a result, the problem is to seek how to exactly obtain  $\bar{P}_k(-)$  and  $\bar{K}_k$ .

**Definition 4.3:** Scalar variable  $\alpha_k$  is defined by

$$\alpha_k = \max \left\{ 1, \frac{1}{m} \text{trace}(\bar{C}_k C_k^{-1}) \right\}$$

$$\text{or } \alpha_k = \max \left\{ 1, \frac{\text{trace}(\bar{C}_k)}{\text{trace}(C_k)} \right\} \quad (18)$$

where the AFEKF uses the relationship  $\bar{C}_k = \alpha_k C_k$ .

In general, the innovation covariance shows the effect of the present error because the innovation is easily affected by the error. Let's think for system with incomplete dynamic equation. The real innovation covariance of the present is estimated as  $\bar{C}_k$ . Then  $\bar{C}_k$  is represented by

$$\bar{C}_k = \alpha_k C_k = H_k \bar{P}_k(-) H_k^T + R_k = \lambda_k H_k P_k(-) H_k^T + R_k \quad (19)$$

The increase of the innovation covariance by  $\alpha_k$  is due to the increase of the error covariance by  $\lambda_k$  because the system has incomplete dynamic equation. On the other hand  $R_k$  remains unchanged. If the increased error covariance is larger than  $R_k$ , we can think that  $\alpha_k$  is almost equal to  $\lambda_k$ .

**Remark 4.1:** Sometimes, we only partially know the dynamic equation of nonlinear stochastic system. Then, the estimation error may be increased by the effect of unknown information. This means that the error increase

is due to incomplete process noise covariance, unknown input bias or incomplete model coefficients. The effects of incomplete information in dynamic equation can be compensated through the increase of the magnitude of  $P_k(-)$ . If we assume the relationship  $\lambda_k = \alpha_k$ , then the error covariance is

$$\bar{P}_k(-) = \alpha_k P_k(-).$$

**Definition 4.4:** A discrete-time adaptive fading extended Kalman filter is given by the following coupled difference equations when the information of nonlinear stochastic system (10) is partially known.

$$\hat{x}_k(-) = f_{k-1}(\hat{x}_{k-1}(+)) \quad (20a)$$

$$\bar{P}_k(-) = \lambda_k [F_{k-1} \bar{P}_{k-1}(+) F_{k-1}^T + Q_{k-1}] \quad (20b)$$

$$\bar{K}_k = \bar{P}_k(-) H_k^T [H_k \bar{P}_k(-) H_k^T + R_k]^{-1} \quad (20c)$$

$$\bar{P}_k(+) = (I - \bar{K}_k H_k) \bar{P}_k(-) \quad (20d)$$

$$\hat{x}_k(+) = \hat{x}_k(-) + \bar{K}_k [z_k - h_k(\hat{x}_k(-))] \quad (20e)$$

where  $\lambda_k = \alpha_k$ .

#### 4.2 Adaptive Two-stage Extended Kalman Filter using the AFEKF

The ATEKF can be easily designed by the proposed AFEKF. This ATEKF is used when the information of  $A_k$  and  $Q_k$  are incomplete. For a compensation of the effects of incomplete information in the bias filter of the TEKF, the calculated innovation covariance and the estimated innovation covariance are defined by (21b) and (21c).

**Definition 4.5:** Several equations related to the innovation are arranged as follows.

$$\bar{\eta}_k^b = \bar{\eta}_k^x - N_k \bar{b}_{k-1}(-) \quad (21a)$$

$$C_k^b = H_k(\hat{x}_k(-), \bar{b}_k(-)) \bar{P}_k^x(-) H_k^T(\hat{x}_k(-), \bar{b}_k(-)) + R_k + N_k \bar{P}_k^b(-) N_k^T \quad (21b)$$

$$\bar{C}_k^b = \frac{1}{M-1} \sum_{i=k-M+1}^k \bar{\eta}_i^b \bar{\eta}_i^{bT} \quad (21c)$$

The  $\alpha_k^b$  is equal to the forgetting factor  $\lambda_k^b$  where  $\bar{C}_k^b = \alpha_k^b C_k^b$ . By the forgetting factor calculated from (21b) and (21c), the state error covariance equation (7b) is changed into

$$\bar{P}_k^b(-) = \lambda_k^b [A_{k-1} \bar{P}_{k-1}^b(+) A_{k-1}^T + Q_{k-1}^b] \quad (22)$$

The modified bias free filter of the TEKF has  $u_k$  and  $\bar{Q}_k^x$ . For convenience, the equation of (8e) and (8f) are rewritten as (23a) and (23b). In (23a),  $u_k$  is related to the incomplete  $A_k$  and  $Q_k^b$ . In (23b),  $\bar{Q}_k^x$  is also related to the incomplete  $Q_k^b$ .

$$\begin{aligned} u_k &= (\bar{U}_{k+1} - U_{k+1}(\hat{x}_k(+), \bar{b}_k(+))) A_k \bar{b}_k(+)) \\ &= (\bar{U}_{k+1} - \bar{U}_{k+1} [I - Q_k^b [\bar{P}_{k+1}^b(-)]^{-1}]) A_k \bar{b}_k(+)) \quad (23a) \\ &= \bar{U}_{k+1} Q_k^b [\bar{P}_{k+1}^b(-)]^{-1} A_k \bar{b}_k(+)) \end{aligned}$$

$$\bar{Q}_k^x = Q_k^x + U_{k+1} Q_k^b \bar{U}_{k+1} \quad (23b)$$

For a compensation of the effects of incomplete information in the modified bias free filter of the TEKF, the each innovation covariance is defined by (24b) and (24c).

**Definition 4.6:** Several equations related to the innovation are arranged as follows.

$$\bar{\eta}_k^x = z_k - H_k(\hat{x}_k(-), \bar{b}_k(-)) \bar{x}_k(-) - E_k \quad (24a)$$

$$C_k^x = H_k(\hat{x}_k(-), \bar{b}_k(-)) \bar{P}_k^x(-) H_k^T(\hat{x}_k(-), \bar{b}_k(-)) + R_k \quad (24b)$$

$$\bar{C}_k^x = \frac{1}{M-1} \sum_{i=k-M+1}^k \bar{\eta}_i^x \bar{\eta}_i^{xT} \quad (24c)$$

The  $\alpha_k^x$  is equal to the forgetting factor  $\lambda_k^x$  where  $\bar{C}_k^x = \alpha_k^x C_k^x$ . By the forgetting factor calculated from (24b) and (24c), the state error covariance equation (6b) is changed into

$$\bar{P}_k^x(-) = \lambda_k^x \begin{bmatrix} F_{k-1}(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) \bar{P}_{k-1}^x(+) \\ F_{k-1}^T(\hat{x}_{k-1}(+), \bar{b}_{k-1}(+)) + \bar{Q}_{k-1}^x \end{bmatrix} \quad (25)$$

The proposed ATEKF is applied to the INS-GPS loosely coupled system with unknown fault bias in next section.

## 5 Application to the INS-GPS Loosely Coupled System with Fault Bias

### 5.1 INS-GPS Loosely Coupled System

The dynamic model of the INS-GPS loosely coupled system can be formed from the differential equations of the navigation error. The time varying stochastic system model is

$$\dot{x}(t) = F_{INS}(t)x(t) + G_{LC}(t)b(t) + G_{LC}(t)w_{LC}(t) \quad (26)$$

$$x = [\delta L \ \delta l \ \delta h \ \delta V_N \ \delta V_E \ \delta V_D \ \phi_N \ \phi_E \ \phi_D]^T \quad (27)$$

$$w_{LC} = [w_{ax} \ w_{ay} \ w_{az} \ w_{gx} \ w_{gy} \ w_{gz}]^T \quad (28)$$

$$z(t) = H_{LC}x(t) + v_{LC}(t) = \begin{bmatrix} P_{INS} \\ V_{INS} \end{bmatrix} - \begin{bmatrix} P_{GPS} \\ V_{GPS} \end{bmatrix} \quad (29)$$

$$H_{LC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0_{1 \times 3} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0_{1 \times 3} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0_{1 \times 3} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0_{1 \times 3} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0_{1 \times 3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1 \times 3} \end{bmatrix} \quad (30)$$

where the error state variable  $x(t)$  represents the position, velocity and attitude errors of vehicle, the bias term  $b(t)$  represents the inertial sensor bias errors, which consist of the accelerometer bias error and the gyro bias error,  $w_{LC} \sim N(0, Q_{LC})$  is white noise of the inertial sensor and  $v_{LC} \sim N(0, R_{LC})$  is white noise of the GPS. (Hong, 2004; Titterton, 1997)

**Remark 5.1:** The accelerometer error consists of white Gaussian noise and the accelerometer bias error, which is assumed as a random constant. The gyroscope error consists of white Gaussian noise and the gyroscope bias error, which is assumed as a random constant.

To analyze the performance of the INS-GPS loosely coupled system, experiments were carried on the campus of the Seoul National University. The Differential GPS (DGPS) trajectory is used as a reference trajectory. The IMU (LP-81) and the GPS receiver (CMC Allstar) were mounted on the test vehicle. LP-81 provides raw IMU measurements at 100Hz, while CMC Allstar provides GPS data at 1Hz. The specifications for LP-81 and CMC Allstar are shown respectively in Table 5.1 and Table 5.2. The initial alignment time is 310 sec and the total navigation time is 1490 sec.

### 5.2 Fault Bias Estimation using the ATEKF

To verify the performance of the ATEKF for the INS-GPS loosely coupled system with unknown fault bias, we consider an accelerometer fault. We insert a fault bias into data obtained from experiment of section 5.1.

**Table 5.1** The specifications of LP-81

Sensor	Accel. ( $1\sigma$ )	Gyro. ( $1\sigma$ )
Bias repeatability	500 $\mu g$	3 deg/h
Scale Factor	500 ppm	500 ppm
Misalignment	60 arcsec	60 arcsec
Noise	0.02 ft/sec	5 arcsec
Correlated Noise	40 $\mu g$	0.1 deg/h

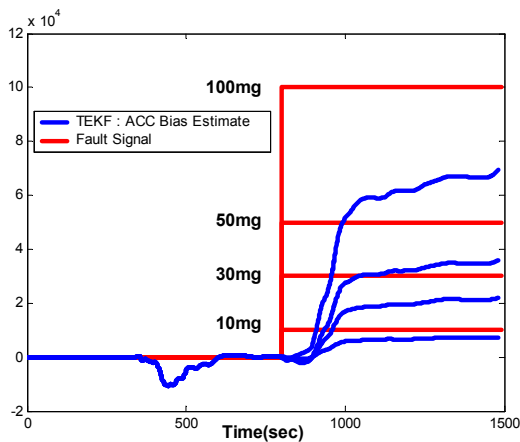
**Table 5.2** The specifications of CMC Allstar

Parameter	Allstar ( $1\sigma$ )
Receive frequency	L1 : 1575.42 MHz
Tracking code	C/A code (SPS)
Position accuracy	16 m CEP
Velocity accuracy	0.2 m/s

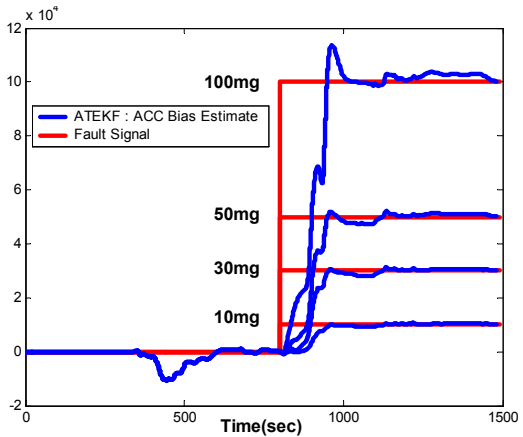
**Table 5.3** The fault of the Accelerometer

Fault position	Magnitude of fault bias	Time
Accel. X-axis	10 mg, 30 mg, 50 mg, 100 mg	800 sec

As shown in Table 5.3, we insert fault biases of 10 mg, 30 mg, 50 mg and 100 mg into X-axis of an accelerometer. The fault occurrence time is 800 sec. When the accelerometer sensor has a fault, the position error of the ATEKF is smaller than that of the TEKF. Fig 5.1 shows the bias estimation results of the TEKF and the ATEKF for several accelerometer fault biases. The tracking performance of the ATEKF is better than that of the TEKF. The ATEKF quickly tracks unknown fault bias. The estimate of fault bias is used for compensation of the sensor fault, so the navigation error in the ATEKF is more decreased than the TEKF. Finally the result of the ATEKF for jumping fault bias is shown in Fig 5.2. Totally, the ATEKF can effectively track unknown fault bias.

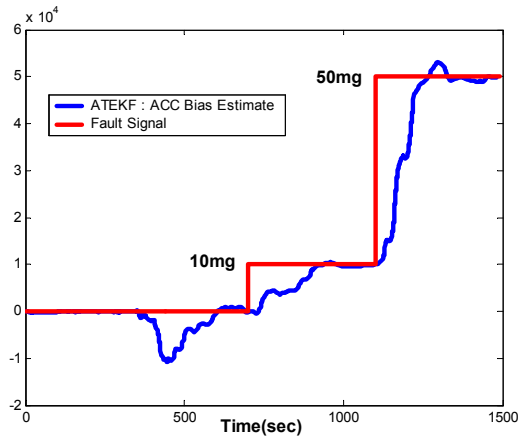


(a) Bias estimation (TEKF)



(b) Bias estimation (ATEKF)

**Fig 5.1** The bias estimation results of the TEKF and the ATEKF for several accelerometer fault biases



**Fig 5.2** The results of bias estimation in the ATEKF for jumping fault bias

## 6 Conclusion

The problem of unknown random bias frequently happens in a number of practical situations. The two-stage Kalman filtering technique considers this problem. The information of unknown random bias is unknown in general. However until now, a designer assumed that this information is known when two-stage extended Kalman filter is designed. So, to solve this problem, this paper proposes the adaptive two-stage extended Kalman filter for nonlinear stochastic system with unknown random bias. To design the adaptive two-stage extended Kalman filter, the adaptive fading extended Kalman filter is firstly designed. This AFEKF can be used for nonlinear stochastic system when the system information is partially known. The AFEKF can be applied to nonlinear stochastic system with unknown random bias. But the AFEKF can only estimate true state, so this cannot eliminate unknown random bias. On the other hand, the

ATEKF can estimate true state and unknown random bias together, so this can eliminate unknown random bias.

The proposed ATEKF can be applied to the problem related a fault compensation. The reliability of vehicle or aircraft depends on integrated navigation system like the INS-GPS loosely coupled system. Thus the fault tolerance is very important in integrated navigation system. The EKF is widely used for integrated navigation system, but the EKF cannot effectively track time varying parameters or unknown parameter. The TEKF has also this problem. This paper applies the ATEKF to the INS-GPS loosely coupled system with unknown fault bias. Unknown fault biases in accelerometer can be effectively eliminated by the ATEKF. As a result, this paper shows that the performance of the ATEKF is better than that of the TEKF in case of unknown fault bias.

## References

- Alouani A.T., Xia P., Rice T.R. and Blair W.D. (1993) *On the Optimality of Two-stage State Estimation In the Presence of Random Bias*. IEEE Transactions Automatic Control, AC-38, pp. 1279–1282.
- Caglayan A.K. and Lancraft R.E. (1983) *A Separated Bias Identification and State Estimation Algorithm for Nonlinear Systems*. Automatica, 19(5), pp. 561–570.
- Friedland B. (1969) *Treatment of bias in recursive filtering*. IEEE Transactions Automatic Control, AC-14, pp. 359–367.
- Hong H.S., Lee J.G. and Park C.G. (2004) *Performance improvement of in-flight alignment for autonomous vehicle under large initial heading error*. IEE Proceedings Radar, Sonar and Navigation, 151(1), pp. 57–62.
- Hsieh C.S. and Chen F.C. (1999) *Optimal Solution of The Two-stage Kalman Estimator*. IEEE Transactions Automatic Control, AC-44, pp. 194–199.
- Ignagni M.N. (1990) *Separate-Bias Kalman Estimator with Bias State Noise*. IEEE Transactions Automatic Control, AC-35, pp. 338–341.
- Ignagni M.N. (2000) *Optimal and Suboptimal Separate-Bias Kalman Estimator for a Stochastic Bias*. IEEE Transactions Automatic Control, AC-45, pp. 547–551.
- Keller J.Y. and Darouach. M. (1997) *Optimal Two-stage Kalman Filter in the Presence of Random Bias*. Automatica, 33(9), pp. 1745–1748.
- Ljung L. (1979) *Asymptotic Behavior of the Extended Kalman Filter as a Parameter Estimator for Linear Systems*. IEEE Transactions Automatic Control, AC-24, pp. 36–50.
- Mendel J.M. (1976) *Extension of Friedland's bias filtering technique to a class of nonlinear systems*. IEEE Transactions Automatic Control, AC-21, pp. 296–298.
- Shreve R.L. and Hedrick W.R. (1974) *Separating bias and state estimates in recursive second-order filter*. IEEE Transactions Automatic Control, AC-19, pp. 585–586.
- Titterton D.H. and Weston J.L. (1997) *Strapdown inertial navigation technology*. Peter Peregrinus, United Kingdom.