Aperture Jitter Effects in Software Radio GNSS Receivers

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Abstract. Increasingly, software radio techniques are being used in the implementation of communications receivers in general, and GNSS receivers in particular. In such a receiver, the received signal is sampled as close to the receive antenna as possible, and all subsequent processing uses digital signal processing (DSP) techniques. The sampling clock will suffer from phase noise instabilities, leading to a phenomenon known as aperture jitter. This paper examines the effects of aperture jitter for a number of "typical" software radio GNSS receivers. A jitter specification is derived which restricts the noisy effects due to jitter to 10dB below thermal noise. It transpires that regardless of the new signals that are selected to accompany it, it is the L1 signal that drives this jitter specification.

Key words: Software radio, GNSS receiver, aperture jitter.

1 Introduction

1.1 Bandpass Sampling

With the increase in the variety of signals required to be processed by radio receivers, versatile receiver designs are gaining in popularity. Software radio (SWR) receivers (which perform the "radio" functions in a processor) and software-defined radio (SDR) receivers (where these procedures are also performed digitally but using hardware controlled and configured in software) are two approaches that deliver this versatility. In both these approaches, the ideal is to convert the received signal to a sampled digital signal as closely as possible to the receive antenna. With the advent of new GPS signals, and a range of new Galileo signals, GNSS is an application where SWR and SDR are likely to have an impact.

Sampling the received signal using low-pass sampling (LPS), the more usual interpretation of Nyquist's sampling theorem, requires a sampling rate of twice the maximum frequency of interest. For GNSS, this is of the order of 3.2Gs/s. Analogue-to-digital converters (ADCs) do exist that will operate at these frequencies (Lee, 2003, Poulton, 2003), but they are expensive and consume a great deal of power. A cheaper, more efficient method of conversion in a SWR receiver is "bandpass sampling" (BPS) (Vaugan, 1991), where, consistent with Nyquist's sampling theorem, it is possible to sample an RF signal at twice the bandwidth of that signal. That signal is the "aliased" into the baseband used for that sampling rate. For bandpass sampling to be successful, out of band signals must be attenuated to reduce aliasing of unwanted signals and noise, requiring a high-Q bandpass filter. This is a more demanding design requirement than the low pass anti-aliasing filter required for LPS at the same sampling frequency. When several bands are required for downconversion, the minimum sampling rate is twice the sum of the bandwidths. However, there are constraints that mean that the sampling rate is usually higher than that minimum.

Software receivers have been designed for the GPS L1 signal (Akos, 1996, Ledvina, 2003, Lin, 2001) and for the L5 signal (Ries, 2002). Bandpass sampling dual band receivers have also been designed, for L1 GPS and Glonass (Akos, 1999), and for GPS L1 and L3 (Thor, 2003).

1.2 Aperture Jitter

Aperture jitter is defined as the sample-to-sample variation in time between the effective points at which the samples are actually taken (Sheingold, 1986). The effects of aperture jitter can be modelled as noise (we will use the terminology of (Sun, 2004). If the signal into the analogue-to-digital converter (ADC) is $y(t)$, then the jitter noise can be modelled as:

$$\varepsilon_\tau(n) = y(t_n + \tau_n) - y(t_n) \quad (1)$$

where t_n is the ideal sampling instant and τ_n is the offset to that instant due to jitter. The power of this jitter noise signal is:

$$N_\tau = E\left[\varepsilon_\tau^2(n)\right] - E\left[\varepsilon_\tau(n)\right]^2 \quad (2)$$

Previous work has identified two formulae for this jitter noise power. Both assume a sinusoidal signal and zero-mean gaussian jitter around fixed "perfect" sampling points. This latter assumption has been found experimentally to be reasonable (Shinagawa, 1990), although the spectrum is not white if a phase-locked loop is used to generate the clock (Da Dalt, 2002), or if the jitter is accumulative(Awad, 1998). The first formula assumes that $2\pi f_c \tau_n << 1$, where f_c is the carrier frequency of the sampled signal, i.e. that the jitter offset is much less than a period of the carrier (Shinagawa, 1990):

$$N_\tau = 2\pi^2 f_c^2 \sigma_\tau^2 A^2 \quad (3)$$

where σ_τ^2 is the variance of τ_n and A is the amplitude of the sinusoid. The second formula makes no assumption about the size of τ_n (Awad, 1998):

$$N_\tau = A^2 \left(1 - e^{-2\pi^2 f_c^2 \sigma_\tau^2}\right) \quad (4)$$

Other simple non-sinusoidal signals such as square and triangular waves have also been examined with regard to their jitter noise power (Kobayashi, 1999). Comparisons have been made between the noise generated by jitter and thermal (Shinagawa, 1990) and quantisation (Kobayashi, 1999) noise.

2 Example GNSS Receivers

2.1 Frequency Bands

Sampling requirements for a number of GNSS example systems are examined, restricting interest to GPS and Galileo signals that are available to commercial users.

Commercial satellite navigation receivers have until recently largely been restricted to using the GPS signal at L1 (carrier 1575.42MHz, chipping rate 1.023Mcps) (ICD, 2003). Soon GPS will provide a similar signal on L2 (carrier 1227.6 MHz) (ICD, 2003, Fontana, 2001) and a new signal on L5 (carrier 1176.45 MHz, chipping rate 10.23Mcps) (ICD, 2002, RTCA, 2000, van Dierendonck 2000).

Galileo will have several signals available to commercial users. Open services (OS) are available on E5 (1164-1215MHz, nominal carrier 1189MHz) and E2-L1-E1, known for convenience as L1 (1559-1592Mz). Commercial services (CS), for which a fee is required but

are not restricted to security services, are available on E6 (1215-1300MHz, carrier 1278.75MHz) and L1. At the time of writing, the above frequency allocations were the latest to have been formally released (Hein, 2002, Hein, 2003), although they will change because of agreements between the Galileo and GPS teams (GPSW, 2004, 2004A). For instance, the OS L1 signal bandwidth has been reduced by a factor of 2 so that it has a 1.013MHz chipping rate, and a binary offset code of 1.013MHz, a so-called BOC(1,1) code.

Combinations of these signals were selected that are likely to be common within a "GNSS" receiver. Example 1 is the most expensive receiver processing GPS L1, L2, L5 and Galileo OS and CS, using L1, E5 and E6. Example 2 uses only the free-to-air GPS L1 and L2 and Galileo OS, on L1 and E5. Example 3 uses L1 and E5 (ignoring L2) and example 4 uses L1 and L2 (i.e. the GPS bands that are "currently" available, and incorporating the Galileo L1). Example 5 is cheapest arrangement, simply using "L1 only" and incorporating the new Galileo signal.

Table 1. Frequency bands for example receiver designs

Band	E5 (L5)	L2	E6	L1
Fmin	1164	1217	1260	1573
fmax	1214	1238	1300	1577
Ex. 1	X	X	X	X
Ex. 2	X	X		X
Ex. 3	X			X
Ex. 4		X		X
Ex. 5				X

The frequencies required for the four examples are shown in Table 1. It can be seen that because the frequency bands are contiguous, in all cases, only two bands are required. Example 1, for instance, must receive in the range 1164-1300MHz and 1573-1577MHz.

2.2 Sampling Rates

For simplicity, as the type of results sought need only to be indicative, bandpass sampling is assumed, but only the minimum sampling rate required by Nyquist's theorem is considered (i.e. the full set of constraints identified in (Vaugan, 1991) are not all accounted for). These minimum sampling rates are shown in Table 2.

2.3 Data Bandwidths

As we have seen, aperture jitter tends to be modelled as noise. This noise can be considered to be additive with the thermal noise present in the receiver. For the new signals, which have data in one channel and a dataless

pilot signal in quadrature, the effects of noise are greater in the data channel, so we will consider the bandwidth of the data channel as being the relevant bandwidth in which thermal noise appears. The data bandwidths of the GNSS signals are shown in Table 3.

Table 2. For the 5 examples, the lower bandwidth, the upper bandwidth, the total bandwidth, and the minimum sampling rate, considering only Nyquist's theorem. All figures in MHz

	BW 1	BW 2	Total BW	f_{smin}
Ex. 1	136	4	140	280
Ex. 2	74	4	78	156
Ex. 3	50	4	54	108
Ex. 4	21	4	25	50
Ex. 5	-	4	4	8

Table 3 GNSS data bandwidths

GPS band [ICD (2003), van Dierendonck (2000)]	Data bandwidth (Hz)
L1	100
L2	100
L5	2000
Galileo band [Hein (2002), Hein (2003)]	
L1	500
E5	500
E6	2000

2.4 Signal Strengths

We will be comparing SNR due to noise to SNR due to jitter. In order to do this, we will need to know the signal levels for each of the signals we are examining. These are shown in Table 4.

Table 4 GNSS signal strengths

GPS signal [ICD (2003), van Dierendonck (2000)]	Signal strength (dBW)
L1	-160
L2	-160
L5	-154
Galileo signal [Hein (2002), Hein (2003)]	
L1	-155
E5	-152
E6	-155

3 Derivation of Acceptable Aperture Jitter

It is assumed that the jitter is relatively small with respect to the carrier frequency, and hence equation (3) rather than (4) is used to evaluate the contribution to the noise power made by aperture jitter. The SNR due to jitter arising from (3) is:

$$SNR_j = \frac{S}{N_\tau} = \frac{\frac{A^2}{2}}{2\pi^2 f_c^2 \sigma_\tau^2 A^2} = \frac{1}{\left(2\pi f_c \, \sigma_\tau\right)^2} \quad (5)$$

In order to keep this contribution to an insignificant size, the jitter noise power is restricted to be 10dB down from the thermal noise power, as in equation (6).

$$SNR_j \geq 10 \frac{S}{N_{th}} \quad (6)$$

where S is the signal power from Table 4, and $N_{th} = kTB$ is the thermal noise power, with the bandwidths of interest B being taken from Table 3. Thus, using equation (6), it is possible to evaluate the required jitter standard deviation allowable for each GNSS band of interest, by using

$$\sigma_\tau \leq \frac{1}{2\pi f_c} \sqrt{\frac{N_{th}}{10S}} \quad (7)$$

Evaluating this expression for the values selected in the previous sections gives the jitter requirements in Table 5.

Table 5 Jitter requirements (standard deviation) for each of the signal bands

GPS signal	Jitter requirement (ps)
L1	2.05
L2	2.63
L5	6.15
Galileo signal	
L1	2.58
E5	2.42
E6	6.35

These jitter requirements do not vary very much with the loosest (6.35ps) being only 3.1 times the tightest (2.05ps). The tightest of these jitter requirements is for the GPS L1 band, which is probably not surprising as it is the oldest of the signals. However, it has been included in all of our examples, so the 2ps jitter requirement is the one that drives the specification for all of the examples. This requirement can be turned into a phase noise standard deviation requirement of the sampling clock:

$$\sigma_{\theta\tau} = 2\pi f_s \sigma_\tau \qquad\qquad (8)$$

where f_s is the bandpass sampling frequency. Interestingly, as the sampling frequency increases, the phase noise requirement gets looser. Therefore, in phase noise terms, the most difficult receiver to design is what in other ways is the easiest receiver to design, the "L1-only" example, giving a phase noise requirement of 50μrads.

4 Conclusion

The analysis evaluated the jitter requirement such that noise due to sampling jitter at the carrier frequencies of GNSS signals was 10dB less than the thermal noise. For all GNSS bands, this requirement was of the order of picoseconds. Several combinations of bands were examined but they all used the L1 band, which has the tightest requirement for jitter, a standard deviation less than 2ps. However, that requirement is not much less than for L2 and E5. The jitter analysis assumed that $2\pi f_c \tau_n \ll 1$, and for the tightest specification $2\pi f_c \sigma_\tau = 0.02$, which satisfies that assumption.

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